MATH 129-020:
SIMS
TEST 1

SPRING 2019

Name	Key
I.D. Number	

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Directions: Show all work on calculating the integrals below, unless you are told you can use the integration table. When you use the integration table, indicate which number you are using.

(1) Calculate the following: Do **not** use an integration table.

$$\int \frac{2x}{\sqrt[3]{4x-1}}\,dx\,.$$

$\boxed{\begin{array}{l}\underline{\text{Substitution}} \\ \text{let } u = 4x-1 \\ \Rightarrow \quad du = 4dx \\ u = 4x-1 \Rightarrow u+1 = 4x \\ \qquad\qquad x = \frac{u+1}{4}\end{array}}$

$$\int \frac{2x}{\sqrt[3]{4x-1}}\,dx = \int \frac{2\cdot\left(\frac{u+1}{4}\right)}{\sqrt[3]{u}} \cdot \frac{1}{4}du$$

$$= \frac{1}{8}\int u^{-1/3}(u+1)\,du$$

$$= \frac{1}{8}\int u^{2/3}\,du + \frac{1}{8}\int u^{-1/3}\,du$$

$$= \frac{1}{8}\,\frac{u^{\frac{5}{3}}}{\frac{5}{3}} + \frac{1}{8}\,\frac{u^{2/3}}{\frac{2}{3}} + C$$

$$= \frac{3}{40}(4x-1)^{\frac{5}{3}} + \frac{3}{16}(4x-1)^{2/3} + C$$

(2) Calculate the following: Do **not** use an integration table.

$$\int x^2 \ln^2(x)\, dx$$

Integration by Parts:

$$\int x^2 \ln^2(x)\,dx = \frac{1}{3}x^3 \ln^2(x) - \int \frac{x^3}{3} \cdot 2\ln(x) \cdot \frac{1}{x}\,dx$$

$$\boxed{\begin{array}{ll} u = \ln^2(x) & dv = x^2\,dx \\ du = 2\ln(x)\cdot\frac{1}{x}\,dx & v = \frac{x^3}{3} \end{array}}$$

$$= \frac{1}{3}x^3 \ln^2(x) - \frac{2}{3}\int x^2 \ln(x)\,dx$$

$$= \frac{1}{3}x^3 \ln^2(x) - \frac{2}{3}\left[\frac{1}{3}x^3 \ln(x) - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx\right]$$

$$\boxed{\begin{array}{ll} u = \ln(x) & dv = x^2\,dx \\ du = \frac{1}{x}\,dx & v = \frac{x^3}{3} \end{array}}$$

$$= \frac{1}{3}x^3 \ln^2(x) - \frac{2}{9}x^3 \ln(x) + \frac{2}{9}\int x^2\,dx$$

$$= \frac{1}{3}x^3 \ln^2(x) - \frac{2}{9}x^3 \ln(x) + \frac{2}{27}x^3 + C$$

(3) Calculate the following integral.

$$\int \frac{t}{\sqrt{4t^2+36}}\,dt\,.$$

<u>Trig. Substitution:</u>

$$\int \frac{t}{\sqrt{4t^2+36}}\,dt = \int \frac{t}{\sqrt{4(t^2+9)}}\,dt$$

$$= \frac{1}{2}\int \frac{t}{\sqrt{t^2+9}}\,dt$$

$t = 3\tan(\theta)$

$dt = 3\sec^2(\theta)\,d\theta$

$t^2+9 = 9\tan^2(\theta)+9$

$= 9\sec^2(\theta)$

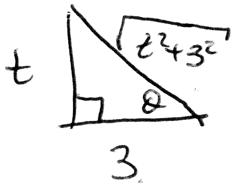
$$= \frac{1}{2}\int \frac{3\tan(\theta)}{\sqrt{9\sec^2(\theta)}}\cdot 3\sec^2(\theta)\,d\theta$$

$$= \frac{3}{2}\int \tan(\theta)\cdot\sec(\theta)\,d\theta$$

<u>Recall:</u>

$\frac{d}{d\theta}\sec(\theta) = \sec(\theta)\tan(\theta)$

$t = 3\tan(\theta) \Rightarrow \tan(\theta) = \frac{t}{3}$



$$= \frac{3}{2}\sec(\theta) + C$$

$$= \frac{3}{2}\,\frac{1}{\cos(\theta)} + C$$

$$= \frac{3}{2}\cdot\frac{\sqrt{t^2+9}}{3} + C$$

$$= \frac{1}{2}\sqrt{t^2+9} + C$$

(4) Integrate the following.

$$\int \frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3} dx\,.$$

Partial Fractions

Must divide 1st

$$\begin{array}{r} 2x \\ x^2-2x-3 \overline{\smash{)}\, 2x^3 - 4x^2 - x - 3} \\ \underline{-2x^3 \overset{+}{-} 4x^2 \overset{+}{-} 6x } \\ 5x - 3 \end{array}$$

$$\Rightarrow \int \frac{2x^3-4x^2-x-3}{x^2-2x-3}dx = \int 2x + \frac{5x-3}{x^2-2x-3}\,dx$$

$$= x^2 + \int \frac{5x-3}{(x-3)(x+1)}\,dx$$

$$\boxed{\begin{aligned} &\frac{5x-3}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1} \\ &5x-3 = A(x+1) + B(x-3) \\ &\quad = (A+B)x + (A-3B) \\ &5 = A+B \\ &-3 = A-3B \\ &8 = 4B \Rightarrow B = 2 \\ &\quad \Rightarrow A = 3 \end{aligned}}$$

$$= x^2 + 3\int \frac{1}{x-3}dx + 2\int \frac{1}{x+1}dx$$

$$= x^2 + 3\ln|x-3| + 2\ln|x+1| + C$$

Let $f(\theta) = \ln(1+\cos^2(\theta))$

θ	$f(\theta)$
0	$\ln(2)$
$\pi/2$	0
π	$\ln(2)$
$3\pi/2$	0
2π	$\ln(2)$

(5) Consider the following integral:

$$\int_0^{2\pi} \ln(1+\cos^2(\theta))\,d\theta$$

Evaluate (with 2 decimal place accuracy):

$\Delta x = \frac{b-a}{2} = \frac{2\pi}{2} = \pi$

$0 \quad \pi \quad 2\pi$

a) LEFT(2)

$= f(0)\cdot\pi + f(\pi)\cdot\pi = 2\pi\ln(2) \cong 4.36$

b) RIGHT(2)

$= f(\pi)\cdot\pi + f(2\pi)\cdot\pi = 2\pi\ln(2) \cong 4.36$

c) MID(2)

$= f(\pi/2)\cdot\pi + f(\frac{3\pi}{2})\cdot\pi = 0 \cong 0.00$

d) TRAP(2)

$= \frac{\text{Left}(2) + \text{Right}(2)}{2} \cong 4.36$

e) Consider the following similar integral:

$$\int_0^{\pi/2} \ln(1+\cos^2(\theta))\,d\theta$$

which is larger LEFT(10) or RIGHT(10)? You need not calculate either. An answer with no justification will receive no credit.

$f(\theta) = \ln(1+\cos^2(\theta))$

$f'(\theta) = \frac{1}{1+\cos^2(\theta)} \cdot 2\cos(\theta)\cdot(-\sin(\theta))$

$= \frac{-2\cos(\theta)\sin(\theta)}{1+\cos^2(\theta)} \leq 0$ for $0 \leq \theta \leq \pi/2$

$\Rightarrow$ LEFT(10) is larger because f is decreasing!