Name:

Section 3.2
The Unit Circle

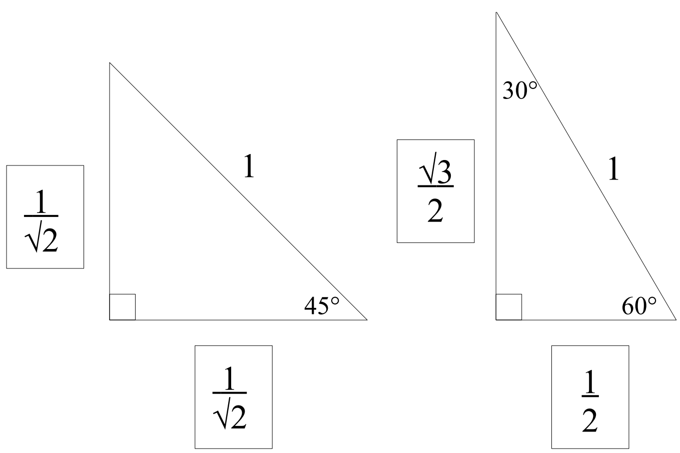
Definition. A **unit circle** is a circle which has a radius of 1 unit. In the coordinate plane, a unit circle centered at the origin has the equation

$$h^2 + v^2 = 1,$$

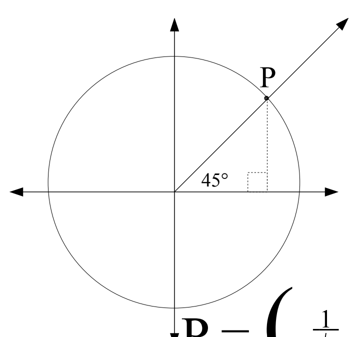
where h is the horizontal coordinate and v is the vertical coordinate. All points (h, v) that lie on the unit circle will satisfy the equation above.

1. Do the following points lie on the unit circle?
   (a) (0.8, -0.6)
   This point lies on the unit circle if it satisfies the equation $h^2 + v^2 = 1$, which we can see it does: $(0.8)^2 + (-0.6)^2 = 0.64 + 0.36 = 1$.
   (b) (-6, 8)
   We can see that this point does not satisfy $h^2 + v^2 = 1$: $(-6)^2 + 8^2 = 36 + 64 = 100 \neq 1$, so this point does not lie on the unit circle.

2. Fill out the remaining side lengths of the following special triangles.

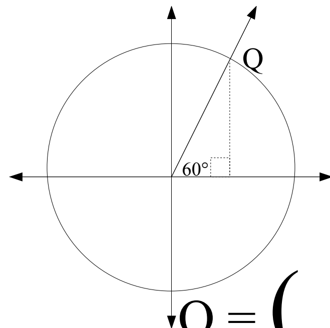


3. Each of these circles has radius 1. Use the special triangles above to find the coordinates of the following points:
   (a) Point P:



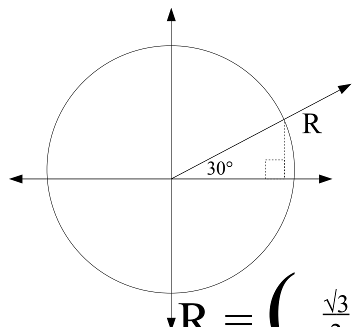
$P = \left(\frac{1}{\sqrt{2}} , \frac{1}{\sqrt{2}} \right)$

(b) Point Q:



$Q = \left(\frac{1}{2} , \frac{\sqrt{3}}{2} \right)$

(c) Point R:



$R = \left(\frac{\sqrt{3}}{2} , \frac{1}{2} \right)$

(d) What do you notice about the relationship between the trig functions of these angles and the coordinates of the points on the unit circle?

If the point is on the intersection of the unit circle and the terminal side of an angle X, the coordinates of the point are $(h, v) = (\cos X, \sin X)$. For example, the point Q is on the terminal side of a $60°$ angle, so its coordinates are $(\cos 60°, \sin 60°) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

4. What is the circumference of a unit circle? $\underline{\quad 2\pi \quad}$

5. Find the lengths of the arcs cut by the following angles (whose measures are given in radians) in a unit circle.

(a) $\angle A = 4.5$

Since arclength $S = Ar$, and $A = 4.5$ and $r = 1$, then $S = (4.5)(1) = 4.5$.
So the arclength of the arc cut is 4.5 units (not radians! Radians measure angles, not lengths).

(b) $\angle B = 3.14$

Similarly to part (a), $S = Ar$, so $S = (3.14)(1) = 3.14$ units.

(c) $\angle C = \frac{\pi}{16}$

Again, $S = Ar$, so $S = \left(\frac{\pi}{16}\right)(1) = \frac{\pi}{16}$ units.

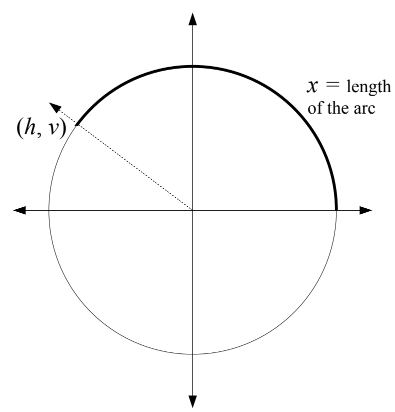
6. What do you notice about the relationship between the angles in a unit circle and the lengths of the arcs cut by them?

   We already know that the length S of an arc cut by central angle A (in radians) in a circle of radius r is $S = Ar$. But a unit circle has radius 1, so

$$S = Ar = A(1) = A,$$

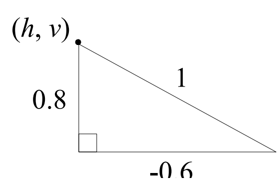
   so $S = A$ in a unit circle. Note that the units are not the same! A is in radians, but S will be in a unit of length, like inches or centimeters.

7. Note that your answer to the previous question means that we can talk about the trig functions of arc lengths on the unit circle, since they correspond to the angle measures in radians. Consider the following picture:



   (a) If $(h, v) = (-0.6, 0.8)$, find $\sin x$, $\cos x$ and $\tan x$.

   Since this is a unit circle, we know that its radius is 1. Also, we know that $S = A$, that is, the arclength x is also equal to the central angle that cuts that arc. So we can find the trig functions of the angle which has measure x, because its terminal side contains the point $(h, v) = (-0.6, 0.8)$. In fact, we can draw a right triangle:



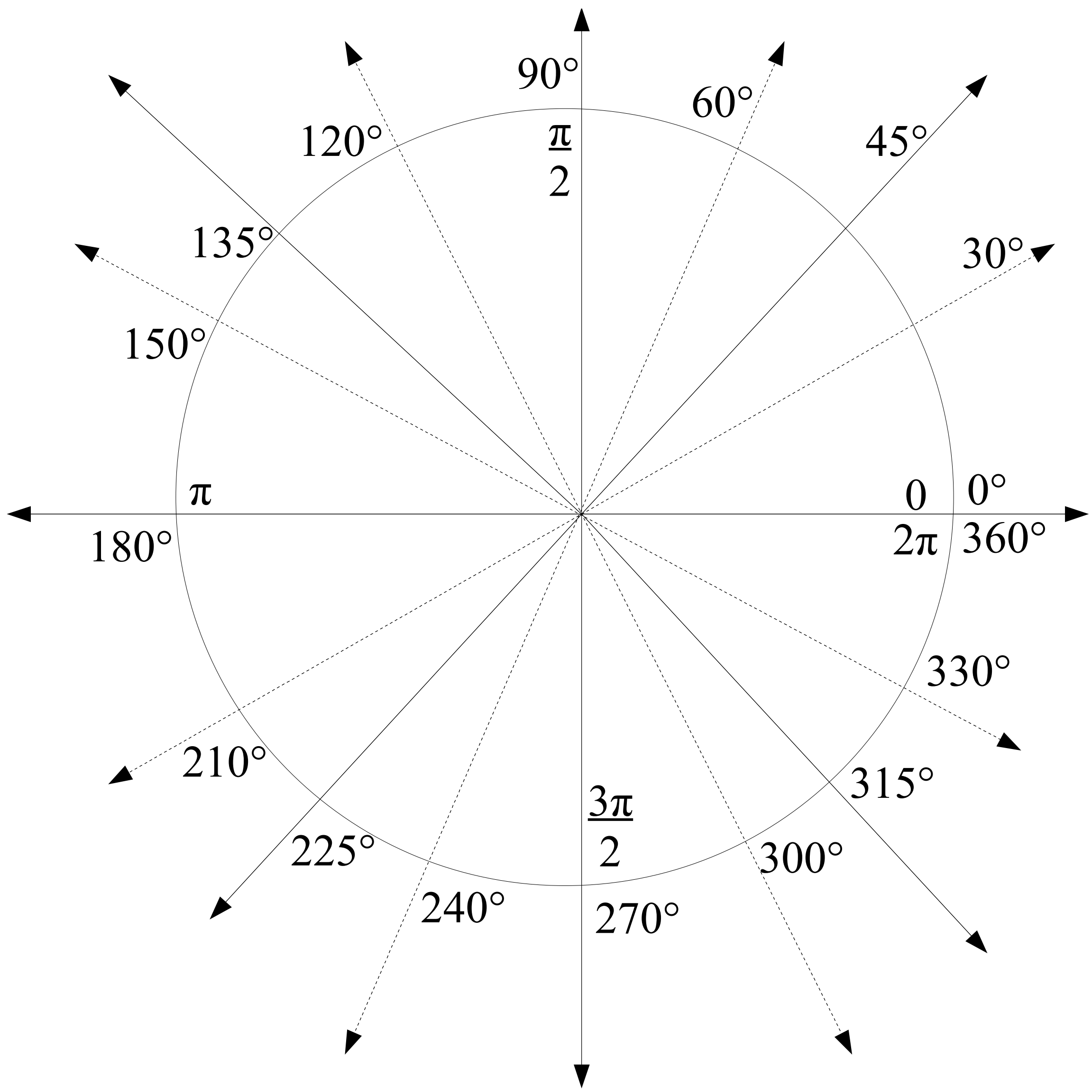
   So using what we know, we can see that:

$$\sin x = 0.8, \quad \cos x = -0.6, \quad \tan x = \frac{0.8}{-0.6} = -\frac{4}{3}.$$

   (b) Suppose $x = 2.5$. Find the coordinates of the point (h, v).

   Since $x = 2.5$, we know that the central angle that cuts it has measure 2.5 radians. We also know that the coordinates of the point $(h, v) = (\cos 2.5, \sin 2.5)$, but these must be calculated in **radian** mode! So $(h, v) = (-0.801, 0.598)$.

8. There are angle measures in degrees shown on the unit circle below. Fill out angle measures in radians. A few have been done for you.



9. Based on what you've learned so far, find the following exactly:

(a) $\sin\frac{\pi}{2} = 1$

(b) $\cos\pi = -1$

(c) $\cos\frac{\pi}{2} = 0$

Notes

- Starting from this homework, we will do all our work in radians.
- Instead of referring to coordinates as x and y we will call them h (horizontal) and v (vertical).
- For a point on the unit circle, given its angle A in standard position, we know that the coordinates of that point are:

  $$(h, v) = (\cos A, \sin A).$$

  What about a point on a circle of radius 3? Note that in general we have

  $$(h, v) = (r \cos A, r \sin A)$$

  as the coordinates of a point on a circle.