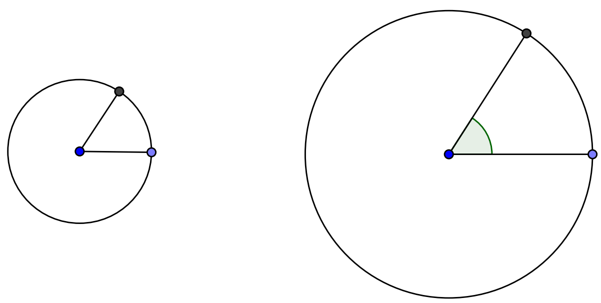
# Lesson Plans - Feb. 25

**Housekeeping**

- New Homework Sheets (3.1, should be 3adfh, not 2adfh)
- WAY TO NOT REVIEW FOR THE TEST

**Section 3.1: Radian Measure**

1. **Definition of a Radian.** Consider a **unit circle**, that is, a circle with radius 1 unit. What is the circumference of this circle? $2\pi$ units.



Now imagine picking up the radius and placing it around part of the circle. That is, we have defined an arc which has an arclength that is the same as the radius. We can see that this defines a central angle. Can you figure out what this angle is? Note that the arc we've defined is a certain fraction of the circumference. In fact, it is

$$\frac{\text{radius}}{\text{circumference}} = \frac{1}{2\pi},$$

This should correspond to a similar fraction of the full rotation, $360°$. So,

$$\frac{\text{radius}}{\text{circumference}} = \frac{\text{central angle}}{\text{full rotation}}; \quad \frac{x}{360°} = \frac{1}{2\pi}, \text{ so } x = \left(\frac{360}{2\pi}\right)^\circ \approx 57.30°.$$

Let us do the same with a circle of radius 2 units. We find that we get that the central angle is the same: $57.30°$. Do you think this is true for a circle of arbitrary radius? Let the radius of a circle be $R$. The circumference is $2\pi R$, so we get the following:

$$\frac{R}{2\pi R} = \frac{x}{360°}; \quad \frac{1}{2\pi} = \frac{x}{360°}; \quad x = \left(\frac{360}{2\pi}\right)^\circ \approx 57.30°.$$

This angle stays the same no matter what the length of the radius. It becomes useful to define this very natural unit of measurement for an angle: we call it a **radian**. How many radians make up one full rotation? $2\pi$ radians. Another way to think of radians is as the number of radii you can fit into a certain arc. For example, if you have a circle with radius 4 cm and we consider the central angle that cuts an arc of length $10cm$, that angle has measure 2.5 radians.

2. **Examples.**

   (a) A central angle $A$ cuts an arc with length 5.67 cm. The radius of the circle is 3.1 cm. Find the measure of angle $A$ in radians.
   *Answer.* Circumference of the circle: $6.2\pi$ cm. So we can set up the following:

   $$\frac{5.67cm}{6.2\pi cm} = \frac{A}{2\pi}; \quad A = 2\pi \cdot \frac{5.67cm}{6.2\pi cm} = \frac{5.67}{3.1} \approx 1.83 \text{ (radians)}.$$

   (b) A central angle of $144°$ is in a circle with radius 2.5 feet. Find the length of the arc cut by the angle.
   *Answer.* Again, we know that the circumference of the circle is $5\pi$. So:

   $$\frac{144°}{360°} = \frac{\text{arc}}{5\pi}; \quad \text{arc} = 5\pi \frac{144°}{360°} = 0.4(5\pi) = 2\pi \text{ feet.}$$

3. We can calculate the **arc length** of an arc along a circle. Note that earlier we set up

   $$\frac{\text{radius}}{\text{circumference}} = \frac{\text{central angle} = 1 \text{ radian}}{\text{full rotation}},$$

   where the central angle in fact had the measure of 1 radian. What about for an angle that is not 1 radian? It should make sense that

   $$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{full rotation}}.$$

   So any time you need to calculate the arc length of an arc cut by a central angle, you can set up this proportion.

4. A **sector** of a circle is the part of the circle bounded by a central angle and the arc it cuts.

5. **Examples.**

   (a) Consider a circle with radius 4.5 in. Find the area of the sector formed by a central angle of measure 3 radians.
   *Answer.* Consider the area of the circle:

   $$\text{Area of the circle } = \pi r^2 = \pi(4.5)^2 = 20.25\pi \text{ in}^2.$$

   The area of the sector is the same fraction of the area of the whole circle as the central angle is of the total rotation. Since we've measured the central angle in radians,

   $$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{central angle}}{\text{total rotation}},$$

   so

   $$\frac{\text{area of sector}}{20.25\pi} = \frac{3}{2\pi}; \quad \text{area of sector} = \frac{3}{2\pi} \cdot 20.25\pi = 30.375 \text{ in}^2.$$

   (b) A circle of radius 8.3 cm has a sector with area 10 cm$^2$. Find the measure of the central angle that forms this sector in radians first, then in degrees. *Answer.* First, we can see that the area of the circle is $\pi r^2 = \pi(8.3)^2 = 68.89\pi$ cm$^2$. Again, we can set up a proportion:

   $$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{central angle}}{\text{total rotation}}; \quad \frac{10}{68.89\pi} = \frac{\text{central angle}}{2\pi}$$

so central angle $= 20/68.89 \approx 0.29$ (radians). We can proceed in two ways to figure out the measure in degrees. We could convert from radians to degrees (which we will talk about later), or we can set up the proportion differently:

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{central angle}}{\text{total rotation}}; \quad \frac{10}{68.89\pi} = \frac{\text{central angle}}{360^\circ}.$$

This shows us that the central angle is approximately $16.6^\circ$.

6. **Converting between radian measure and degrees.** What is the measure of $90^\circ$ in radians? We know that $90^\circ$ is one-quarter of the total rotation, so

$$\frac{1}{4} \cdot 2\pi = \frac{\pi}{2}.$$

However, we can see the following:

$$\frac{90^\circ}{360^\circ} \cdot 2\pi = \frac{\pi}{2}.$$

What is the measure of $\frac{\pi}{6}$ radians in degrees? We know that this is some fraction of the total rotation in radians ($2\pi$), and that the degree measure will be the same fraction of $360^\circ$, so we can write

$$\frac{\pi/6}{2\pi} = \frac{x}{360^\circ}, \quad x = \frac{\pi/6}{2\pi} \cdot 360^\circ.$$

So we can see that $x = 360/12^\circ = 30^\circ$. Note that in the first conversion, we basically multiplied by $2\pi/360^\circ$, while in the second conversion, we multiplied by $360^\circ/2\pi$. These simplify to $\pi/180^\circ$ and $180^\circ/\pi$.

**Homework**

Read pages 127-132 in the book, and do the following problems:
Section 3.1: #1W, 2abce, 3adfh, 4abf, 5bgh, 6, 9a, 10, 11, 12