MATH 129-011:
TEST 1
MAKE-UP

SPRING 2018

Name	
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

Directions: This work is an optional assignment for those who took the first test on Friday, February 2nd. It is due on Monday, February 12th at the beginning of class. No late work will be accepted. If you turn this in, I will grade it (with a score out of 100) and your new grade on test 1 will be the average of the two scores you have received. If you do not turn this in, your grade on test 1 will stay the same.

Show all work on calculating the integrals below, unless you are told you can use the integration table. When you use the integration table, indicate which number you are using.

(1) Calculate the following: Do **not** use an integration table.

a)

$$\int x^5 \cos(x^3)\, dx$$

Substitute

$u = x^3$

$du = 3x^2 dx$

$$\Rightarrow \int x^5 \cos(x^3)dx = \int x^3 \cos(x^3) \cdot x^2 dx$$

$$= \frac{1}{3}\int u \cos(u)\, du$$

Integration by parts

$$= \frac{1}{3}\left[u \sin(u) - \int \sin(u) du\right]$$

$$= \frac{1}{3}x^3 \sin(x^3) + \frac{1}{3}\cos(x^3) + C$$

b)

$$\int \frac{3t-2}{\sqrt[5]{1-\pi t}}\, dt\,.$$

$$\boxed{\begin{array}{l}\text{Substitute} \\ u = 1-\pi t \Rightarrow t = \frac{1}{\pi}(1-u) \\ du = -\pi dt\end{array}}$$

$$\int \frac{3t-2}{\sqrt[5]{1-\pi t}} dt = \int \frac{\frac{3}{\pi}(1-u) - 2}{u^{1/5}} \cdot \left(\frac{-1}{\pi}\right) du$$

$$= \frac{-1}{\pi}\int u^{-1/5}\left(\left(\frac{3}{\pi}-2\right) - \frac{3}{\pi}u\right) du$$

$$= \frac{-1}{\pi}\left(\frac{3}{\pi}-2\right)\frac{u^{4/5}}{4/5} + \frac{3}{\pi^2}\frac{u^{9/5}}{9/5} + C = -\frac{5}{4\pi}\left(\frac{3}{\pi}-2\right)(1-\pi t)^{4/5} + \frac{5}{3\pi^2}(1-\pi t)^{9/5} + C$$

(2) Suppose that f has three continuous derivatives on $[0, b]$. If you know that $f''(b) = 2$, $f'(b) = -3$, $f(b) = 1$, and $f(0) = 0$, calculate

$$\int_0^b x^2 f'''(x)\, dx\,.$$

$$\int_0^b x^2 f'''(x)\,dx = x^2 f''(x)\Big|_{x=0}^{x=b} - \int_0^b f''(x)\cdot 2x\,dx$$

$$\boxed{\begin{array}{ll} u = x^2 & dv = f'''(x)dx \\ du = 2x\,dx & v = f''(x) \end{array}}$$

$$= b^2 f''(b) - 2\int_0^b x f''(x)\,dx$$

$$\boxed{\begin{array}{ll} u = x & dv = f''(x)dx \\ du = dx & v = f'(x) \end{array}}$$

$$= 2b^2 - 2\left[x f'(x)\Big|_0^b - \int_0^b f'(x)\,dx \right]$$

$$= 2b^2 - 2b f'(b) + 2\int_0^b f'(x)\,dx$$

$$= 2b^2 + 6b + 2\big(f(b) - f(0)\big)$$

$$= 2b^2 + 6b + 2$$

(3) Calculate the following integrals. (You **may** use an integration table.)

a)

$$\int \frac{1}{\sqrt{3-t^2+7t}}\,dt\,.$$

Complete the square

$3-t^2+7t = -t^2+7t+3$

$= -\left(t^2-7t+\left(\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2\right)+3$

$= -\left(t-\frac{7}{2}\right)^2+\frac{49}{4}+\frac{12}{4}$

$= -(t-7/2)^2+61/4$

$\int \frac{1}{\sqrt{3-t^2+7t}}\,dt$

$= \int \frac{1}{\sqrt{\frac{61}{4}-\left(t-\frac{7}{2}\right)^2}}\,dt$

#28 $a=\sqrt{\frac{61}{4}}$

$= \arcsin\left(\frac{t-7/2}{\sqrt{\frac{61}{4}}}\right)+C$

b)

$$\int (2x^5-3x^2+1)\sin(\pi x)\,dx\,.$$

$p(x) = 2x^5-3x^2+1$

$p'(x) = 10x^4-6x$

$p''(x) = 40x^3-6$

$p'''(x) = 120x^2$

$p^{(4)}(x) = 240x$

$p^{(5)}(x) = 240$

#15 with $p(x)=2x^5-3x^2+1$ and $a=\pi$

$\int (2x^5-3x^2+1)\sin(\pi x)\,dx$

$= -\frac{1}{\pi}(2x^5-3x^2+1)\cos(\pi x)$

$+\frac{1}{\pi^2}(10x^4-6x)\sin(\pi x)$

$+\frac{1}{\pi^3}(40x^3-6)\cos(\pi x)$

$-\frac{1}{\pi^4}(120x^2)\sin(\pi x)$

$-\frac{1}{\pi^5}(240x)\cos(\pi x)$

$+\frac{1}{\pi^6}(240)\sin(\pi x)+C$

(4) Integrate the following. You may **not** use an integration table. For full credit, show your work.

$$\int \frac{e^{2x}}{(e^{4x}+1)(e^{2x}-2)^2}\,dx\,.$$

Let $u = e^{2x}$

$du = 2e^{2x}\,dx$

$$\Rightarrow \int \frac{e^{2x}}{(e^{4x}+1)(e^{2x}-2)^2}dx = \int \frac{1}{(u^2+1)(u-2)^2}\,\tfrac{1}{2}du$$

Partial fractions

$$\frac{1}{(u^2+1)(u-2)^2} = \frac{Au+B}{u^2+1} + \frac{C}{u-2} + \frac{D}{(u-2)^2}$$

$$\Rightarrow \int \frac{e^{2x}\,dx}{(e^{4x}+1)(e^{2x}-2)^2} = \frac{1}{2}\int \frac{1}{(u^2+1)(u-2)^2}du$$

$$= \frac{A}{2}\int \frac{u}{u^2+1}du + \frac{B}{2}\int \frac{1}{u^2+1}du + \frac{C}{2}\int \frac{1}{u-2}du + \frac{D}{2}\int \frac{1}{(u-2)^2}du + C'$$

$$= \frac{A}{4}\ln|u^2+1| + \frac{B}{2}\arctan(u) + \frac{C}{2}\ln|u-2| - \frac{D}{2}\frac{1}{u-2} + C'$$

Now all we need are the coefficients.

$$= \frac{A}{4}\ln|e^{4x}+1| + \frac{B}{2}\arctan(e^{2x}) + \frac{C}{2}\ln|e^{2x}-2| - \frac{D}{2}\frac{1}{e^{2x}-2} + C'$$

For the coefficients:

$$\frac{1}{(u^2+1)(u-2)^2} = \frac{Au+B}{u^2+1} + \frac{C}{u-2} + \frac{D}{(u-2)^2}$$

$$\Rightarrow \quad 1 = (Au+B)(u-2)^2 + C(u^2+1)(u-2) + D(u^2+1)$$

$$\Rightarrow \quad 1 = (A+C)u^3 + (-4A+B-2C+D)u^2 + (4A-4B+C)u + (4B-2C+D)$$

$$\Rightarrow \quad 0 = A+C \quad \Rightarrow \quad A = -C$$

$$0 = -4A + B - 2C + D \quad \Rightarrow \quad \boxed{0 = B + 2C + D}$$

$$0 = 4A - 4B + C \quad \Rightarrow \quad \boxed{0 = -4B - 3C} \quad \Rightarrow \quad 4B = -3C$$

$$1 = 4B - 2C + D \quad \Rightarrow \quad \boxed{1 = 4B - 2C + D}$$

$$\Rightarrow \quad 0 = -\frac{3}{4}C + 2C + D \quad \Rightarrow \quad 0 = \frac{5}{4}C + D$$

$$1 = -3C - 2C + D \qquad 1 = -5C + D$$

$$\Rightarrow \quad -1 = \frac{5}{4}C + 5C$$

$$= \frac{25}{4}C$$

$$C = \frac{-4}{25}$$

$$D = \frac{-5}{4}C = \frac{5}{25}$$

$$\Rightarrow A = \frac{4}{25}$$

$$4B = -3\left(\frac{-4}{25}\right)$$

$$B = \frac{3}{25}$$

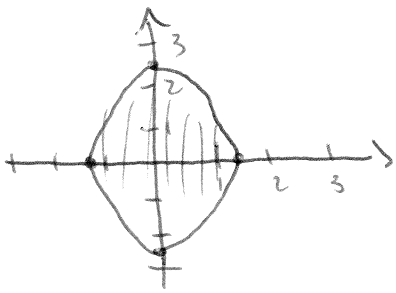
$$A = \frac{4}{25}, \quad B = \frac{3}{25}, \quad C = -4/25, \quad D = \frac{5}{25}$$

(5) Sketch the following graph and find the area enclosed by it:

$$16x^2 + 4y^2 = 25$$

$$16x^2+4y^2=25 \Rightarrow \frac{16x^2}{25}+\frac{4y^2}{25}=1 \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$a=\frac{5}{4} \qquad b=\frac{5}{2}$$



$$16x^2+4y^2=25 \Rightarrow 4y^2=25-16x^2$$

$$y=\pm\sqrt{\frac{25}{4}-4x^2}$$

$$\text{Area of ellipse} = 4\cdot\text{Area in } Q_1 = 4\cdot\int_0^{5/4}\sqrt{\frac{25}{4}-4x^2}\,dx$$

$$x=\frac{5}{4}\sin(\theta)$$

$$dx=\frac{5}{4}\cos(\theta)\,d\theta$$

$$= 8\cdot\int_0^{5/4}\sqrt{\frac{25}{16}-x^2}\,dx$$

$$= 8\cdot\int_0^{\pi/2}\sqrt{\frac{25}{16}\cos^2(\theta)}\cdot\frac{5}{4}\cos(\theta)\,d\theta$$

$$= 8\cdot\frac{25}{16}\int_0^{\pi/2}\cos^2(\theta)\,d\theta$$

#18 or table

$$= \frac{25}{2}\left[\frac{1}{2}\cos(\theta)\sin(\theta)\Big|_0^{\pi/2}+\frac{1}{2}\int_0^{\pi/2}1\,d\theta\right]$$

$$= \frac{25}{2}\left[0+\frac{\pi}{4}\right]$$

$$= \frac{25\pi}{8}$$

(6) Consider the function

$$f(x) = \frac{1}{3 + x + x^2}.$$

a) On the interval $[1, 3]$, determine whether the function f given above is increasing or decreasing? Based on your answer, order LEFT(n), RIGHT(n), and $\int_1^3 f(x)dx$ from smallest to largest. Check your result by calculating LEFT(2) and RIGHT(2).

$$f(x) = \frac{1}{3+x+x^2}$$

$$f'(x) = (-1)(3+x+x^2)^{-2}(1+2x) = \frac{-1-2x}{(3+x+x^2)^2}$$

+ + + 0 - - - - - - f'

$-3 \quad -1 \quad -\frac{1}{2} \quad 1 \quad 3$

f is decreasing on $[1,3]$

$$\Rightarrow \text{Right}(n) \le \int_1^3 f(x)dx \le \text{Left}(n)$$

$$\text{Left}(2) = f(1)\cdot 1 + f(2)\cdot 1 = \frac{14}{45} \approx .3111$$

$$\text{Right}(2) = f(2)\cdot 1 + f(3)\cdot 1 = \frac{8}{45} \approx .1778$$

b) Repeat part a) on the interval $[-3, -1]$.

f is increasing on $[-3,-1]$

$$\Rightarrow \text{Left}(n) \le \int_{-3}^{-1} f(x)dx \le \text{Right}(n)$$

$$\text{Left}(2) = f(-3)\cdot 1 + f(-2)\cdot 1 = \frac{14}{45} \approx .3111$$

$$\text{Right}(2) = f(-2)\cdot 1 + f(-1)\cdot 1 = \frac{8}{15} \approx .5333$$

c) On the interval $[1, 3]$, find MID(2) and TRAP(2).

$$\text{Mid}(2) = f(\tfrac{3}{2})\cdot 1 + f(\tfrac{5}{2})\cdot 1 = \frac{296}{1269} \approx .2333$$

$$\text{Trap}(2) = \frac{\text{Left}(2) + \text{Right}(2)}{2} = \frac{11}{45} \approx .2445$$