Lesson Plans - Jan. 19

Housekeeping

- Email address
- Office hours
- Grade breakdown (Homework: 15%, 2 Tests, each worth: 25%, Final Exam: 35%)
- Homework assignment sheet
- History of trig

Section 1.2: Similar Triangles

1. Proportion

   (a) Definition: A *proportion* is an equality between two ratios. For example,

   $$\frac{2}{3} = \frac{6}{9}.$$

   (b) Solve:

   $$\frac{4}{7} = \frac{x}{140}.$$

   There are two ways to solve this. First, we can realize that $140 = 7 \times 20$, to see that if we multiply both the top and the bottom by the same number, we get

   $$\frac{4}{7} \times \frac{20}{20} = \frac{80}{140}.$$

   So, this tells us that $x = 80$. Or, we could do this algebraically.

   $$\begin{aligned} (140)\frac{4}{7} &= \frac{x}{140}(140) \\ \frac{560}{7} &= x \\ 80 &= x. \end{aligned}$$

   Note that both methods work, but the second is more reliable.

   (c) Solve:

   $$\frac{y+2}{12} = \frac{17}{48}.$$

   $$\begin{aligned} \frac{y+2}{12} &= \frac{17}{48} \\ (48)(y+2) &= 12 \times 17 \\ 48y + 96 &= 204 \\ 48y &= 108 \\ y &= \frac{108}{48} = 2.25. \end{aligned}$$

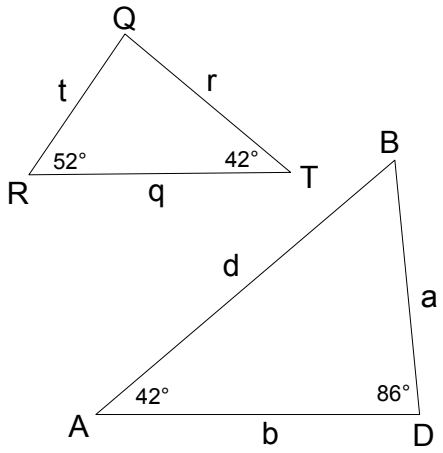
2. Similarity

(a) Definition: Two shapes are *similar* if they have the same shape.

(b) Definition: Two shapes are *congruent* if they have the same shape AND size.

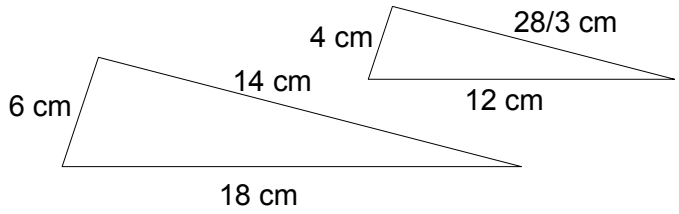
(c) How can you tell if two triangles are similar? What needs to be true if they are congruent? Two triangles are similar if they have all the same angles, OR if their corresponding sides are proportional. They are congruent if they have all the same side lengths.

(d) Consider ΔRQT and ΔABD. They are similar (how can you prove that this is true?).



Note that we label a side of a triangle with the lowercase letter of the angle opposite it. The corresponding sides are d and q, a and t, and r and b. We write this as $\Delta TRQ \sim \Delta ABD$ - with the congruent angles in the same order.

(e) Similar shapes are just scaled-up or scaled-down versions of each other. So, if you multiply the sides of ΔTRQ by the same number, called a *scaling factor*, you should get the sides of ΔABD.
For example, let us prove that the following triangles are similar.



This means that all the corresponding sides should be in the same proportion (or, that they all have the same scaling factor). Clearly, the longest sides should correspond with each other, as should the shortest. So, we have the following ratios:

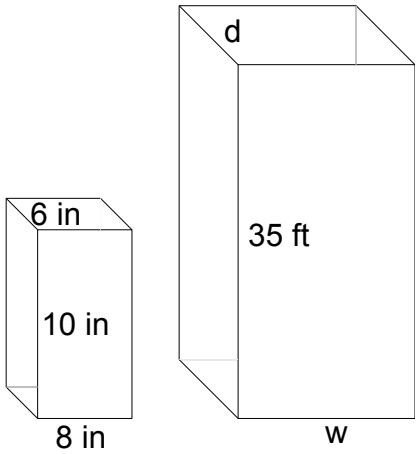
$$\begin{aligned} \frac{4}{6} &= \frac{2}{3} \\ \frac{12}{18} &= \frac{2}{3} \\ \frac{28/3}{14} &= \frac{2}{3}, \end{aligned}$$

which implies that the corresponding sides are proportional.

$$\frac{4}{6} = \frac{12}{18} = \frac{28/3}{14}.$$

3. Examples:

   (a) You are making a scale model of a box-shaped building. Your model is 10 inches high, 8 inches wide, and 6 inches deep. If the actual building is going to be 35 feet high, how wide and how deep is the actual building going to be?
   Answer. A scale model is similar to the original building, which means all the dimensions should be in proportion.



   So we have that

$$\frac{\text{height of model}}{\text{height of building}} = \frac{\text{width of model}}{\text{width of building}} = \frac{\text{depth of model}}{\text{depth of building}}.$$

   Since the height of the building is 35 feet and the height of the model is 10 inches, we can see that

$$\frac{10 \text{ inches}}{35 \text{ feet}} = \frac{\text{width of model}}{\text{width of building}}.$$

   We also know that the model is 8 inches wide, so we can solve for the width of the building.

$$\begin{aligned}
\frac{10 \text{ inches}}{35 \text{ feet}} &= \frac{8 \text{ inches}}{\text{width of building}} \\
\frac{10 \text{ inches}}{35 \text{ feet}} &= \frac{8 \text{ inches}}{w} \\
10w \text{ inches} &= 35 \times 8 \text{ (feet } \times \text{ inches)} \\
10w \text{ inches} &= 280 \text{ (feet } \times \text{ inches)} \\
w &= \frac{280 \text{ (feet } \times \text{ inches)}}{10 \text{ inches}} \\
w &= 28 \text{ feet.}
\end{aligned}$$

   So the building is **28 feet wide**. We can do a similar thing to find the depth of the building, since we know that

$$\frac{\text{height of model}}{\text{height of building}} = \frac{\text{depth of model}}{\text{depth of building}},$$

   so,

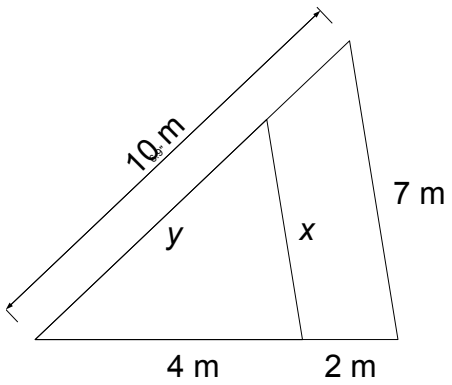
$$\frac{10 \text{ inches}}{35 \text{ feet}} = \frac{d \text{ inches}}{\text{depth of building}}$$

$$\begin{aligned}
\frac{10 \text{ inches}}{35 \text{ feet}} &= \frac{6 \text{ inches}}{d} \\
10d \text{ inches} &= 35 \times 6 \text{ (feet } \times \text{ inches)} \\
10d \text{ inches} &= 210 \text{ (feet } \times \text{ inches)} \\
d &= \frac{210 \text{ (feet } \times \text{ inches)}}{10 \text{ inches}} \\
d &= 21 \text{ feet.}
\end{aligned}$$

So the **depth** of the building is **21 feet**.

(b) (Writing - think individually, compose answers in pairs - write up answers on the board.) ΔABC is a right triangle with angle A having measure $18°$. Right triangle ΔDEF has right angle F and angle E has measure $72°$. Are these two triangles similar? Prove your answer.
Answer.

(c) A landscaper has a triangular plot of grass that she would like to expand outward, but she would like the plot to remain the same shape. She is planning to extend a side of the plot that is 4 meters long by another 2 meters. If the sides of the plot are 6 meters, 7 meters and 10 meters after the expansion, what were the original lengths of the sides of the plot?
Answer. It's helpful to draw a picture of the situation.



From the picture, we can see that the 4 m side of the smaller plot corresponds to the 6 m side of the larger plot. So we can set up the proportion like this:

$$\frac{4 \text{ m}}{6 \text{ m}} = \frac{x}{7 \text{ m}} = \frac{y}{10 \text{ m}}.$$

First we can solve for x:

$$\begin{aligned}
\frac{4 \text{ m}}{6 \text{ m}} &= \frac{x}{7 \text{ m}} \\
\frac{4 \times 7 \text{ m}}{6} &= x \\
\frac{28 \text{ m}}{6} &= x \\
4.66 \text{ m} &\approx x.
\end{aligned}$$

Then we can solve for y:

$$\frac{4 \text{ m}}{6 \text{ m}} = \frac{y}{10 \text{ m}}$$

$$\begin{aligned} \frac{4 \times 10 \text{ m}}{6} &= y \\ \frac{40 \text{ m}}{6} &= y \\ 6.66 \text{ m} &\approx y. \end{aligned}$$

Homework
Read pages 11-16 in the book, and do the following problems:
Section 1.2: #2, 3, 5, 7, 8, 15, 18, 19, 21