MATH 355-002:
SIMS
TEST 2 MAKE-UP

SPRING 2019

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Directions: This work is an optional assignment for those who took the second test on Thursday, April 11th. It is due on Tuesday, April 23rd at the beginning of class. No late work will be accepted. If you turn this in, I will grade it and your new grade on test 2 will be the average (out of 100%) of the two scores you have received. If you do not turn this in, your grade on test 2 will stay the same.

Where indicated, show work to illustrate you are using the method requested in the particular question.

(1) Find the first order perturbation approximation $p_1(t) = k_0(t)+k_1(t)\epsilon$ for the solution of the following initial value problem:

$$x' = 1 - xe^{\epsilon x} \quad \text{with} \quad x(0) = 1\,.$$

(2) Consider the following linear, homogeneous system:

$$\tilde{x}' = A\tilde{x} \quad \text{with} \quad \tilde{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} -4 & 6 \\ -3 & 2 \end{pmatrix}.$$

a) Use your favorite method to find a fundamental solution matrix for the above system. In addition to stating your answer, explain also which method you used.

b) Use your result above to solve the initial value problem with $\tilde{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$. For full credit, do all the corresponding matrix arithmetic to find formulas for both components of this solution.

(3) Consider the following 2^{nd}-order equation:

$$x'' + 4x = \sin(3t)\,.$$

a) Convert this 2^{nd}-order equation to a 1^{st}-order system.

b) Use the Variation of Constants formula to find the general solution of the resulting system (show your work). For full credit, also give an explicit formula for the general solution of the original 2^{nd}-order equation.

(4) Consider the following linear, homogeneous system:

$$\begin{aligned} x' &= 3x \\ y' &= -x + 3y \end{aligned}$$

a) Use the Putzer algorithm (show your work) to find a normalized solution matrix.

b) Use your result above to find the solution of the initial value problem with $\tilde{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(5) For each of the matrices below, do the following:

a) Sketch the phase plane portrait of the associated, homogeneous linear systems: $\tilde{x}' = A\tilde{x}$. Include a hand-drawn sketch and a computer printout of a more accurate picture. (For the computer assisted sketch, draw all eigenorbits (if relevant) and at least 2 other orbits.)

b) State whether or not the origin is stable.

Do a) and b) above for each of the following matrices:

$$A_1 = \begin{pmatrix} 0 & 1 \\ -4 & 5 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 3 & 2 \\ -4 & -1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 9 & -17 \\ 5 & -9 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$$