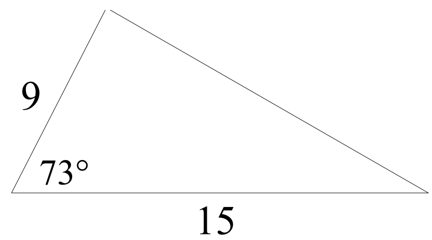
Lesson Plans - Apr. 20

Section 5.1: The Law of Sines (Part 1)

1. (Warmup). Find the area of the triangle shown:



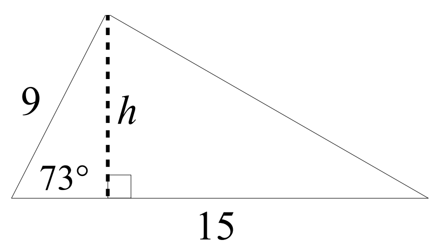
Answer. The area of a triangle is

$$A = \frac{1}{2}bh.$$

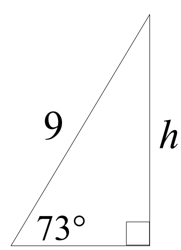
Remember that we can call any side of a triangle the base of the triangle. However, in this case, let us call the side that is on the bottom the base of the triangle. Then we know so far:

$$A = \frac{1}{2}(15)h,$$

but we still need to find the height. It helps to draw an altitude from the top vertex down to the base:



Notice that we have a right triangle:



We know how to find h in this situation.

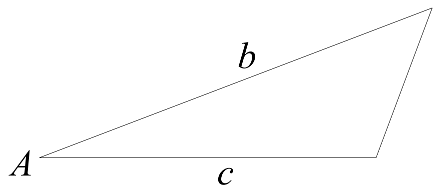
$$\sin(73^\circ) = \frac{h}{9}, \quad h = 9\sin(73^\circ) \approx 8.61.$$

So then we can find the area of the triangle:

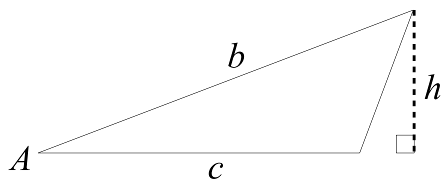
$$A = \frac{1}{2}(15)(8.61) = 64.58.$$

2. Note that what we did in the above situation, we picked a base and then we knew another side length and the angle between the two sides. We then used those two to find the height of the triangle, which we then used to find the area of the triangle.

   Find area of the following triangle:



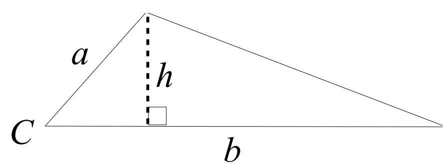
   Answer. We can draw the altitude of this triangle



   Note that $\sin A = \frac{h}{b}$, so $h = b \sin A$. Therefore, the area of the triangle is

$$\text{Area} = (b \sin A)(c) = bc \sin A.$$

   However, if we changed the orientation of this triangle, we can see that the area of the triangle is as follows:



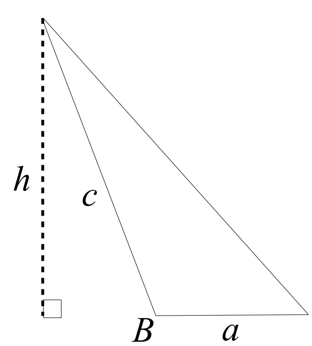
   Now we can see that if we call b the base of the triangle, we know that the height $h = a \sin C$, so the area of this triangle is

$$\text{Area} = (b)(a \sin C) = ab \sin C.$$

   Notice that we did nothing to the original triangle aside from rotate it, so the area stays the same. So, we so far know

$$\text{Area} = bc \sin A = ab \sin C.$$

   We can rotate the triangle one more time to find:



However, we should notice this one thing: the right triangle we've created actually includes an angle of measure $(180^\circ - B)$, so we have

$$\sin(180^\circ - B) = \frac{h}{c}, \quad h = c\sin(180^\circ - B).$$

However, we can simplify this a little, since we have an identity for the sine of a difference:

$$\sin(180^\circ - B) = \sin(180^\circ)\cos(B) - \sin(B)\cos(180^\circ) = (0)\cos(B) - \sin(B)(-1) = -(-1)\sin B = \sin B.$$

So, in fact, we can see that $h = c\sin B$, so

$$\text{Area} = (a)(c\sin B) = ac\sin B.$$

3. So, note that finally, we have that

$$\text{Area} = bc\sin A = ab\sin C = ac\sin B.$$

Let us divide through as follows by abc:

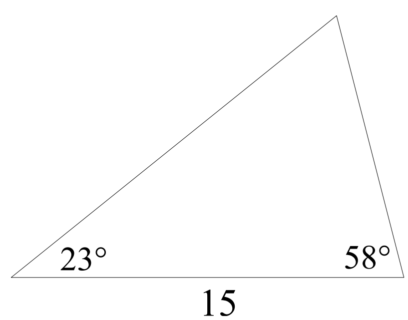
$$\frac{bc\sin A}{abc} = \frac{ab\sin C}{abc} = \frac{ac\sin B}{abc},$$

which gives us

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c},$$

which is the Law of Sines.

4. **Example.** Use the Law of Sines to solve the following triangle:



Answer. Note that the final angle has measure 99°. So we can set up the following proportion:

$$\frac{\sin(99^\circ)}{15} = \frac{\sin(23^\circ)}{b} = \frac{\sin(58^\circ)}{c}.$$

So we can see that

$$\frac{\sin(99^\circ)}{15} = \frac{\sin(23^\circ)}{b}, \quad b = \frac{\sin(23^\circ)}{\sin(99^\circ)} \cdot 15 \approx 5.93,$$

and

$$\frac{\sin(99^\circ)}{15} = \frac{\sin(58^\circ)}{c}, \quad c = \frac{\sin(58^\circ)}{\sin(99^\circ)} \cdot 15 \approx 12.88.$$

Homework
Read pages 183-185 in the book, and do the following problems:
Section 5.1: #