Math Homework #4

1. Compute the Fourier transform of the following function

$$f(x) = e^{-2|x|} + \mathbb{1}_{[0,2]}(x)$$

(Recall that $\mathbb{1}_{[a,b]}$ is the function whose value is 1 for $a < x < b$ and 0 else.) Use the following identities to get your answer (you will not have to compute any integral).

$f(x)$	$\widehat{f}(\xi)$
$e^{-\|x\|}$	$\frac{1}{\pi(1+\xi^2)}$
$\mathbb{1}_{[-a,a]}(x)$	$\frac{\sin(a\xi)}{\pi\xi}$
$f(x-c)$	$e^{-ic\xi}\widehat{f}(\xi)$
$f(ax)$	$\frac{1}{a}\widehat{f}\left(\frac{\xi}{a}\right)$

2. Use the Cooley Tukey algorithm to calculate (by hand, as done in class) the discrete Fourier transform of the following sample (use a hand calculator at each step, rounding to two decimals; actually only the last step will really want a calculator).

t	0	$\pi/4$	$2\pi/4$	$3\pi/4$	$4\pi/4$	$5\pi/4$	$6\pi/4$	$7\pi/4$
$x(t)$	1.23	0.45	−0.23	−0.10	−0.79	−0.43	0.25	0.15

3. Compute the indefinite integral from zero to x of the distribution

$$x + \delta_{-2}(x) - 3\delta_5(x)$$

Your answer will be a piecewise function.

4. Compute the distributional derivative of the function

$$f(x) = |x| + 2\mathbb{1}_{[4,7]}(x)$$

5. Suppose X, Y are RV's with joint density

$$f_{X,Y} = \begin{cases} 2e^{-x-y} & \text{if } 0 < x < y \\ 0 & \text{else} \end{cases}$$

   (a) Compute the density of Y.
   (b) Compute the conditional density $f_{X|Y}(x|y)$
   (c) Calculate the conditional probability $P\{X > 5|Y = 6\}$
   (d) Compute the conditional probability $P\{X > 5|Y > 6\}$.