MATH 129-020:
SIMS
TEST 2

SPRING 2019

Name	Key
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

(1) Determine if the following integral converges or diverges. If it converges, find its value.

$$\int_5^8 \frac{1}{\sqrt[3]{x^2 - 10x + 25}}\, dx$$

This is an improper integral of Type 2.

let
$u = x - 5$
$du = dx$

$$\int_5^8 \frac{1}{\sqrt[3]{x^2-10x+25}}\,dx = \lim_{a \to 5^+} \int_a^8 \frac{1}{\sqrt[3]{(x-5)^2}}\,dx$$

$$= \lim_{a \to 5^+} \int_{a-5}^{3} \frac{1}{\sqrt[3]{u^2}}\,du$$

$$= \lim_{a \to 5^+} \int_{a-5}^{3} u^{-2/3}\,du$$

$$= \lim_{a \to 5^+} \left. \frac{u^{1/3}}{\frac{1}{3}} \right|_{a-5}^{3}$$

$$= 3\sqrt[3]{3}$$

Thus, this integral converges and it converges to $3\sqrt[3]{3}$.

(2) Consider the following integral

$$\int_4^\infty \frac{3t^2}{\sqrt{2t^7+5}}dt$$

Does this integral converge or diverge? For full credit, you must justify your claim. **You need not calculate this integral.**

This is an improper integral of type 1.

Let

$$f(t) = \frac{3t^2}{\sqrt{2t^7+5}}.$$

For large t,

$$f(t) = \frac{3t^2}{\sqrt{2t^7+5}} = \frac{t^2 \cdot 3}{\sqrt{t^7(2+\frac{5}{t^7})}} = \frac{t^2 \cdot 3}{t^{7/2}\sqrt{2+\frac{5}{t^7}}}$$

$$\cong \frac{3}{\sqrt{2}} \cdot \frac{1}{t^{3/2}}$$

Since $p = 3/2 > 1$, we expect this integral to converge.

Note that:

$$2t^7 \le 2t^7+5$$

$$\Rightarrow \sqrt{2t^7} \le \sqrt{2t^7+5}$$

$$\Rightarrow \frac{1}{\sqrt{2t^7+5}} \le \frac{1}{\sqrt{2t^7}}$$

$$\Rightarrow \frac{3t^2}{\sqrt{2t^7+5}} \le \frac{3t^2}{\sqrt{2t^7}} = \frac{3}{\sqrt{2}} \cdot \frac{1}{t^{3/2}}$$

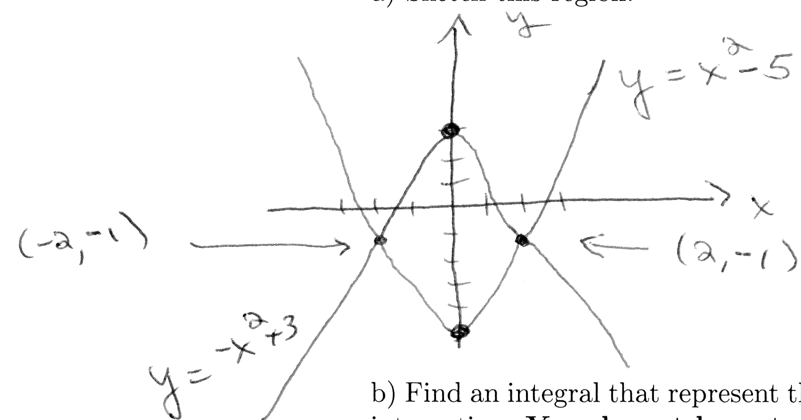
Since $g(t) = \frac{3}{\sqrt{2}} \cdot \frac{1}{t^{3/2}}$ has a convergent integral and $f(t) \le g(t)$,

$$\int_4^\infty \frac{3t^2}{\sqrt{2t^7+5}} dt$$

converges by comparison.

(3) Consider the region bounded by the curves $y = -x^2 + 3$ and $y = x^2 - 5$.

a) Sketch this region.



Intersection?

$$-x^2 + 3 = x^2 - 5$$
$$8 = 2x^2$$
$$x = \pm 2$$

b) Find an integral that represent the area of this region using x-axis integration. **You do not have to calculate the integral.**

$$\text{Area} = \int_{-2}^{2} \left[(-x^2 + 3) - (x^2 - 5) \right] dx$$

c) Find an integral (or integrals) that represent the area of this region using y-axis integration. **You do not have to evaluate the integral(s).**

Note: $y = -x^2 + 3 \Rightarrow x^2 = 3 - y$
$x = \pm\sqrt{3-y}$

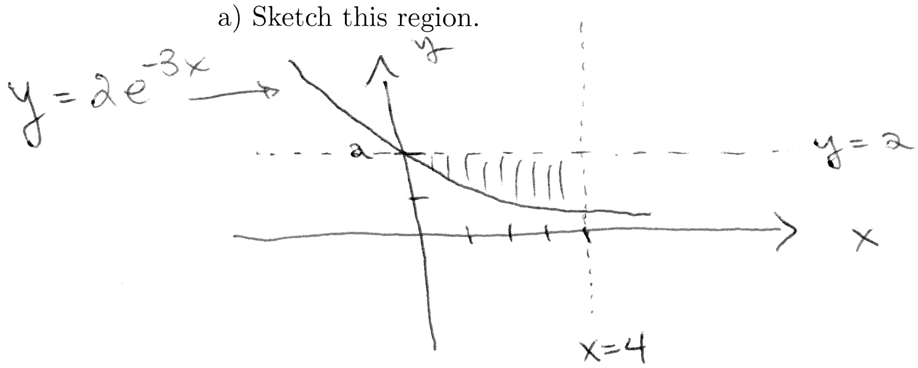
$y = x^2 - 5 \Rightarrow x^2 = y + 5$
$x = \pm\sqrt{y+5}$

Need 2 integrals for this area:

$$\text{Area} = \int_{-5}^{-1} \left[\sqrt{y+5} - (-\sqrt{y+5}) \right] dy$$
$$+ \int_{-1}^{3} \left[\sqrt{3-y} - (-\sqrt{3-y}) \right] dy$$

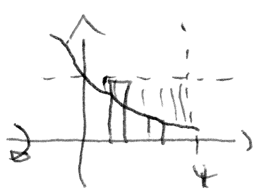
(4) Consider the region bounded by $y = 2e^{-3x}$, $y = 2$, and $x = 4$.

a) Sketch this region.



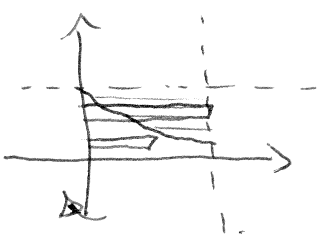
Set up an integral (or integrals) which determines the volume of the solid obtained by revolving this region about the:

b) x-axis



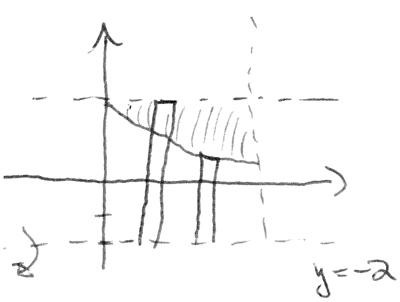
$$\text{Volume} = \pi \int_0^4 (2)^2\,dx - \pi \int_0^4 (2e^{-3x})^2\,dx$$

c) y-axis



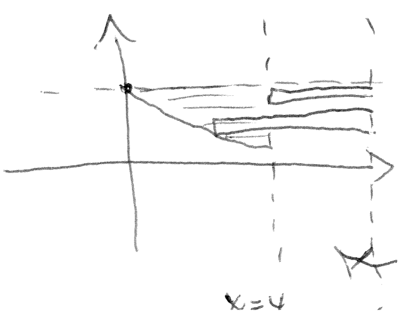
$$\text{Volume} = \pi \int_{2e^{-12}}^{2} (4)^2\,dy - \pi \int_{2e^{-12}}^{2} \left(-\tfrac{1}{3}\ln(y/2)\right)^2 dy$$

d) line $y = -2$



$$\text{Volume} = \pi \int_0^4 (2-(-2))^2\,dx - \pi \int_0^4 (2e^{-3x}-(-2))^2\,dx$$

e) line $x = 7$

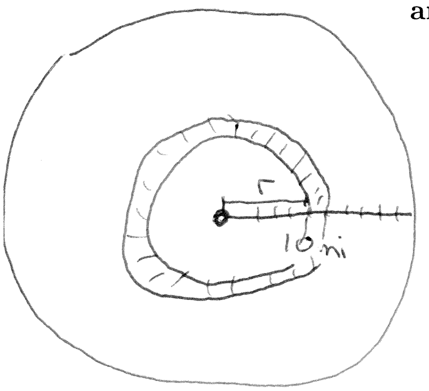


$$\text{Volume} = \pi \int_{2e^{-12}}^{2} \left[7-\left(-\tfrac{1}{3}\ln(y/2)\right)\right]^2 dy$$

$$- \pi \int_{2e^{-12}}^{2} [7-4]^2\,dy$$

(5) At ground level, pollution emanates circularly from a factory with a density $f(r) = 0.237e^{-.32r^2}\ \frac{\text{kg}}{\text{mi}^2}$ with r the distance from the factory in miles.

a) Write a Riemann sum which approximates the total mass of the pollution within a 10 mile radius of the factory. **To receive any partial credit, draw a sketch illustrating your calculation and label your variable.**



$$\text{Riemann sum for Total mass} \cong \sum \text{mass on each ring}$$

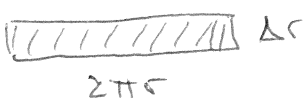
$$= \sum (\text{density})(\text{area})$$

$$= \sum f(r) \cdot 2\pi r \cdot \Delta r$$

$$\frac{\text{kg}}{(\text{mi})^2} \quad \text{mi} \quad \text{mi}$$

↑ units

Area of ring $\cong 2\pi r \cdot \Delta r$



b) Write a definite integral which corresponds to the Riemann sum found in part a) above. Find the exact value of this integral and then approximate it's value with 2 decimal-place accuracy.

$$\rightarrow \quad 2\pi \int_0^{10} f(r) \cdot r\,dr = 2\pi(0.237) \int_0^{10} e^{-.32r^2} \cdot r\,dr$$

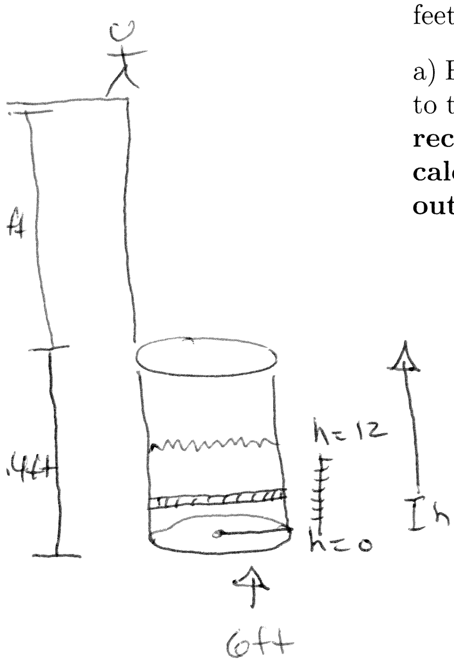
$$u = -0.32r^2 \qquad du = -0.64r\,dr$$

$$= \frac{2\pi(0.237)}{-(0.64)} \int_0^{-32} e^u\,du$$

$$= \frac{2\pi(0.237)}{0.64}\left(1 - e^{-32}\right)$$

$$\cong 2.33$$

(6) A cylindrical tank half-filled with oil is standing vertically with its top 15 feet under ground. The tank has a radius of 6 feet and is 24 feet tall. The density of the oil is 35 lb/ft^3.



a) Find an integral which describes the work done in pumping the oil to the top of the tank. **You need not evaluate the integral. To receive any partial credit, draw a sketch illustrating your calculation, indicate where your variable is zero, and write out the corresponding Riemann sum.**

Total work Done $= \sum$ work done on each slice

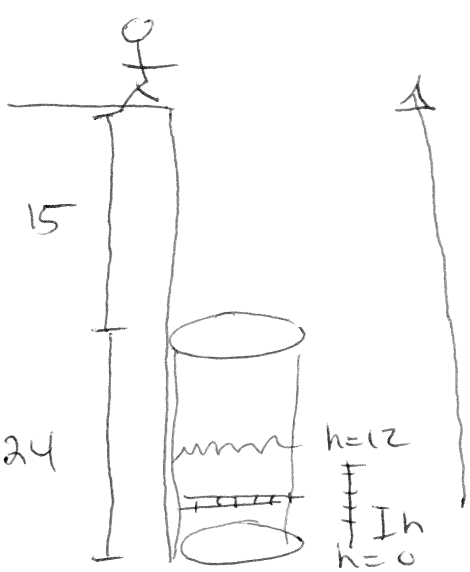
$= \sum$ (Force)(distance) on each slice

$= \sum$ [(density)(volume)](distance)

$$= \sum \left[(35\,\text{lb}/\text{ft}^3)(\pi(6\text{ft})^2 \underset{\text{ft}}{\Delta h})\right](12-h+12)$$

$$\rightarrow \int_0^{12} 35 \cdot \pi \cdot 36\ (24-h)\, dh$$

b) Find an integral which describes the work done in pumping the oil to ground level. **You need not evaluate the integral. To receive any partial credit, draw a sketch illustrating your calculation, indicate where your variable is zero, and write out the corresponding Riemann sum.**



Total work done $= \sum$ work done on each slice

$= \sum$ [(density)(volume)] (distance)

$$= \sum \left[(35\,\text{lb}/\text{ft}^3)(\pi(6\text{ft})^2 \Delta h)\right](12-h+12+15)$$

$$\rightarrow \int_0^{12} 35 \cdot \pi \cdot 36\ (39-h)\, dh$$