Lesson Plans - Feb. 2

Housekeeping

- Office hours: M 2-3, W 3-4, R 9-10
- Quiz Thursday, Feb. 4 on Ch. 1
- Chapter 1 was the easiest chapter - things are going to get harder from here
- Chapter 1 Review Problems: http://math.arizona.edu/ trig/

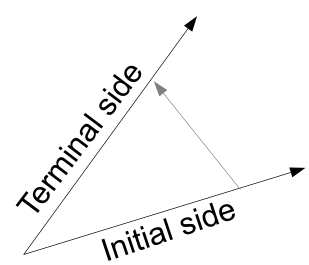
Review Problems

1.

2.

Section 2.1: Angles in Standard Position

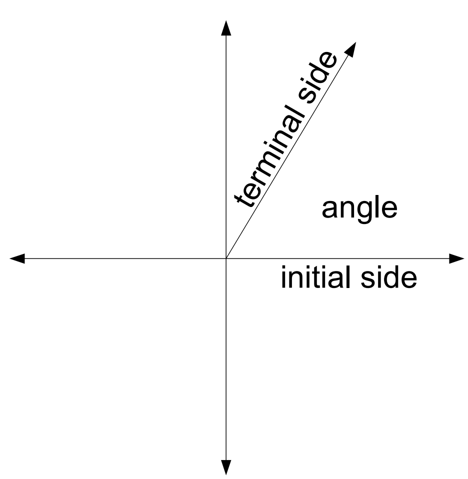
1. Def: *angle*. Amount of rotation in one rotational direction (that is, clockwise or counterclockwise). An angle starts at one ray and *rotates* to the other ray.



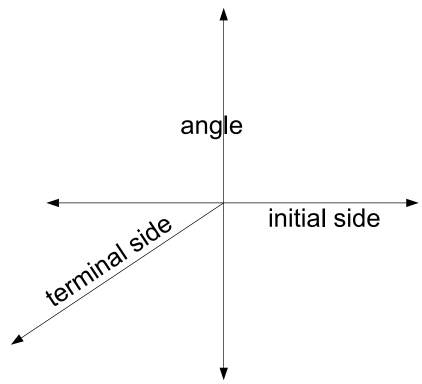
If we drew an angle in the xy-plane (the *Cartesian* plane), we say that it is in *standard position* if

- the angle's vertex is at the origin $(0, 0)$,
- and it's initial side is along the positive horizontal axis.

For example, the following angle is in standard position:

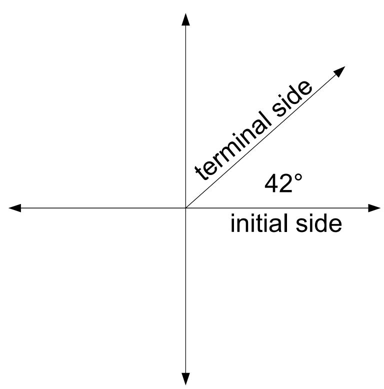


So is this next angle:

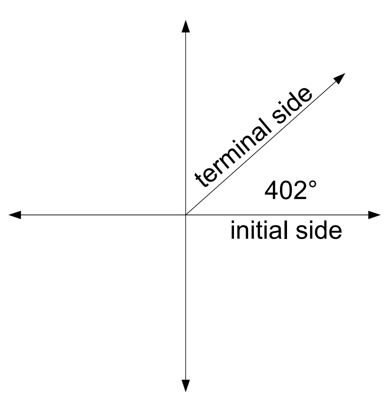


Notice that this angle is not onlt not an acute angle, it has measure greater than 180°. We have 360° of rotation before we begin to retrace our steps.

2. *Coterminal angles*: So, let us look at an angle with measure 42°. We would draw it in standard position like this:

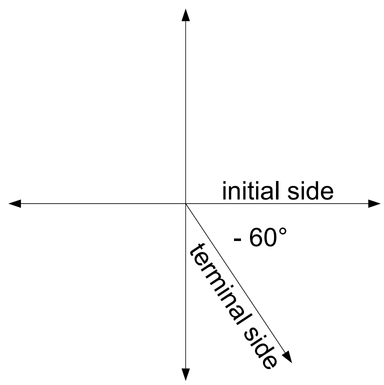


How would we draw an angle with measure 402°?

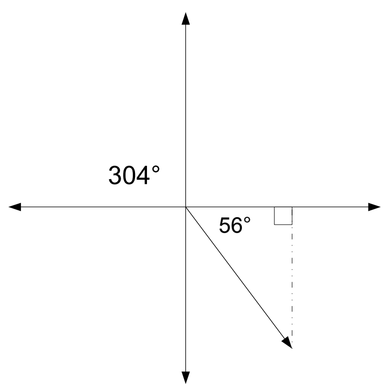


Note that both angles have their terminal sides in the same place, that is, 42° counterclockwise from the positive horizontal axis. Since their initial and terminal sides match up, we call these angles *coterminal* angles. By how much do coterminal angles differ?

3. *Negative angles*: Note that we can rotate in two directions: clockwise or counterclockwise. Rotating counterclockwise means rotating up, so these are positive angles. Rotating clockwise is rotating down, so angles measured this way are negative.



4. *Reference angles.* Eventually we want to talk about the trig functions of angles in standard position, but so far we only know how to talk about these functions in terms of right triangles. Consider the angle 304°. It would terminate in Quadrant IV.

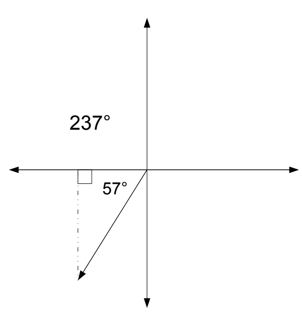


We can draw a perpendicular line from the terminal side to the horizontal axis, forming a right triangle. The angle in the triangle with its vertex on the origin $(0, 0)$ measures 56°. We call this angle the *reference angle* of 304°.

5. Examples:

   (a) What is a positive angle which is coterminal to $-53°$?
   Answer. $-53° + 360° = 307°$.

   (b) Find the reference angle of 237°.
   Answer. If we sketch this angle, we can see that it terminates in Quadrant III.

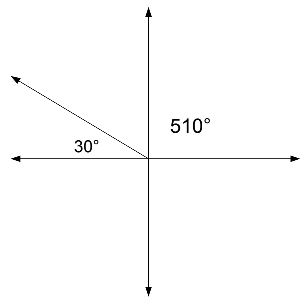


So the angle between the horizontal axis and the terminal side of the angle is

$$237^\circ - 180^\circ = 57^\circ.$$

So the reference angle is 57°.

(c) Find the reference angle of 510°.
Answer.

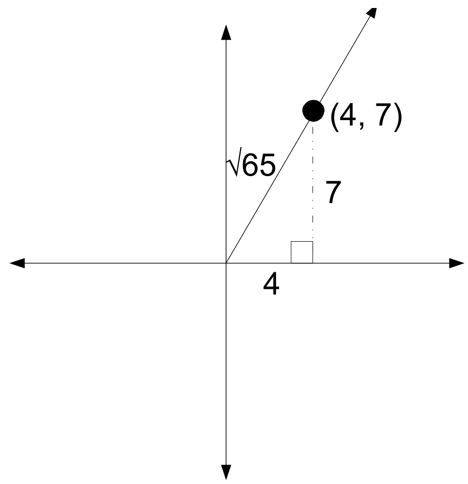


It has reference angle 30°.

Homework
Read pages 69-73 in the book, and do the following problems:
Section 2.1: #1acfg, 2ac, 3, 4abceg, 6, 7, 9, 10W, 12, 13

Section 2.2: The Sine Function for Angles in Standard Position

1. Suppose $\angle A$ is in standard position, and its terminal side contains the point $(4, 7)$.



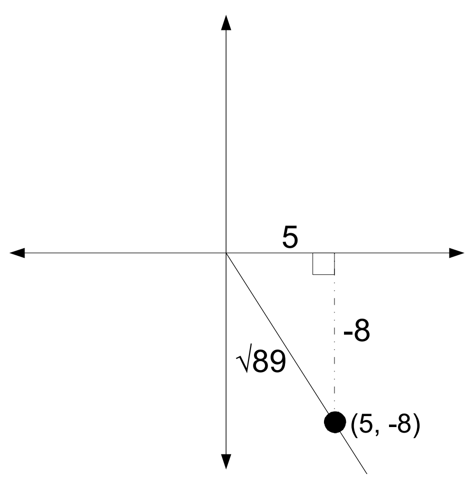
We can draw a perpendicular line and form a right triangle. We can see that from this information we can calculate the sine of $\angle A$:

$$tandar \sin A = \frac{7}{\sqrt{65}}.$$

So, given $\angle A$ in standard position and a point on the terminal side of $\angle A$, we can define

$$\sin A = \frac{\text{vertical coordinate of the point}}{\text{distance between the point and the origin}} = \frac{y}{d}, \text{ where } d = \sqrt{x^2 + y^2}.$$

2. Now consider $\angle C$, where $(5, -8)$ is a point on the terminal side of $\angle C$. We can draw the angle:

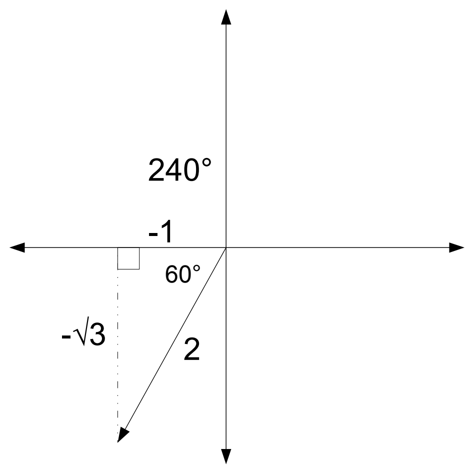


Note that we can talk about the sine of the angle in the triangle that has its vertex on the origin, but let us consider the opposite side to have length -8, instead of positive 8. So,

$$\sin(\text{angle}) = \frac{\text{vertical coordinate}}{\text{distance between point and origin}} = \frac{-8}{\sqrt{89}}.$$

So we can see that the sine of this angle is negative. In what quadrants is sine going to be negative?

3. Find the exact value of $\sin 240°$.
   Note that the reference angle of $\sin 240°$ is $60°$. So we can sketch our standard 30-60-90 triangle onto the angle. However, notice that we are going in a negative direction both horizontally and vertically.



   So,
$$\sin 240° = -\frac{\sqrt{3}}{2}.$$

4. If two angles A and B are coterminal, then $\sin A = \sin B$. Is the converse true? Consider $\sin 45°$ and $\sin 135°$.