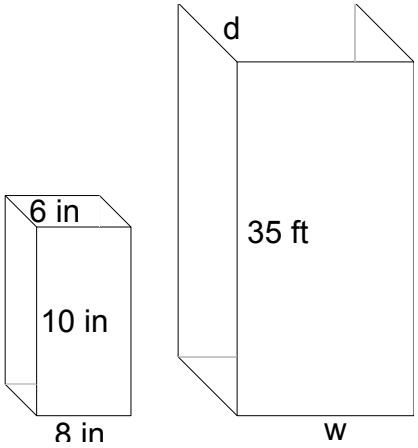
Lesson Plans - Jan. 19 and Jan. 21

Housekeeping

- Homework: stapled, separate sheet of paper, questions
- Set up appointments to talk about grades

Review

- You are making a scale model of a box-shaped building. Your model is 10 inches high, 8 inches wide, and 6 inches deep. If the actual building is going to be 35 feet high, how wide and how deep is the actual building going to be?
  Answer. A scale model is similar to the original building, which means all the dimensions should be in proportion.



So we have that

$$\frac{\text{height of model}}{\text{height of building}} = \frac{\text{width of model}}{\text{width of building}} = \frac{\text{depth of model}}{\text{depth of building}}.$$

Since the height of the building is 35 feet and the height of the model is 10 inches, we can see that

$$\frac{10 \text{ inches}}{35 \text{ feet}} = \frac{\text{width of model}}{\text{width of building}}.$$

We also know that the model is 8 inches wide, so we can solve for the width of the building.

$$\begin{aligned}
\frac{10 \text{ inches}}{35 \text{ feet}} &= \frac{8 \text{ inches}}{\text{width of building}} \\
\frac{10 \text{ inches}}{35 \text{ feet}} &= \frac{8 \text{ inches}}{w} \\
10w \text{ inches} &= 35 \times 8 \text{ (feet } \times \text{ inches)} \\
10w \text{ inches} &= 280 \text{ (feet } \times \text{ inches)} \\
w &= \frac{280 \text{ (feet } \times \text{ inches)}}{10 \text{ inches}} \\
w &= 28 \text{ feet.}
\end{aligned}$$

So the building is **28 feet wide**. We can do a similar thing to find the depth of the building, since we know that

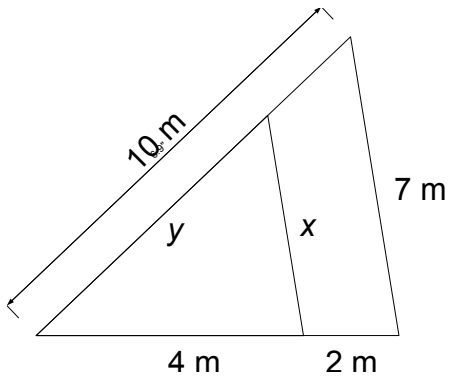
$$\frac{\text{height of model}}{\text{height of building}} = \frac{\text{depth of model}}{\text{depth of building}},$$

so,

$$\begin{aligned}
\frac{10 \text{ inches}}{35 \text{ feet}} &= \frac{d \text{ inches}}{\text{depth of building}} \\
\frac{10 \text{ inches}}{35 \text{ feet}} &= \frac{6 \text{ inches}}{d} \\
10d \text{ inches} &= 35 \times 6 \text{ (feet } \times \text{ inches)} \\
10d \text{ inches} &= 210 \text{ (feet } \times \text{ inches)} \\
d &= \frac{210 \text{ (feet } \times \text{ inches)}}{10 \text{ inches}} \\
d &= 21 \text{ feet.}
\end{aligned}$$

So the **depth** of the building is **21 feet**.

- A landscaper has a triangular plot of grass that she would like to expand outward, but she would like the plot to remain the same shape. She is planning to extend a side of the plot that is 4 meters long by another 2 meters. If the sides of the plot are 6 meters, 7 meters and 10 meters after the expansion, what were the original lengths of the sides of the plot?
Answer. It's helpful to draw a picture of the situation.



From the picture, we can see that the 4 m side of the smaller plot corresponds to the 6 m side of the larger plot. So we can set up the proportion like this:

$$\frac{4 \text{ m}}{6 \text{ m}} = \frac{x}{7 \text{ m}} = \frac{y}{10 \text{ m}}.$$

First we can solve for x:

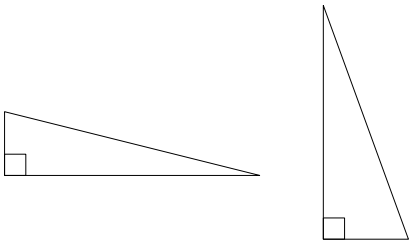
$$\begin{aligned}
\frac{4 \text{ m}}{6 \text{ m}} &= \frac{x}{7 \text{ m}} \\
\frac{4 \times 7 \text{ m}}{6} &= x \\
\frac{28 \text{ m}}{6} &= x \\
4.66 \text{ m} &\approx x.
\end{aligned}$$

Then we can solve for y:

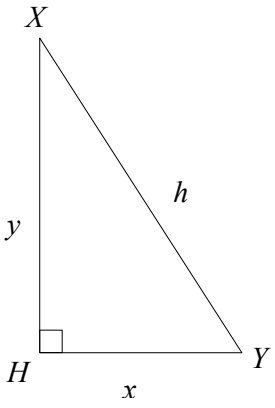
$$\begin{aligned} \frac{4 \text{ m}}{6 \text{ m}} &= \frac{y}{10 \text{ m}} \\ \frac{4 \times 10 \text{ m}}{6} &= y \\ \frac{40 \text{ m}}{6} &= y \\ 6.66 \text{ m} &\approx y. \end{aligned}$$

Section 1.3: Right Triangle Trigonometry

1. There are relationships between the acute angles in a right triangle and the lengths of the sides.



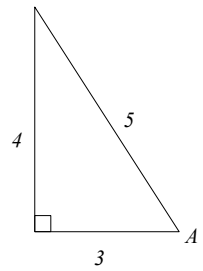
We can see that a shorter side is across from a smaller angle, and a larger side is across from a larger angle. This tells us that there should be some relationship between the length of the side and the angle across from it. We define these relationships with trigonometric functions.



$$\begin{aligned} \sin X &= \frac{\text{side opposite } \angle X}{\text{hypotenuse}} = \frac{x}{h} \\ \cos X &= \frac{\text{side adjacent to } \angle X}{\text{hypotenuse}} = \frac{y}{h} \\ \tan X &= \frac{\text{side opposite } \angle X}{\text{side adjacent to } \angle X} = \frac{x}{y}. \end{aligned}$$

2. Examples

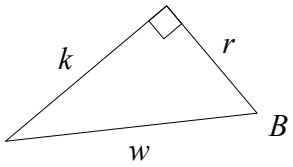
   (a) Solve for $\sin A$.



Answer.

$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{4}{5}.$$

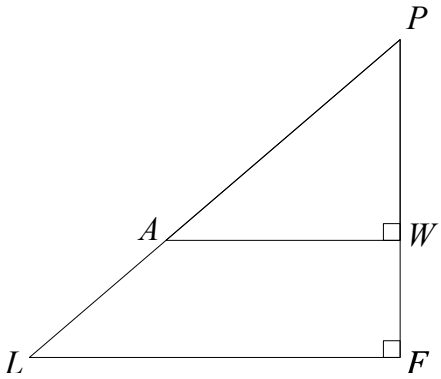
(b) Solve for $\sin B$.



Answer.

$$\sin B = \frac{\text{side opposite } \angle B}{\text{hypotenuse}} = \frac{k}{w}.$$

(c) (Writing, in groups) Use the ratios of sides and what you know about similar triangles to explain why $\sin(\angle PAW) = \sin(\angle PLF)$.



Answer. Since $\angle APW$ is in both triangles, and both triangles have another angle which is a right angle, we can say that ΔPLF and ΔPAW are similar. This means that

$$\frac{PA}{PL} = \frac{PW}{PF}.$$

We know that

$$\sin(\angle PAW) = \frac{PW}{PA}$$

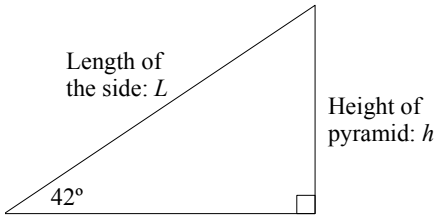
and

$$\sin(\angle PLF) = \frac{PF}{PL}.$$

So, we can say that $PF = PL\sin(\angle PLF)$ and $PW = PA\sin(\angle PAW)$. If we substitute these into the proportion above, we get

$$\begin{aligned} \frac{PA}{PL} &= \frac{PA\sin(\angle PAW)}{PL\sin(\angle PLF)} \\ 1 &= \frac{\sin(\angle PAW)}{\sin(\angle PLF)} \\ \sin(\angle PLF) &= \sin(\angle PAW). \end{aligned}$$

(d) A pyramid makes a slope of 42° with the ground, and it rises to a height of 146.5 meters. Find the length of the side of the pyramid.
Answer. We can draw a right triangle to illustrate this situation:



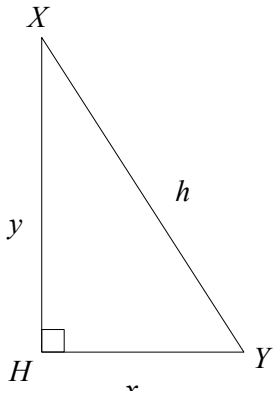
We can see that

$$\sin 42^\circ = \frac{h}{L} = \frac{146.5}{L},$$

so

$$L = \frac{146.5}{\sin 42^\circ} \approx 218.94 \text{ meters.}$$

3. There are 3 more trig functions called the Reciprocal functions. They are defined as:



$$\begin{aligned} \csc X &= \frac{1}{\sin X} = \frac{\text{hypotenuse}}{\text{side opposite } \angle X} = \frac{h}{x} \\ \sec X &= \frac{1}{\cos X} = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle X} = \frac{h}{y} \\ \cot X &= \frac{1}{\tan X} = \frac{\text{side adjacent to } \angle X}{\text{side opposite } \angle X} = \frac{y}{x}. \end{aligned}$$

Homework
Read pages 23-28 in the book, and do the following problems:
Section 1.3: #1, 2, 4, 6, 7, 10, 12, 15, 16, 17, 18, 19, 20