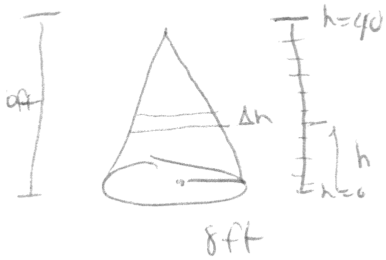
MATH 129-011:
TEST 3- MAKE UP

SPRING 2018

Name	Key
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total	40	

Directions: This work is an optional assignment for those who took the third test on Monday, April 2, 2018. It is due on Friday, April 13, 2018 before the beginning of class. **No late work will be accepted; one minute late to class is late.** If you turn this in, I will grade it (with a score out of 100) and your new grade on test 3 will be the average of the two scores you have received. If you do not turn this in, your grade on test 3 will stay the same.



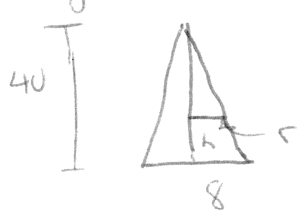
(1) A water tank is in the shape of a cone with height 40 ft and base radius 8 ft. The cone rests on its base. Recall that 1 cubic foot of water weighs 62.4 pounds. For the problems below, you do not need to calculate the integrals. To receive any partial credit, you must show the work on the Riemann sum you use to determine the integral.

a) If the tank is full, find an integral which calculates the work required to pump the water to the top (the height of the tip) of the tank.

Total work done $\simeq \sum$ work done on slices $= \sum$ (Force)(distance) on each slice $= \sum$ [(density)(volume)](distance) on each slice

$= \sum \left[(62.4 \text{ lbs}/\text{ft}^3)(\pi r^2 \Delta h) \right] (40-h)$

By similar triangles



$\Rightarrow \frac{\text{back}}{\text{base}} = \frac{40}{8} = \frac{40-h}{r}$

$r = \frac{1}{5}(40-h)$

$\rightarrow \int_0^{40} \frac{62.4 \cdot \pi}{5^2} (40-h)^3 \, dh$

7.8414

b) If the tank is filled to half its height, find an integral which calculates the work required to pump the water 5 ft above the top of the tank.

Total work done $\simeq \sum$ work done on slices $= \sum$ [(density)(volume)](distance)

$= \sum \left[(62.4 \text{ lbs}/\text{ft}^3)\left(\pi \left(\tfrac{1}{5}(40-h)\right)^2 \Delta h\right) \right] (45-h)$

$\rightarrow \int_0^{20} \frac{62.4 \, \pi}{5^2} (40-h)^2 (45-h) \, dh$

(2) a) Write the following as a finite geometric sum and find its value.

$$\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+\frac{3}{32}+\cdots+\frac{3}{2^{14}}$$

$$\sum_{m=2}^{14} 3\left(\tfrac{1}{2}\right)^m = 3\left(\tfrac{1}{2}\right)^2 \cdot \sum_{m=0}^{12}\left(\tfrac{1}{2}\right)^m$$

$$= \frac{3}{4}\cdot\left(\frac{1-\left(\frac{1}{2}\right)^{13}}{1-\frac{1}{2}}\right)$$

$$= \frac{3}{2}\left(1-\frac{1}{2^{13}}\right)$$

Also

$$\boxed{\sum_{m=2}^{14} 3\left(\tfrac{1}{2}\right)^m = \frac{3\left(1-\left(\frac{1}{2}\right)^{15}\right)}{1-\frac{1}{2}} - 3 - \frac{3}{2}}$$

b) Calculate

$$\sum_{n=5}^{\infty}\frac{(-2)^n+(2e)^n}{\pi^{2n}}$$

$$\sum_{n=5}^{\infty}\frac{(-2)^n+(2e)^n}{\pi^{2n}} = \sum_{n=5}^{\infty}\left(\frac{-2}{\pi^2}\right)^n + \sum_{n=5}^{\infty}\left(\frac{2e}{\pi^2}\right)^n$$

$$= \left(\frac{-2}{\pi^2}\right)^5\cdot\frac{1}{1+\frac{2}{\pi^2}} + \left(\frac{2e}{\pi^2}\right)^5\cdot\frac{1}{1-\frac{2e}{\pi^2}}$$

(3) Determine whether the following series converge or diverge. Write a sentence describing the convergence test you used and state your conclusion. For full/partial credit, show all work necessary to reach your conclusions.

a)

$$\sum_{n=3}^{\infty} \frac{5n^2+4}{7n^5+2n^3}$$

Let $a_n = \dfrac{5n^2+4}{7n^5+2n^3} = \dfrac{n^2\left(5+\frac{4}{n^2}\right)}{n^5\left(7+\frac{2}{n^2}\right)} \approx \dfrac{5}{7n^3}$

Apply limit comparison

Let $b_n = \dfrac{5}{7} \cdot \dfrac{1}{n^3}$

By p-test, $\sum_{n=3}^{\infty} b_n$ converges since $p=3>1$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{5n^2+4}{7n^5+2n^3} \cdot \frac{7n^3}{5}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}\left(5+\frac{4}{n^2}\right)\cdot 7\cancel{n^3}}{\cancel{n^5}\left(7+\frac{2}{n^2}\right)\cdot 5}$$

$$= \frac{(5+0)\cdot 7}{(7+0)\cdot 5} = 1 > 0$$

Thus $\sum_{n=3}^{\infty} a_n$ converges by limit comparison.

b)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}}$$

Use Alternating Series test.

i) $a_n = \dfrac{1}{\sqrt{n^2+3}} > 0$.

ii) $n < n+1 \Rightarrow n^2 < (n+1)^2$

$\Rightarrow n^2+3 < (n+1)^2+3$

$\Rightarrow \dfrac{1}{(n+1)^2+3} < \dfrac{1}{n^2+3}$

$\Rightarrow a_{n+1} = \sqrt{\dfrac{1}{(n+1)^2+3}} < \sqrt{\dfrac{1}{n^2+3}} = a_n$

iii) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{1}{\sqrt{n^2+3}} = 0$

Since we checked all the conditions, this series converges by the alternating series test.

(4) Consider the following power series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{3^{2n}}(x+2)^n$$

a) Find the radius of convergence.

let $a_n = \dfrac{(-1)^n n^2 (x+2)^n}{3^{2n}}$

Then $\displaystyle\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)^2 |x+2|^{n+1}}{3^{2(n+1)}} \cdot \frac{3^{2n}}{n^2 |x+2|^n} = \frac{|x+2|}{3^2}$

$\Rightarrow R = 3^2 = 9.$

b) Find the interval of convergence. You do not need to consider the endpoints.

The center is $a = -2$.

The interval of convergence is then:

$(a-R, a+R) = (-2-9, -2+9)$

$= (-11, 7)$