MATH 464
EXAMPLES OF COMMON DISCRETE RANDOM VARIABLES

SPRING 2016

The following is a list of common discrete random variables.

(1) **The Bernoulli Random Variable:** Let $0 \le p \le 1$. We say that X is a Bernoulli random variable if the range of X is $\{0, 1\}$ and

$$P(X = 1) = p \quad \text{and} \quad P(X = 0) = 1 - p\,.$$

Sometimes we say thas this is a one parameter Bernoulli random variable with parameter p. It is common to denote by $q = 1 - p$. In this case, the associated probability mass function is simple:

$$f_X(x) = \begin{cases} p & \text{if } x = 1, \\ q = 1 - p & \text{if } x = 0, \\ 0 & \text{otherwise.} \end{cases}$$

One often thinks of Bernoulli random variables as corresponding to the experiment of flipping a coin. Typically, one takes X to be the function which assigns the value 1 to heads and 0 to tails.

In this case, the mean and variance of X are given by

$$\mu = E(X) = p \quad \text{while} \quad \sigma^2 = \text{var}(X) = p(1 - p)$$

Moreover, the moment generating function associated to X is given by

$$M_X(t) = pe^t + (1 - p)$$

and it is defined for all $t \in \mathbb{R}$.

(2) **The Binomial Random Variable:** Let $0 \le p \le 1$ and n be a positive integer. We say that X is a Binomial random variable if the range of X is $\{0, 1, 2, 3, \cdots, n\}$ and

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad \text{for any } 0 \le k \le n\,.$$

One often thinks of Binomial random variables as corresponding to the experiment of flipping an unfair coin n times. In this case, p is the probability of heads on a single flip, and X is the number of heads we get in n flips. The parameter n is often called the number of trials.

In this case, the mean and variance of X are given by

$$\mu = E(X) = np \quad \text{while} \quad \sigma^2 = \text{var}(X) = np(1-p)$$

Moreover, the moment generating function associated to X is given by

$$M_X(t) = [pe^t + (1-p)]^n$$

and it is defined for all $t \in \mathbb{R}$.

Note that the Bernoulli random variable with parameter p is the simplest case of the binomial random variable with $n = 1$ and $p = p$, see above. In fact, any binomial random variable can be thought of as a sum of n independent Bernoulli trials.

(3) **The Geometric Random Variable:** Let $0 < p \leq 1$ be given. We say that X is a Geometric random variable if the range of X is $\{1, 2, 3, \cdots\}$ and

$$P(X = k) = p(1-p)^{k-1} \quad \text{for any integer } k \geq 1\,.$$

One often thinks of the Geometric random variable as corresponding to the experiment where you flip an unfair coin until you get heads. Here p is the probability of getting heads and $X = k$ corresponds to the event that you got heads in k flips; the flip of heads being counted. *Caution:* Some books use a different convention and take X to be the number of tails before we get the first heads . . .

In this case, the mean and variance of X are given by

$$\mu = E(X) = \frac{1}{p} \quad \text{while} \quad \sigma^2 = \text{var}(X) = \frac{1-p}{p^2}$$

Moreover, the moment generating function associated to X is given by

$$M_X(t) = \frac{pe^t}{1-(1-p)e^t}$$

and it is defined for all $t \in \mathbb{R}$ for which $(1-p)e^t < 1$.

(4) **The Poisson Random Variable:** Let $\lambda > 0$ be given. We say that X is a Poisson random variable if the range of X is $\{0, 1, 2, 3, \cdots\}$ and

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad \text{for any integer } k \geq 0\,.$$

There is no simple experiment that produces a Poisson random variable, but it comes up often in applications. It is, for example, a limiting case of the Binomial random variable.

In this case, the mean and variance of X are given by

$$\mu = E(X) = \lambda \quad \text{while} \quad \sigma^2 = \text{var}(X) = \lambda$$

Moreover, the moment generating function associated to X is given by

$$M_X(t) = \exp\left[\lambda\left(\exp[t] - 1\right)\right]$$

and it is defined for all $t \in \mathbb{R}$.

(5) **The Negative Binomial Random Variable:** Let $0 < p \leq 1$ and n be a positive integer. We say that X is a Negative Binomial random variable if the range of X is $\{n, n+1, n+2, \cdots\}$ and

$$P(X = k) = \binom{k-1}{n-1} p^n (1-p)^{k-n} \quad \text{for any } k \geq n\,.$$

One often thinks of the Negative Binomial random variable as corresponding to the experiment where you flip an unfair coin until you get heads a total of n times. Here p is the probability of heads, and X is the total number of flips it takes (including the n heads).

In this case, the mean and variance of X are given by

$$\mu = E(X) = \frac{n}{p} \quad \text{while} \quad \sigma^2 = \text{var}(X) = \frac{n(1-p)}{p^2}$$

Moreover, the moment generating function associated to X is given by

$$M_X(t) = \left[\frac{pe^t}{1-(1-p)e^t}\right]^n$$

and it is defined for all $t \in \mathbb{R}$ for which $(1-p)e^t < 1$.