MATH 355-002:
SIMS
TEST 2

SPRING 2019

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Directions: Solve all the problems below. Where indicated, show work to illustrate you are using the method requested in the particular question.

(1) Consider the following system:

$$\begin{aligned} x' &= \ln(x+y+1)+4 \\ y' &= \sin(x+y-1)+2 \end{aligned}$$

a) Determine whether our fundamental existence and uniqueness results apply to the initial condition: Why or Why not?

$$x(0) = 1 \quad \text{and} \quad y(0) = -1\,.$$

b) Determine whether our fundamental existence and uniqueness results apply to the initial condition: Why or Why not?

$$x(0) = 0 \quad \text{and} \quad y(0) = -1\,.$$

Let a be a real parameter and consider the following system:

$$\begin{aligned} x' &= 3x+5y+ax^2 \\ y' &= 2x-y+e^t \end{aligned}$$

c) For what values of a do our fundamental existence and uniqueness results apply for all initial conditions? Explain.

d) For what values of a are we guaranteed existence and uniqueness of solutions (to an initial value problem) that are defined for all t? Explain.

(2) Consider the following linear, homogeneous system:

$$\tilde{x}' = A\tilde{x} \quad \text{with} \quad \tilde{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} -2 & 3 \\ 2 & 3 \end{pmatrix}.$$

a) Show that

$$\tilde{x}_1(t) = \begin{pmatrix} e^{4t} \\ 2e^{4t} \end{pmatrix} \quad \text{and} \quad \tilde{x}_2(t) = \begin{pmatrix} -3e^{-3t} \\ e^{-3t} \end{pmatrix}$$

are two linearly independent solutions of this linear, homogeneous system on the interval $(-\infty, \infty)$.

b) Use your result above to solve the initial value problem with $\tilde{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. For full credit, do all the corresponding matrix arithmetic to find formulas for both components of this solution.

(3) Consider the following linear, non-homogeneous system:

$$\begin{aligned} x' &= -3x + 5y - 1 \\ y' &= -2x + 3y + 1 \end{aligned}$$

a) Verify that

$$\Phi(t) = \begin{pmatrix} 5\cos(t) & 5\sin(t) \\ 3\cos(t) - \sin(t) & \cos(t) + 3\sin(t) \end{pmatrix}$$

is a fundamental solution matrix for the corresponding homogeneous system.

b) Use your result above and the Variation of Constants formula to find the general solution of this non-homogeneous system. For full credit, do all the corresponding matrix arithmetic to find formulas for both components of this general solution.

(4) Consider the following linear, homogeneous system:

$$\begin{aligned} x' &= 2x + y \\ y' &= -2x \end{aligned}$$

a) Use the Putzer algorithm to find a normalized solution matrix.

b) Use your result above to find the solution of the initial value problem with $\tilde{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(5) Consider the following 2^{nd}-order equation:

$$x'' - x' - 6x = 0\,.$$

a) Convert this 2^{nd}-order equation to a 1^{st}-order system.

b) Find the eigenvalues and corresponding eigenvectors for the resulting coefficient matrix.

c) Sketch the corresponding phase plane portrait.

d) Based on your sketch in part c), determine whether the origin is stable or unstable.