MATH 355-002:
TEST 1
MAKE-UP

SPRING 2019

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Directions: This work is an optional assignment for those who took the first test on Thursday, February 21st. It is due on Tuesday, March 12th at the beginning of class. No late work will be accepted. If you turn this in, I will grade it and your new grade on test 1 will be the average (out of 100%) of the two scores you have received. If you do not turn this in, your grade on test 1 will stay the same.

Where indicated, show work to illustrate you are using the method requested in the particular question.

(1) In the following initial value problems, the number a is a real parameter. Determine the values of a for which our fundamental theorem on existence and uniqueness of solutions applies. Explain your answer.

(a)
$$x' = \ln(a - x) \quad \text{with} \quad x(0) = 0\,.$$

(b)
$$x' = \sqrt{a^2 - x^2} \quad \text{with} \quad x(1) = 2\,.$$

(c)
$$x' = \tan(ax) \quad \text{with} \quad x(0) = \frac{\pi}{2}\,.$$

(2) Using the variation of constants formula, solve the following initial value problem:

$$x' = \frac{2}{t}x + t^2(1 + e^{-at}) \quad \text{with} \quad x(1) = 1\,.$$

Here $a \neq 0$ is a real parameter.

(3) Using the super position principle and the method of undetermined coefficients, find the general solution of the following differential equation:

$$x' = 2x - 3te^{2t} + 5t^3 \,.$$

(4) Let $0 < a < b$ be real parameters. Consider the following differential equation:

$$x' = (x+a)(x-a)^3(x-b)^4(x+b)^3$$

Sketch the phase line portrait.
Find and classify all equilibria.
Find the linearization at any hyperbolic equilibrium.

(5) Consider the following differential equation:

$$x' = (x^2 - 4)(p - x^2)$$

Here p is a parameter.

Sketch a bifurcation diagram for this differential equation which indicates the type of equilibria in each branch of the diagram. Also indicate all bifurcation values and classify the type of each bifurcation.