Test 2 Review

Concepts to Remember

Converting degree measure to radian measure
Converting radian measure to degree measure
Unit circle
Coordinates of points on the unit circle [for angle x, the point on the terminal side of the angle which intersects the unit circle has coordinates $(\cos x, \sin x)$]
Quadrants in which sine, cosine and tangent are negative
Circumference/area of a circle
Finding arc length/finding the area of a sector of a circle (using proportions)
Graphs of sine, cosine and tangent (including domain and range)
Transformations of the form $y = d + a\sin(b(x - c))$ [know what a, b, c, d represent]
Transformations of the form $y = d + a\cos(b(x - c))$
Transformations of the form $y = \tan(b(x - c))$
Negative angle identities [$\sin(-x) = -\sin(x)$, $\cos(-x) = \cos(x)$, $\tan(-x) = -\tan(x)$]
Why do we restrict the domain of $y = \sin(x)$ in order to define $y = \sin^{-1}(x)$?
What is the restricted domain of $y = sin(x)$? $y = \cos(x)$? $y = \tan(x)$?
Graphs of $y = \arcsin(x)$, $y = \arccos(x)$, $y = \arctan(x)$

Definitions

radian	central angle	arc
sector	radius	unit circle
translation	amplitude	average value
period	maximum value	minimum value
high point	low point	aymptote
$x-$intercept	odd symmetry	even symmetry
odd function	even function	inverse function
one-to-one	restricted domain	inverse sine
inverse cosine	inverse tangent	

Things to Remember from Previous Chapters

- SOH-CAH-TOA
- Trig functions of special angles (and how they relate to the unit circle)
- Properties of right triangles

Exercises

Basics

1. Sketch a central angle with the given radian measure in standard position.
   (a) -1 radians
   (b) 3.5 radians
2. Convert the following degree measures into radians **exactly**.
   (a) $45°$
   (b) $-100°$
   (c) $50°$
3. Convert the following radian measures into degrees **exactly**.
   (a) $\frac{2\pi}{3}$ radians
   (b) 2 radians
4. Find the length of the arc cut by a central angle of 4.5 radians in a circle of radius 10 cm.
5. Find the length of the arc cut by a central angle of $120°$ in a circle of radius 8 cm.
6. Find the area of the sector determined by a central angle of 3 radians in a circle of 5 inches.
7. Find the area of the sector determined by a central angle of $60°$ in a circle of 7 inches.
8. The point (h, v) lies on the unit circle. If $h = -0.6$, find **two** possible values for v.
9. Consider the interval of $x-$values: $[3.15, 4.71]$ (where the interval represents angles in radians), fill in the blanks with $<$, $>$ or $=$.
   (a) $\sin x$ _____ 0
   (b) $\cos x$ _____ 0
   (c) $\tan x$ _____ 0
10. Graph the function $y = -3 - 2\cos\left(\frac{\pi}{6}(x-2)\right)$.
11. Graph the function $y = \tan\left(\frac{\pi}{4}(x+1)\right)$.
12. Find a formula for $f^{-1}(x)$ if $f(x) = 9x - 8$.
13. Find exact values **in radians** for the following:
    (a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
    (b) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

(c) $\tan^{-1}(-1)$

14. Solve the equation for w: $2\sin(3w) - 5 = z$.

15. Sketch $\tan^{-1}\left(\frac{20}{21}\right)$ in standard position.

Problems

1. Angle D terminates in Quadrant II, and angle E terminates in Quadrant III. Both angles have the same reference angle. If $\sin D = 0.21$, what is $\sin E$? Explain your answer.

2. A function involving **sine** has a low points at $(3, 2)$ and a high point $(11, 6)$. There are no other high or low points between these two given points.

   (a) Find the average value of this function.

   (b) Find the amplitude of this function.

   (c) Find the period of this function.

   (d) Give the coordinates of one more high point.

   (e) Give the coordinates of one more low point.

   (f) Graph one period of this function.

   (g) Find a formula for this function.

3. A transformation of the function $y = \tan(x)$ has consecutive asymptotes at $x = -2$ and $x = 6$. Find a formula for this function.

4. Suppose $\cos(0.6) \approx 0.83$. Without your calculator, approximate $\cos(-0.6)$.

5. Suppose $\sin(0.6) \approx 0.56$. Without your calculator, approximate $\sin(-0.6)$.

6. Use the previous two problems to find $\tan(-0.6)$. (Do not use a calculator, which means you may need to use long division.)

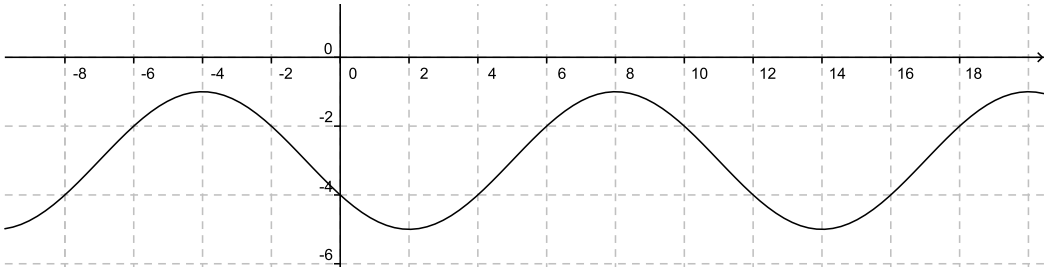
7. If $g(x) = \frac{2x-1}{8}$, then find $[g^{-1}(4^{-1})]^{-1}$.

8. Find $\tan\left(\arccos\left(\frac{7}{25}\right)\right)$. You may want to draw a right triangle first.

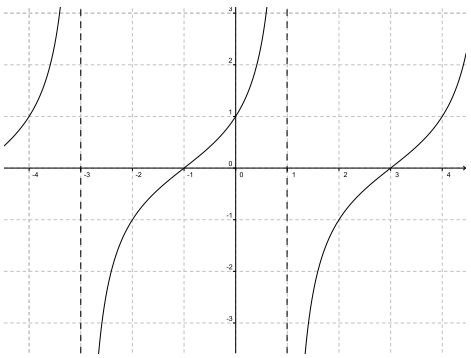
Solutions

Basics

1. You should end up with angles with the following descriptions:
   (a) Angle rotates clockwise, terminates in Quadrant IV, closer to $-\frac{\pi}{2}$ than to 0.
   (b) Angle rotates counterclockwise, terminates in Quadrant II, very close to π
2. (a) $\frac{\pi}{4}$
   (b) $-\frac{5\pi}{18}$
   (c) $\frac{5\pi}{36}$
3. (a) 120°
   (b) $\left(\frac{360}{\pi}\right)^\circ$
4. 45 cm
5. $\frac{16\pi}{3}$ cm ≈ 16.76 cm
6. $\frac{75}{2}$ square inches $= 37.5$ square inches
7. $\frac{49\pi}{6}$ square inches ≈ 25.66 square inches
8. $v = 0.8$ or $v = -0.8$
9. (a) $\sin x < 0$
   (b) $\cos x < 0$
   (c) $\tan x > 0$
10. Graph:



11. Graph:

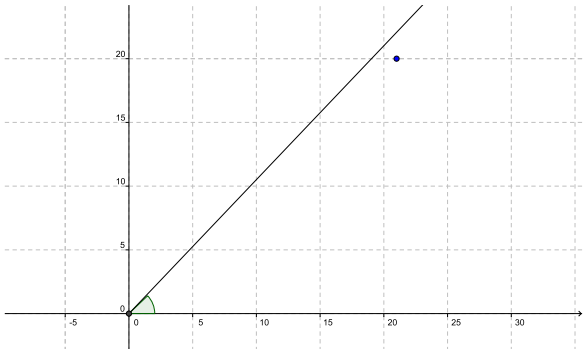


12. $f^{-1}(x) = \frac{x+8}{9}$

13. (a) $-\frac{\pi}{3}$
    (b) $\frac{3\pi}{4}$
    (c) $-\frac{\pi}{4}$

14. $\frac{\sin^{-1}\left(\frac{z+5}{2}\right)}{3} = w$

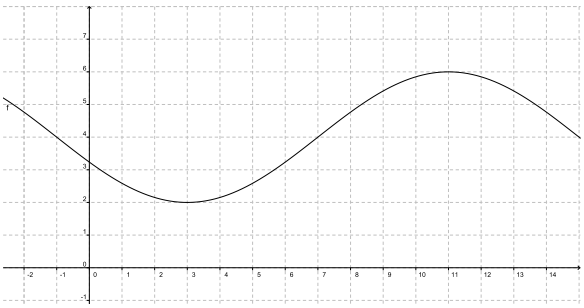
15. Sketch: Note that the terminal side goes through the point $(21, 20)$.



Problems

1. $\sin E = -0.21$. For explanation, see Chapter 2.

2. (a) Average value is the average of the maximum value and minimum value: $\frac{2+6}{2} = 4$.
   (b) The amplitude is the difference between the maximum value and the average value: $6 - 4 = 2$.
   (c) The horizontal distance between the high point and the low point is half the period. Since that distance is 8, the period is 16.
   (d) High point: 16 away from the high point we have: $(27, 6)$ or $(-5, 6)$
   (e) Low point: 16 away from the low point we have: $(19, 2)$ or $(-13, 2)$
   (f) Graph:



(g) We know that the amplitude $a = 2$, period is 16, so $b = \frac{2\pi}{16} = \frac{\pi}{8}$, and the average value $d = 4$. Since we know that the function hits the average value at $(7, 4)$ and rises to the right, so we can consider this to be a horizontal shift 7 units right. So an equation is:

$$y = 4 + 2\sin\left(\frac{\pi}{8}(x-7)\right).$$

3. Since the asymptotes are at $x = -2$ and $x = 6$, we know that the period of this function is 8. This also tells us that there is an $x-$intercept at $x = 2$. So, the horizontal shift is 2 units to the right, and the $b = \frac{\pi}{8}$. So the equation is

$$y = \tan\left(\frac{\pi}{8}(x-2)\right).$$

4. Since the negative angle identities tell us that $\cos(-x) = \cos(x)$, we know that $\cos(-0.6) \approx 0.83$.

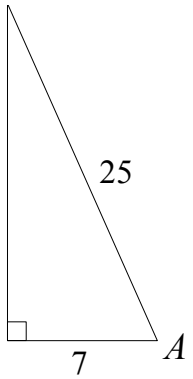
5. Since the negative angle identities tell us that $\sin(-x) = -\sin(x)$, we know that $\sin(-0.6) \approx -0.56$.

6. Since the negative angle identities tell us that $\tan(-x) = -\tan(x)$, we know that $\tan(-0.6) \approx \frac{\sin(-0.6)}{\cos(-0.6)} = \frac{-0.56}{0.83} \approx 0.67$.

7. We can see that $g^{-1}(x) = \frac{8x+1}{2}$, so

$$[g^{-1}(4^{-1})]^{-1} = \left[g^{-1}\left(\frac{1}{4}\right)\right]^{-1} = \left[\frac{8(1/4)+1}{2}\right]^{-1} = \left[\frac{2+1}{2}\right]^{-1} = \left[\frac{3}{2}\right]^{-1} = \frac{2}{3}.$$

8. First let us draw a right triangle. Since we are looking for $\arccos\left(\frac{7}{25}\right)$, we know that we want to set up one angle A so that $\cos(A) = \frac{7}{25}$ (that is $A = \arccos\left(\frac{7}{25}\right)$).



We can use the Pythagorean Theorem to show that the remaining side has length 24. So, we can see that

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{24}{7}.$$