Math Homework #3

1. Let X be a random variable with density

$$f_X(t) = \begin{cases} Cx^{-2} & \text{if } x > 3 \\ 0 & \text{else.} \end{cases}$$

   (a) Calculate the value of the constant C so that f_X is a probability density.

   (b) Calculate the cumulative distribution function for X.

   (c) Calculate the expectation value of X.

   (d) Show that the expectation value of $E[X^2] = \infty$. Conclude from this that also $Var(X) = \infty, \sigma(X) = \infty$.

2. Consider the following gambling game: player A pays n dollars to player B. Then player A flips a fair coin, if it's "heads" he flips again; he keeps flipping until it falls "tails". Then player B pays 2^k dollars to player A, where k is the numbers of flips that were "heads".

   (a) Show that the expected value for player B's profit for one game is $-\infty$.

   (b) Suppose that the following additional rule is added to the game: if A suceeds in flipping 40 heads in a row, then the game ends. Calculate the expected value of the new game for player B.

   (c) Despite the expected profit of $-\infty$, most mathematicians would play the original game as B if n is sufficiently large, say $n = 40$. Do you think this is a good idea, and why/ why not?

3. (Problem 5.21E from *A first course in probability* by Sheldon Ross; exact wording altered) Suppose that the height of a man is a normal random variable with parameters (measured in inches) $\mu = 71, \sigma = 2.5$. According to this model, calculate the percentage of men taller than $74''$, and $77''$.

4. Suppose that the random variable X has density

$$f_X(x) = \begin{cases} 4e^{-4x} & \text{if } x \geq 0 \\ 0 & \text{else} \end{cases}$$

   Calculate the expected value of $f(X)$, where the function f is defined by

$$f(t) = \begin{cases} 1 & \text{if } t \leq 5 \\ 0 & \text{else} \end{cases}$$

5. Suppose that X is a random variable distributed uniformly on the interval $[1, 3]$. Let Y be the random variable defined by $Y = e^X$. Calculate the density of Y.

6. Calculate the Fourier transforms of each of the following functions.

$$f(x) = \begin{cases} 1 & \text{if } -2 < x < -1 \\ -3 & \text{if } 4 < x < 7 \\ 0 & \text{else} \end{cases}$$

$$g(x) = \begin{cases} e^{ix} & \text{if } -1 < x < 1 \\ 0 & \text{else .} \end{cases}$$

7. Suppose the Fourier transform of $a(x)$ is $\widehat{a}(\xi) = e^{-10\xi^2}$. Calculate the function $a(x)$.