MATH 129-006:
TEST 2

FALL 2015

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

(1) Determine if the following integral converges or diverges. If it converges, find its value.

$$\int_2^{10} \frac{1}{\sqrt[3]{x^2-4x+4}}\,dx$$

$$\int_2^{10} \frac{1}{\sqrt[3]{x^2-4x+4}}\,dx = \lim_{a\to 2^+} \int_a^{10} \frac{1}{\sqrt[3]{(x-2)^2}}\,dx$$

Let
$u = x-2$
$du = dx$

$$= \lim_{a\to 2^+} \int_{a-2}^{8} \frac{1}{\sqrt[3]{u^2}}\,du$$

$$= \lim_{a\to 2^+} \left.\frac{u^{1/3}}{\frac{1}{3}}\right|_{a-2}^{8}$$

$$= 3\cdot\sqrt[3]{8} = 6$$

This integral converges.

Its value is 6.

(2) Consider the following integral

$$\int_4^\infty \frac{t^2+3t+1}{\sqrt{2t^7+5}}\,dt$$

Does this integral converge or diverge? For full credit, you must justify your claim.

This is an improper integral of type I.

Let $f(t) = \dfrac{t^2+3t+1}{\sqrt{2t^7+5}}$

Behavior of f at $+\infty$:

$$f(t) = \frac{t^2\left(1+\frac{3}{t}+\frac{1}{t^2}\right)}{\sqrt{2t^7}\cdot\sqrt{1+\frac{5}{2t^7}}} \cong \frac{1}{\sqrt{2}}\,\frac{t^{4/2}}{t^{7/2}} = \frac{1}{\sqrt{2}}\cdot\frac{1}{t^{3/2}}$$

Since $p = 3/2 > 1$, we think convergence.

<u>Note:</u>

$$2t^7 \le 2t^7+5$$

$$\Rightarrow \sqrt{2t^7} \le \sqrt{2t^7+5}$$

$$\Rightarrow \frac{1}{\sqrt{2t^7+5}} \le \frac{1}{\sqrt{2t^7}}$$

Since $\int_4^\infty g(t)\,dt$ converges $\left(\begin{matrix}\text{all have}\\ p>1\end{matrix}\right)$

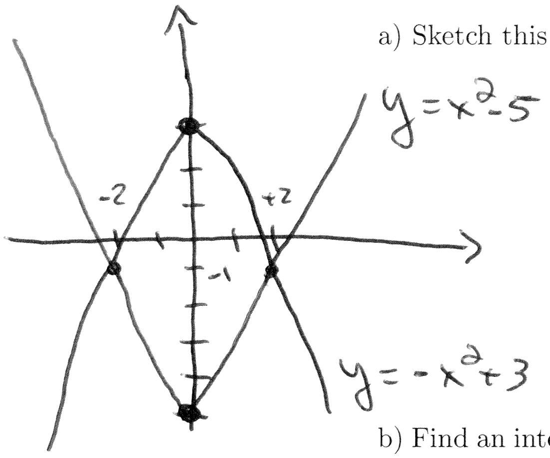
then $\int_4^\infty f(t)\,dt$ converges by comparison.

$$\Rightarrow f(t) = \frac{t^2+3t+1}{\sqrt{2t^7+5}} \le \frac{t^2+3t+1}{\sqrt{2t^7}} = \frac{1}{\sqrt{2}}\cdot\frac{1}{t^{3/2}} + \frac{3}{\sqrt{2}}\,\frac{1}{t^{5/2}} + \frac{1}{\sqrt{2}}\,\frac{1}{t^{7/2}}$$

$= g(t)$

(3) Consider the region bounded by the curves $y = -x^2 + 3$ and $y = x^2 - 5$.

a) Sketch this region.



$-x^2 + 3 = x^2 - 5$

$0 = 2x^2 - 8$

$= 2(x^2 - 4)$

$x = \pm 2$

b) Find an integral that represent the area of this region using x-axis integration. **You do not have to calculate the integral.**

$$\text{Area} = \int_{-2}^{2} \left[-x^2 + 3 - (x^2 - 5) \right] dx$$

c) Find an integral (or integrals) that represent the area of this region using y-axis integration. **You do not have to evaluate the integral(s).**

Need two integrals here:

$$\text{Area} = \int_{-5}^{-1} \left[\sqrt{y+5} - (-\sqrt{y+5}) \right] dy$$

$$+ \int_{-1}^{3} \left[\sqrt{3-y} - (-\sqrt{3-y}) \right] dy$$

$y = x^2 - 5$

$x^2 = y + 5$

$x = \pm\sqrt{y+5}$

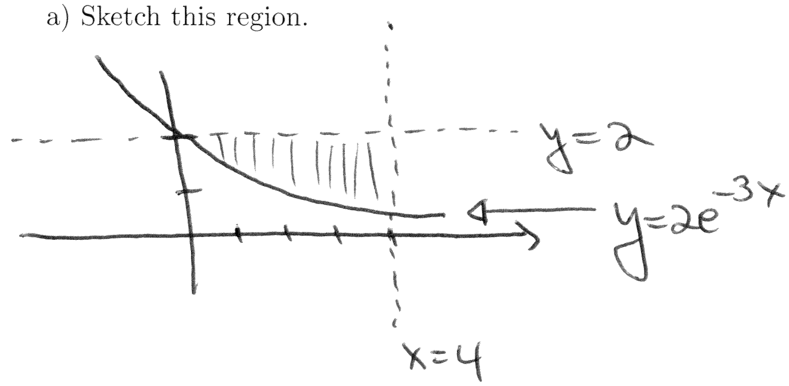
$y = -x^2 + 3$

$x^2 = 3 - y$

$x = \pm\sqrt{3-y}$

(4) Consider the region bounded by $y = 2e^{-3x}$, $y = 2$, and $x = 4$.

a) Sketch this region.



Write an integral (or integrals) which determines the volume of the solid obtained by revolving this region about the:

b) x-axis

$$\text{Volume} = \pi \int_0^4 (2)^2\,dx - \pi \int_0^4 (2e^{-3x})^2\,dx$$

c) y-axis

$$\text{Volume} = \pi \int_{2e^{-12}}^{2} (4)^2\,dy - \pi \int_{2e^{-12}}^{2} \left(-\tfrac{1}{3}\ln(y/2)\right)^2 dy$$

d) line $y = -2$

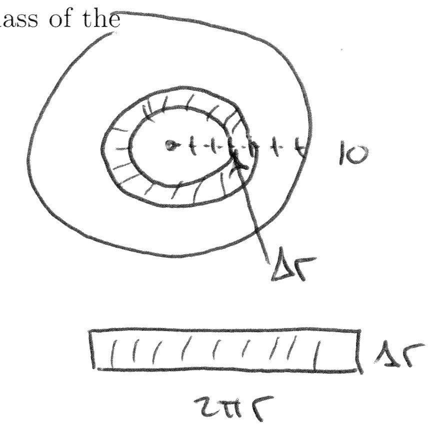
$$\text{Volume} = \pi \int_0^4 (2-(-2))^2\,dx - \pi \int_0^4 (2e^{-3x}-(-2))^2\,dx$$

e) line $x = 7$

$$\text{Volume} = \pi \int_{2e^{-12}}^{2} \left[7-\left(-\tfrac{1}{3}\ln(y/2)\right)\right]^2 dy - \pi \int_{2e^{-12}}^{2} [7-4]^2\,dy$$

(5) At ground level, pollution emanates circularly from a factory with a density $f(r) = 0.237e^{-.32r^2}\ \frac{\text{kg}}{\text{mi}^2}$ with r the distance from the factory in miles.

a) Write a Riemann sum which approximates the total mass of the pollution within a 10 mile radius of the factory.



Riemann sum for total mass $= \sum$ sum of mass on each ring

$= \sum$ (density)(area)

$$= \sum \underset{\frac{kg}{mi^2}}{f(r)} \cdot \underset{mi}{2\pi r} \cdot \underset{mi}{\Delta r}$$

b) Write a definite integral which corresponds to the Riemann sum found in part a) above. Find the exact value of this integral and then approximate it's value with 2 decimal-place accuracy.

$$2\pi \int_0^{10} r f(r)\,dr = 2\pi(0.237)\int_0^{10} e^{-.32r^2} r\,dr$$

$$= \frac{2\pi(0.237)}{-(0.64)} \int_0^{-32} e^u\,du \qquad u = -.32r^2, \quad du = -.64r\,dr$$

$$= \frac{2\pi(0.237)}{0.64}\left(1 - e^{-32}\right)$$

$$\simeq 2.33$$