Directions: Show all work on calculating the integrals below, unless you are specifically asked to use a table.

(1) Calculate the following: Do **not** use an integration table.

a)

$$\int \frac{1-4x}{\sqrt[3]{2x+1}}\,dx\,.$$

$$\int \frac{1-4x}{\sqrt[3]{2x+1}}dx = \int \frac{1-(2u-2)}{u^{1/3}} \cdot \frac{1}{2}du = \frac{1}{2}\int u^{-1/3}(3-2u)du$$

$$= \frac{3}{2}\int u^{-1/3}du - \int u^{2/3}du$$

$$= \frac{3}{2}\cdot \frac{u^{2/3}}{2/3} - \frac{u^{5/3}}{5/3} + C$$

$$= \frac{9}{4}(2x+1)^{2/3} - \frac{3}{5}(2x+1)^{5/3} + C$$

Let $u = 2x+1$

$\Rightarrow du = 2dx$

$\Rightarrow 2x = u-1$

$\Rightarrow 4x = 2(u-1)$
$= 2u-2$

b)

$$\int_0^{\pi/3} \sin(3t)e^{-\cos(3t)}\,dt$$

$$\int_0^{\pi/3} \sin(3t)e^{-\cos(3t)}dt = \int_{-1}^{1} e^u \cdot \frac{1}{3}du$$

$$= \frac{1}{3}(e - e^{-1})$$

Let $u = -\cos(3t)$

$\Rightarrow du = 3\sin(3t)dt$

If $t=0$, $u = -\cos(0) = -1$

If $t = \pi/3$, $u = -\cos(\pi) = 1$

(2) Calculate

$$\int x^3 \ln^2(x)\, dx\,.$$

Do **not** use an integration table.

$$\int x^3 \ln^2(x)\,dx = \frac{x^4}{4}\ln^2(x) - \int \frac{x^4}{4}\, 2\ln(x)\cdot\frac{1}{x}\,dx$$

$$\boxed{\begin{array}{ll} u = \ln^2(x) & dv = x^3\,dx \\ du = 2\ln(x)\cdot\frac{1}{x}\,dx & v = \frac{x^4}{4} \end{array}}$$

$$= \frac{x^4}{4}\ln^2(x) - \frac{1}{2}\int x^3 \ln(x)\,dx$$

$$\boxed{\begin{array}{ll} u = \ln(x) & dv = x^3\,dx \\ du = \frac{1}{x}\,dx & v = \frac{x^4}{4} \end{array}}$$

$$= \frac{x^4}{4}\ln^2(x) - \frac{1}{2}\left[\frac{x^4}{4}\ln(x) - \int \frac{x^4}{4}\cdot\frac{1}{x}\,dx\right]$$

$$= \frac{x^4}{4}\ln^2(x) - \frac{x^4}{8}\ln(x) + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}x^4 + C$$

$\frac{1}{32}$

(3) Calculate the following. You **may** use the integration table.

$$\int \frac{1}{t^3\sqrt{25t^2+4}}\,dt$$

$$\int \frac{1}{t^3\sqrt{25t^2+4}}\,dt = \int \frac{1}{(u/5)^3\sqrt{u^2+2^2}}\,\frac{1}{5}du$$

Let $u = 5t$
$\Rightarrow du = 5dt$
Note: $t = u/5$

$$= 5^2 \cdot \int \frac{1}{u^3\sqrt{u^2+2^2}}\,du$$

Let $u = 2\tan(\theta)$
$du = 2\sec^2(\theta)\,d\theta$
$u^2+2^2 = 2^2\sec^2(\theta)$

$$= 5^2 \cdot \int \frac{1}{2^3\tan^3(\theta)\sqrt{2^2\sec^2(\theta)}}\,2\sec^2(\theta)\,d\theta$$

$$= \frac{5^2}{2^3}\int \frac{\sec(\theta)}{\tan^3(\theta)}\,d\theta = \frac{5^2}{2^3}\int \frac{\cos^2(\theta)}{\sin^3(\theta)}\,d\theta$$

use $\cos^2(\theta) = 1-\sin^2(\theta)$

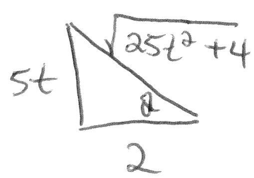
$$= \frac{5^2}{2^3}\underbrace{\int \frac{1}{\sin^3(\theta)}\,d\theta}_{\#19} - \frac{5^2}{2^3}\underbrace{\int \frac{1}{\sin(\theta)}\,d\theta}_{\#20}$$

$$= \frac{5^2}{2^3}\left[-\frac{1}{2}\frac{\cos(\theta)}{\sin^2(\theta)} + \frac{1}{2}\int\frac{1}{\sin(\theta)}\,d\theta\right] - \frac{5^2}{2^3}\cdot\frac{1}{2}\ln\left|\frac{\cos(\theta)-1}{\cos(\theta)+1}\right| + C$$

Note:

$u = 2\tan(\theta)$

$\Rightarrow \tan(\theta) = \frac{u}{2} = \frac{5t}{2}$



$\Rightarrow$ $\sin(\theta) = \frac{5t}{\sqrt{25t^2+4}}$

$\cos(\theta) = \frac{2}{\sqrt{25t^2+4}}$

use these above

(4) Integrate

$$\int \frac{x^4 - 2x^3 - 4x^2 + 1}{x^3 - 2x^2 - 4x + 8}\, dx\,.$$

You **may** use an integration table.

$$\begin{array}{r|l} & \quad x \\ \hline x^3-2x^2-4x+8 & x^4-2x^3-4x^2+1 \\ & \overset{-}{}x^4 \overset{+}{-} 2x^3 \overset{+}{-} 4x^2 \overset{-}{+} 8x \\ \hline & \qquad -8x+1 \end{array}$$

$$\Rightarrow \int \frac{x^4-2x^3-4x^2+1}{x^3-2x^2-4x+8}\,dx = \int x + \frac{-8x+1}{x^2(x-2)-4(x-2)}\,dx$$

$$= \frac{x^2}{2} + \int \frac{-8x+1}{(x^2-4)(x-2)}\,dx$$

$$\frac{-8x+1}{(x+2)(x-2)^2} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \qquad = \frac{x^2}{2} + A\ln|x+2| + B\ln|x-2| - \frac{C}{x-2} + C$$

<u>Now</u>

$$-8x+1 = A(x-2)^2 + B(x+2)(x-2) + C(x+2)$$

$$= (A+B)x^2 + (-4A+C)x + (4A-4B+2C)$$

$A+B=0 \qquad A=-B$

$-4A+C=-8 \qquad \Rightarrow 4B+C=-8 \quad \Rightarrow 4C=-15 \qquad 4B = \frac{-32}{4}+\frac{15}{4}$

$4A-4B+2C=1 \quad \Rightarrow -8B+2C=1 \qquad C=\frac{-15}{4} \qquad B=-\frac{17}{16}$

$A = 17/16$

θ	$\cos(\theta)$
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

(5) Consider the following integral:

$$\int_0^{2\pi} \ln\left(1+\cos^2(\theta)\right)\, d\theta$$

Evaluate (with 2 digits accuracy):

$\Delta x = \frac{2\pi - 0}{2} = \pi$

a) LEFT(2)

$f(\theta) = \ln(1+\cos^2(\theta))$

$LEFT(2) = \pi\left(f(0) + f(\pi)\right)$

$= \pi\left(\ln(2) + \ln(2)\right) = 2\pi\ln(2) \cong 4.36$

b) RIGHT(2)

$RIGHT(2) = \pi\left(f(\pi) + f(2\pi)\right) = 2\pi\ln(2) \cong 4.36$

c) MID(2)

$MID(2) = \pi\left(f(\pi/2) + f(3\pi/2)\right) = 0 \cong 0.00$

d) TRAP(2)

$TRAP(2) = \frac{LEFT(2) + RIGHT(2)}{2} = 2\pi\ln(2) \cong 4.36$

e) Considering the following similar integral:

$$\int_0^{\pi/2} \ln\left(1+\cos^2(\theta)\right)\, d\theta$$

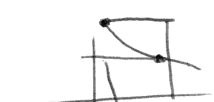
which is larger: LEFT(10) or RIGHT(10)? You need not calculate either. An answer with no justification will receive no credit.

$f(\theta) = \ln(1+\cos^2(\theta))$

$f'(\theta) = \frac{1}{1+\cos^2(\theta)} \cdot 2\cos(\theta)(-1)\sin(\theta)$

≤ 0 for $0 \leq \theta \leq \pi/2$

i.e. f is decreasing



$\Rightarrow$

LEFT(10) is <u>larger</u> than RIGHT(10)!