MATH 129:
TEST 2 MAKE-UP

SPRING 2018

Name	Key
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Directions: This work is an optional assignment for those who took the second test on Wednesday, February 28, 2018. It is due on Friday, March 16, 2018 before the beginning of class. **No late work will be accepted; one minute late to class is late.** If you turn this in, I will grade it (with a score out of 100) and your new grade on test 2 will be the average of the two scores you have received. If you do not turn this in, your grade on test 2 will stay the same.

Show all work on calculating the integrals below, unless you are told you can use the integration table. When you use the integration table, indicate which number you are using.

(1) Determine if the following integral converges or diverges. If it converges, find its value.

$$\int_{-\infty}^{-4} e^{2x+3} dx$$

This is a type I improper integral:

Let $u = 2x+3$
$du = 2dx$

$$\int_{-\infty}^{-4} e^{2x+3} dx = \lim_{a \to -\infty} \int_{a}^{-4} e^{2x+3} dx$$

$$= \lim_{a \to -\infty} \int_{2a+3}^{-5} e^{u} \cdot \tfrac{1}{2} du$$

$$= \lim_{a \to -\infty} \tfrac{1}{2}\left(e^{-5} - e^{2a+3}\right)$$

$$= \frac{e^{-5}}{2}$$

This integral converges and this is its value.

(2) Consider the following integral.

$$\int_0^1 \frac{3x+5}{\sqrt{x}+x^3}\,dx\,.$$

This is a type II improper integral. There is a vertical asymptote at $x=0$.

a) Predict whether or not this integral converges or diverges.

Near $x=0$

$$\frac{3x+5}{\sqrt{x}+x^3} = \frac{3x+5}{\sqrt{x}(1+x^{5/2})} \simeq \frac{5}{\sqrt{x}}$$

← since this is integrable at $x=0$ we expect convergence.

b) Use comparison to prove that your prediction is correct. State the function you use in comparison, state whether it is larger or smaller than the integrand above, and give a reason why it converges or diverges.

For all $0 \le x \le 1$

$$\sqrt{x} \le \sqrt{x}+x^3 \Rightarrow \frac{1}{\sqrt{x}+x^3} \le \frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{3x+5}{\sqrt{x}+x^3} \le \frac{3x+5}{\sqrt{x}} = 3\sqrt{x} + \frac{5}{\sqrt{x}} = g(x)$$

It is clear that $\int_0^1 g(x)\,dx$ converges by p-test, and so

$\int_0^1 \frac{3x+5}{\sqrt{x}+x^3}\,dx$ converges by comparison.

(3) Consider a solid whose base is the region bounded by the curves $y = -x^2 + 3$ and $y = 2x - 5$, with cross-sections perpendicular to the y-axis that are squares.

a) Sketch the base of this solid.

Graphs meet when:

$-x^2 + 3 = 2x - 5$

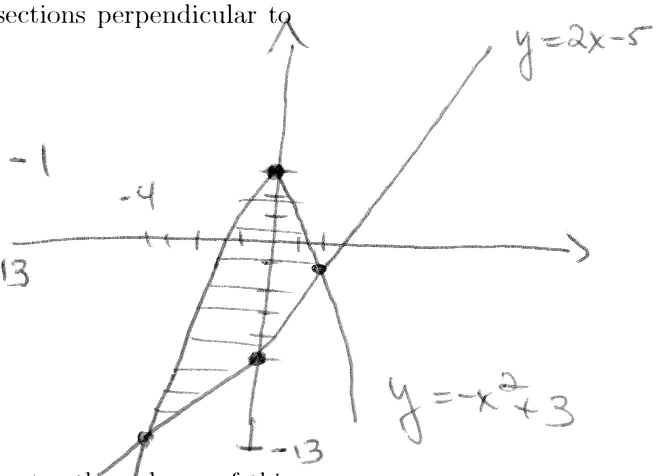
$0 = x^2 + 2x - 8$

$= (x+4)(x-2)$

$x = 2, -4$

at $x = 2$, $2(2) - 5 = -1$

at $x = -4$ $2(-4) - 5 = -13$



b) Find a Riemann sum which approximates the volume of this solid.

$$\text{Riemann sum} = \sum (\text{area})\,\Delta y = \sum (\text{side})^2 \cdot \Delta y$$

$$= \sum \left(\frac{y+5}{2} - (-\sqrt{3-y}) \right)^2$$

$$+ \sum \left(\sqrt{3-y} - (-\sqrt{3-y}) \right)^2$$

Note:

$y = 2x - 5 \Rightarrow 2x = y + 5$

$x = y + 5/2$

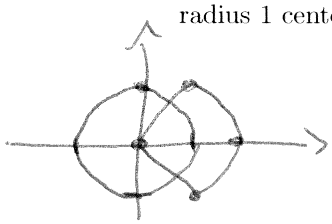
$y = -x^2 + 3 \Rightarrow x^2 = 3 - y$

$x = \pm\sqrt{3-y}$

c) Write a definite integral that calculates this volume precisely. **You do not have to calculate the integral.**

$$\text{Volume} = \int_{-13}^{-1} \left(\frac{y+5}{2} + \sqrt{3-y} \right)^2 dy + \int_{-1}^{3} \left(2\sqrt{3-y} \right)^2 dy$$

(4) Sketch the circle of radius 1 centered at the origin and the circle of radius 1 centered at the point (1,0) both on the same axis.



$x^2+y^2=1$ and $(x-1)^2+y^2=1$

$y=\pm\sqrt{1-x^2}$ $\qquad$ $y=\pm\sqrt{1-(x-1)^2}$

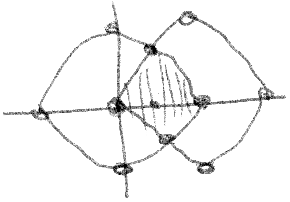
Intersection? $\pm\sqrt{1-x^2}=\pm\sqrt{1-(x-1)^2} \Rightarrow 1-x^2=1-(x-1)^2$

$1-x^2=1-[x^2-2x+1]$

$0=2x-1$

$x=1/2$.

a) Write an integral which represents the area of the intersection of these circles (with x-axis integration). **You need not evaluate the integral.**



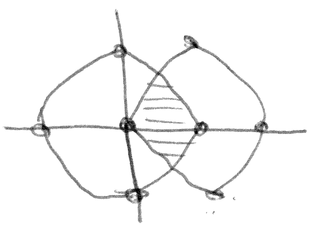
$$\text{Area} = \int_0^{1/2}\left(\sqrt{1-(x-1)^2}-\left(-\sqrt{1-(x-1)^2}\right)\right)dx + \int_{1/2}^{1}\left(\sqrt{1-x^2}-\left(-\sqrt{1-x^2}\right)\right)dx$$

b) Write an integral which represents the area of the intersection of these circles (with y-axis integration). **You need not evaluate the integral.**

$x^2+y^2=1$

$x^2=1-y^2$

$x=\pm\sqrt{1-y^2}$



$$\text{Area} = \int_{-\sqrt{3}/2}^{\sqrt{3}/2}\left(\sqrt{1-y^2}-\left(1-\sqrt{1-y^2}\right)\right)dy$$

$(x-1)^2+y^2=1$

$(x-1)^2=1-y^2$

$x=1\pm\sqrt{1-y^2}$

$x^2+y^2=1$ at $x=1/2$

$(\frac{1}{2})^2+y^2=1$

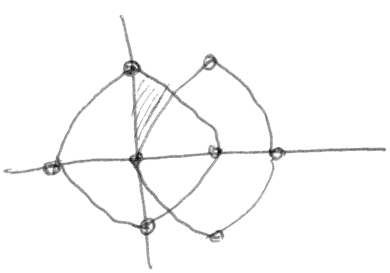
$y^2=1-\frac{1}{4}=\frac{3}{4}$

$y=\pm\frac{\sqrt{3}}{2}$

c) Write an integral which represents the volume of the solid obtained by revolving this region of intersection about the y-axis. **You need not evaluate the integral.**

$$\text{Volume} = \pi\int_{-\frac{\sqrt{3}}{2}}^{\sqrt{3}/2}\left(\sqrt{1-y^2}\right)^2dy - \pi\int_{-\frac{\sqrt{3}}{2}}^{\sqrt{3}/2}\left(1-\sqrt{1-y^2}\right)^2dy$$

d) Consider the region in the first quadrant inside the circle centered at the origin and outside the circle centered at (1,0). Write an integral which represents the volume of the solid obtained by revolving this region about the x-axis. **You need not evaluate the integral.**



$$\text{Volume} = \pi\int_0^{1/2}\left(\sqrt{1-x^2}\right)^2dx - \pi\int_0^{1/2}\left(\sqrt{1-(x-1)^2}\right)^2dx$$

(5) a) Write an integral for the arc length of the curve $y = \frac{2}{5}\sqrt{25 - x^2}$ from $x = 0$ to $x = 4$. **You need not evaluate the integral.** Approximate your answer with LEFT(2) and RIGHT(2).

$$f(x) = \frac{2}{5}\sqrt{25-x^2}$$

$$f'(x) = \frac{2}{5} \cdot \frac{1}{2}(25-x^2)^{-1/2}(-2x)$$

$$(f'(x))^2 = \frac{4}{25}\,\frac{x^2}{(25-x^2)}$$

$$\text{Arc length} = \int_0^4 \sqrt{1 + \frac{4x^2}{25(25-x^2)}}\,dx$$

$$\text{Left}(2) = \sqrt{1+(f'(0))^2} \cdot 2 + \sqrt{1+f'(2)^2} \cdot 2$$

$$= 2 + \sqrt{1 + \frac{16}{25 \cdot 21}} \cdot 2$$

$$\cong 4.03$$

$$\text{Right}(2) = \sqrt{1+f'(2)^2} \cdot 2 + \sqrt{1+f'(4)^2} \cdot 2$$

$$= \sqrt{1 + \frac{16}{25 \cdot 21}} \cdot 2 + \sqrt{1 + \frac{64}{25 \cdot 9}} \cdot 2$$

$$\cong 4.30$$

b) Find an exact value for the arc length of the parametrized curve: $x = \cos(e^t)$ and $y = \sin(e^t)$ when $0 \le t \le 1$.

$$\text{Arclength} = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \int_0^1 \sqrt{(\cancel{\cos}\,\sin(e^t) \cdot e^t)^2 + (\cos(e^t) \cdot e^t)^2}\,dt$$

$$= \int_0^1 \sqrt{e^{2t}(\sin^2(e^t) + \cos^2(e^t))}\,dt$$

$$= \int_0^1 e^t\,dt = e^t\Big|_0^1 = e - 1$$