MATH 355-002:
SIMS
TEST 1

SPRING 2019

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Directions: Solve all the problems below. Where indicated, show work to illustrate you are using the method requested in the particular question.

(1) Consider the following initial value problems. In each case, determine whether or not our fundamental theorem on existence and uniqueness of solutions applies. Each part below is worth 2 points. 1 point for answering yes the theorem applies or no it does not; and 1 point for a sentence explaining why.

(a)
$$x' = \ln(x^2 - t^2 + 1) \quad \text{with} \quad x(1) = 2\,.$$

(b)
$$x' = \ln(x^2 - t^2 + 1) \quad \text{with} \quad x(2) = 1\,.$$

(c)
$$x' = \sqrt{x} - 4t \quad \text{with} \quad x(1) = 0\,.$$

(d)
$$x' = \sqrt{x} - 4t \quad \text{with} \quad x(0) = 1\,.$$

(e)
$$x' = \frac{\sin(x)}{\cos(t)} \quad \text{with} \quad x(0) = \frac{\pi}{2}\,.$$

(2) Using the variation of constants formula, solve the following initial value problem:

$$x' = \sin(at)x + \sin(at) \quad \text{with} \quad x(0) = 1\,.$$

Here $a \neq 0$ is a real parameter.

(3) Using the super position principle and the method of undetermined coefficients, find the general solution of the following differential equation:

$$x' = -2x + 3ae^{-2t} - 4at^2$$

Here a is a real parameter.

(4) Consider the following differential equation:

$$x' = x(x+1)^2(x-2)^3(x+3)^4$$

Sketch the phase line portrait.
Find and classify all equilibria.
Find the linearization at any hyperbolic equilibrium.

(5) Consider the following differential equation:

$$x' = x^3(x^2 - p)$$

Here p is a parameter.

Sketch a bifurcation diagram for this differential equation which indicates the type of equilibria in each branch of the diagram. Also indicate all bifurcation values and classify the type of each bifurcation.