Lesson Plans - Feb. 2

Housekeeping

- Test Next Tuesday (can people stay until 9 am?)
- Yellowish Handout
- Midterm Review
- Homework collected on Thursday will be corrected and placed outside my office

Section 2.5: More Identities

1. **Cofunction Identity revisited**. We know that for an acute angle X, we have

   $$\sin X = \cos(90^\circ - X).$$

   We can extend this identity beyond just acute angles. Fill in the following table:

X	$\sin X$	$\cos(90^\circ - X)$
-90	-1	-1
-50	-0.77	-0.77
0	0	0
90	1	1
100	0.98	0.98
150	0.5	0.5
200	-0.34	-0.34
250	-0.94	-0.94
300	-0.87	-0.87

   We can see that this identity will be true for all angles X (not just acute angles). Another way to check if this is true to graph the following graphs:

   $$\begin{aligned} Y1 &= \sin X \\ Y2 &= \cos(90 - X). \end{aligned}$$

   You can only see one graph because they are in fact both graphing the same function. So,

   $$\sin X = \cos(90^\circ - X), \text{ for all angles } X.$$

   Keep in mind that this is not a proof, but merely an illustration of the point!

2. Example: If $\sin(3A) = \cos(4A + 20^\circ)$, solve for A.
   Answer. Consider that $\sin X = \cos(90^\circ - X)$. We can use substitution to say

   $$X = 3A \qquad \text{and} \qquad 4A + 20^\circ = 90^\circ - X.$$

   This is a system of two equations in two variables, so we can solve this algebraically:

   $$\begin{aligned} 4A + 20^\circ &= 90^\circ - X \\ 4A + 20^\circ &= 90^\circ - 3A \\ 7A &= 70^\circ \\ A &= 10^\circ. \end{aligned}$$

3. **Sine of a sum or difference**. Now let us similarly see that another identity works out: Graph the following functions:

$$\begin{aligned} Y1 &= \sin(30+X) \\ Y2 &= \sin(30)\cos(X)+\cos(30)\sin(X). \end{aligned}$$

Again we see they both produce the same graph. What do you think is true then?

$$\sin(30+X)=\sin(30)\cos(X)+\cos(30)\sin(X).$$

Now, let us try to graph the following with another number in for 30 (pick any angle).

$$\begin{aligned} Y1 &= \sin(93+X) \\ Y2 &= \sin(93)\cos(X)+\cos(93)\sin(X). \end{aligned}$$

Does this suggest that a generalization is true? In fact, we have the following identity: for any angles A and B,

$$\sin(A+B)=(\sin A)(\cos B)+(\cos A)(\sin B).$$

Also, we have a similar identity for the sine of the difference of two angles: or any angles A and B,

$$\sin(A-B)=(\sin A)(\cos B)-(\cos A)(\sin B).$$

4. **Cosine of a sum or difference**. Also, we have the following identites for cosine:

$$\cos(A+B)=(\cos A)(\cos B)-(\sin A)(\sin B)$$

and

$$\cos(A-B)=(\cos A)(\cos B)+(\sin A)(\sin B).$$

5. Examples.

   (a) Find an exact value for $\sin(15^\circ)$.
   Answer. We know that $45^\circ - 30^\circ = 15^\circ$. So,

$$\sin(45^\circ-30^\circ)=(\sin 45^\circ)(\cos 30^\circ)-(\cos 45^\circ)(\sin 30^\circ)=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)=\frac{\sqrt{3}-1}{2\sqrt{2}}.$$

   (b) Find an exact value for $\cos(75^\circ)$.
   Answer. We know that $45^\circ + 30^\circ = 75^\circ$. So,

$$\cos(45^\circ+30^\circ)=(\cos 45^\circ)(\cos 30^\circ)-(\sin 45^\circ)(\sin 30^\circ)=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)=\frac{\sqrt{3}-1}{2\sqrt{2}}.$$

6. **Double-angle identities**. What can you say about $\sin(2A)$? Remember that $\sin(2A) = \sin(A + A)$.

$$\sin(2A)=\sin(A+A)=\sin A\cos A+\cos A\sin A=2\sin A\cos A.$$

Similarly,

$$\begin{aligned} \cos(2A)=(\cos A)(\cos A)-(\sin A)(\sin A) &= \cos^2 A-\sin^2 A \\ &= 1-2\sin^2 A \\ &= 2\cos^2 A-1. \end{aligned}$$

Homework
Read pages 95-99 and 117-12980 in the book, and do the following problems:
Section 2.5