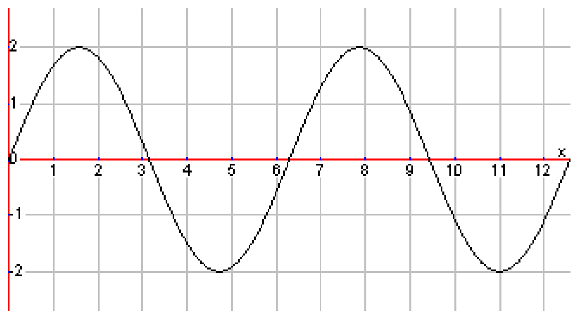
Lesson Plans - Mar. 11

Section 3.5: Stretches and Shrinks of the Graphs of Sine and Cosine

1. **Example 1.** Draw a graph of two periods of the function $y = 2\sin x$. Identify the amplitude of the graph, the average value, the range and the period.
   Answer. It's always a good start to make a table of the important values:

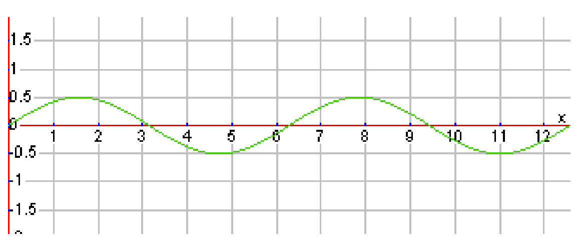
x	$\sin(x)$	$2\sin(x)$
0	0	0
$\frac{\pi}{2}$	1	2
π	0	0
$\frac{3\pi}{2}$	-1	-2
2π	0	0



   We have the following:
   Amplitude: 2
   Average value: 0
   Range: $[-2, 2]$
   Period: 2π.

2. **Example 2.** Draw a graph of two periods of the function $y = \frac{1}{2}\sin x$. Identify the amplitude of the graph, the average value, the range and the period.
   Answer. It's always a good start to make a table of the important values:

x	$\sin(x)$	$\frac{1}{2}\sin(x)$
0	0	0
$\frac{\pi}{2}$	1	$\frac{1}{2}$
π	0	0
$\frac{3\pi}{2}$	-1	$-\frac{1}{2}$
2π	0	0



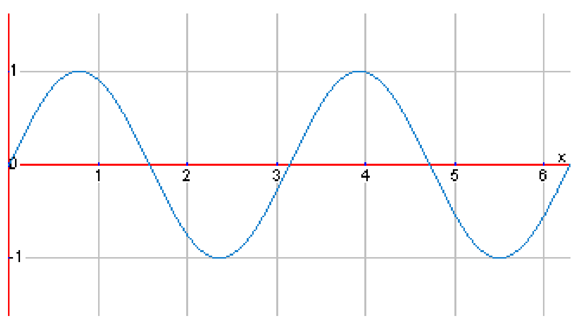
We have the following:
Amplitude: $\frac{1}{2}$
Average value: 0
Range: $[-\frac{1}{2}, \frac{1}{2}]$
Period: 2π.

3. **Example 3.** Draw a graph of two periods of the function $y = \sin(2x)$. Identify the amplitude of the graph, the average value, the range and the period.
Answer. Again, we can make a table. However, we can see that we no longer can see the high

x	$2x$	$\sin(2x)$
0	0	0
$\frac{\pi}{2}$	π	0
π	2π	0
$\frac{3\pi}{2}$	3π	0
2π	4π	0

or low points of the graph. So let us include a few more points in between the ones we have to help us graph this function:

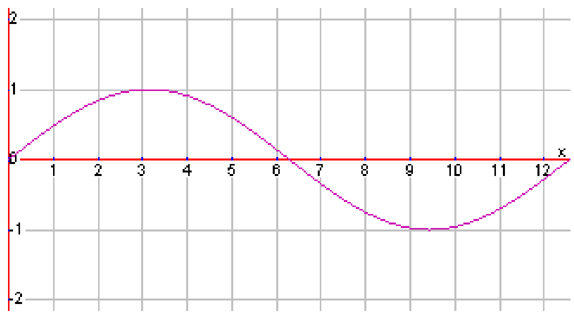
x	$2x$	$\sin(2x)$
0	0	0
$\frac{\pi}{4}$	$\frac{\pi}{2}$	1
$\frac{\pi}{2}$	π	0
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	-1
π	2π	0
$\frac{5\pi}{4}$	$\frac{5\pi}{2}i$	1
$\frac{3\pi}{2}$	3π	0
$\frac{7\pi}{4}$	$\frac{7\pi}{2}$	-1
2π	4π	0



We have the following:
Amplitude: 1
Average value: 0
Range: $[-1, 1]$
Period: π.

4. **Example 4.** Draw a graph of two periods of the function $y = \sin(\frac{x}{2})$. Identify the amplitude of the graph, the average value, the range and the period.
Answer. Again, we can make a table. However, now we will not have enough values, so we need to add more points.

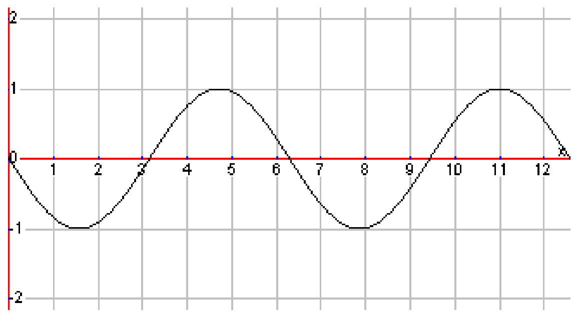
x	$\frac{x}{2}$	$\sin(\frac{x}{2})$
0	0	0
$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
π	$\frac{\pi}{2}$	1
$\frac{3\pi}{2}$	$\frac{3\pi}{4}i$	$\frac{\sqrt{2}}{2}$
2π	π	0
$\frac{5\pi}{2}$	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$
3π	$\frac{3\pi}{2}$	-1
$\frac{7\pi}{2}$	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$
4π	2π	0



We have the following:
Amplitude: 1
Average value: 0
Range: $[-1, 1]$
Period: 4π.

5. **Example 5.** Draw a graph of two periods of the function $y = -\sin(x)$. Identify the amplitude of the graph, the average value, the range and the period.
Answer. This time our table is simple:

x	$\sin(x)$	$-\sin(x)$
0	0	0
$\frac{\pi}{2}$	1	-1
π	0	0
$\frac{3\pi}{2}$	-1	1
2π	0	0



We have the following:
Amplitude: 1
Average value: 0
Range: $[-1, 1]$
Period: 2π.

6. **Summary.** We can see that we have the following for a parent graph $y = f(x)$:

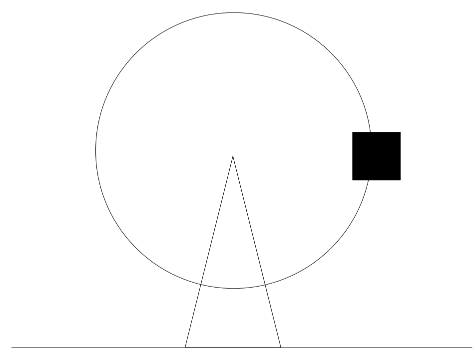
   Vertical expansions and compressions:
   Form: $y = af(x)$, when $a > 1$ this is an expansion, and when $0 < a < 1$ this is a compression
   Changes amplitude, range, max/min values.

   Vertical expansions and compressions:
   Form: $y = f(bx)$, when $0 < b < 1$ this is an expansion, and when $b > 1$ this is a compression
   Changes period. For an original period of 2π, the new period is $\frac{2\pi}{b}$.

7. **Examples.**

   (a) Graph $y = -3\sin(4x)$.

   (b) A toy Ferris wheel has radius 1 foot, and it center is 2 feet above the floor. The wheel rotates counterclockwise. Assume the basket is initially in standard position.



   i. Say the Ferris wheel rotates one radian. How far along the circumference of the Ferris wheel does the basket travel? How high off the ground is the basket? How far to the right or left of center is the basket?
   Answer. 1 foot. Height $= \sin(1) + 2 = 2.84$ ft. Right of center by $\cos(1) = 0.54$ ft.

   ii. What if the basket rotates 4 radians from its starting position?
   Answer. 4 ft. Height $= \sin(4) + 2 = 2.24$ ft. Left of center by $\cos(4) = 0.65$ ft.

   iii. How much will the basket rotate to get to the highest point in the Ferris wheel? How high off the ground will it be?
   Answer. It will rotate $\frac{\pi}{2}$ radians, and it will be 3 feet off the ground.

   iv. How much will the basket rotate to get to the lowest point in the Ferris wheel? How high off the ground will it be?
   Answer. It will rotate $\frac{3\pi}{2}$ radians, and it will be 1 foot off the ground.

Homework
Read pages 173-177 in the book, and do the following problems:
Section 3.5: # 1bdef, 2ad, 3, 4bcd, 6, 7, 8, 9, 10