Test 1 Review

Identities

Quotient:

$$\tan A = \frac{\sin A}{\cos A}$$

Pythagorean:

$$\sin^2 A + \cos^2 A = 1, \quad \tan^2 A + 1 = \sec^2 A, \quad 1 + \cot^2 A = \csc^2 A$$

Reciprocal:

$$\csc A = \frac{1}{\sin A}, \quad \sec A = \frac{1}{\cos A}, \quad \cot A = \frac{1}{\tan A}$$

Cofunction:

$$\begin{aligned} \sin B &= \cos(90^\circ - B) \text{ (This is true for every angle B.)} \\ \cos B &= \sin(90^\circ - B) \text{ (This is true only if B is an acute angle.)} \end{aligned}$$

Sums and Differences:

$$\begin{aligned} \sin(A + B) &= \sin A \cos B + \sin B \cos A \\ \sin(A - B) &= \sin A \cos B - \sin B \cos A \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \end{aligned}$$

Double Angle:

$$\begin{aligned} \sin(2A) &= 2 \sin A \cos A \\ \cos(2A) &= \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 \end{aligned}$$

Trigonometric Functions of Special Angles

$$\begin{array}{lll} \sin 30^\circ = \frac{1}{2} & \sin 60^\circ = \frac{\sqrt{3}}{2} & \sin 45^\circ = \frac{1}{\sqrt{2}} \\ \cos 30^\circ = \frac{\sqrt{3}}{2} & \cos 60^\circ = \frac{1}{2} & \cos 45^\circ = \frac{1}{\sqrt{2}} \\ \tan 30^\circ = \frac{1}{\sqrt{3}} & \tan 60^\circ = \sqrt{3} & \tan 45^\circ = 1 \end{array}$$

Definitions

ray	angle	acute angle
obtuse angle	right angle	acute triangle
obtuse triangle	right triangle	complementary
supplementary	scalene	isosceles
equilateral	legs of right triangle	hypotenuse of right triangle
similar	sine	cosine
tangent	inverse sine/arcsin	inverse cosine/arccos
inverse tangent/arctan	standard position	coterminal
reference angle	converse	periodic function

Exercises

Basics

1. A triangle has angles $37°$ and $110°$. Find the third angle.

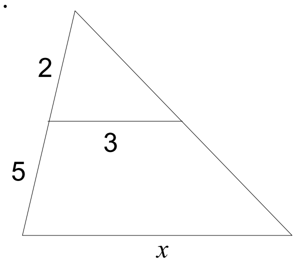
2. Find the angle complementary to $28°$.

3. Find the angle supplementary to $112°$.

4. The letter c stands for an unknown constant with $0° < c < 180°$. Write an expression for the angle which is supplementary to c.

5. A right triangle has leg $x = \sqrt{5}$ and hypotenuse $h = 10$. Find the length of leg y.

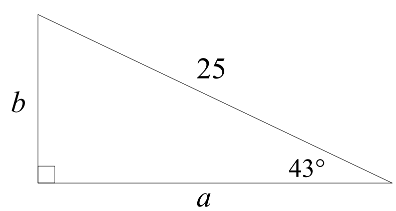
6. Find x.



7. Triangle RMW has $M = 90°$, $r = 3$ cm and $m = 7$ cm. Find $\sin W$, $\cos W$ and $\tan W$.

8. The hypotenuse of a right triangle is 10 cm and one acute angle is $40°$. Find the lengths of the legs.

9. Find the lengths of a and b.



10. Consider ΔXYH where H is a right angle. Solve the triangle if $X = 58°$ and $x = 12$ cm.

11. Find exact values for $\cos^{-1}(\sqrt{3}/2) = A$ and $\tan^{-1}(\sqrt{3}) = B$ if A and B are acute angles.

12. Angle D is acute, and $\sin D = 0.87$. Find approximate values for the other five trigonometric functions.

13. If $\cos B = -0.31$, use identites to find two possible values for $\sin B$.

14. Consider ΔXYH with $\angle H = 90°$. If side $x = 6$ m and $\angle X = 30°$, find y and h exactly.

15. Find the reference angle of $-240°$.

16. The point $(10, -1)$ lies on the terminal side of angle A in standard position. Find the exact value of $\sin A$, $\cos A$ and $\tan A$.
17. Suppose the $y = h(x)$ is periodic with period 7. If you know that $h(-3) = 12$, find four more values of x such that $h(x) = 12$ (at least one of these values should be negative).
18. Find the exact value of $\cos 15°$.
19. Find the exact value of $\sin 105°$.

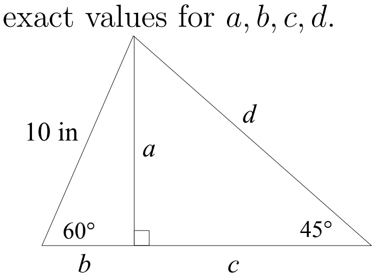
Problems

1. Make sure you understand what you did wrong on the quiz. You should be able to do all problems on the quiz.
2. Sketch ΔPQR with $\angle Q$ a right angle, and label the angles and sides. Refer to the sketch to explain why $\sin P = \cos(90° - P)$, if P is acute.
3. Consider ΔXYH with right angle H. The Pythagorean Theorem tells us that

$$x^2 + y^2 = h^2.$$

Divide both sides of the equation by y^2 to derive the third Pythagorean Identity. When you convert from ratios of sides to trigonmetric functions, focus on angle X.

4. Find exact values for a, b, c, d.



5. When viewed from below, the light beam from a lighthouse is rotating clockwise one revolution every 12 seconds. Through what angle (in degrees) does the beam rotate in one second? In one minute? In one hour?
6. Let x be a real number such that $x > 1$.
   (a) True or False: If $x > 1$, then x is positive.
   (b) State the converse of the statement in part (a).
   (c) Is the statement in part (b) true? Provide an example to support your answer.
7. Draw a graph of $\sin x$ from $0°$ to $720°$.
   (a) What is the period of $\sin x$?
   (b) Find all values of x in this interval for which $\sin x = 1$.
   (c) Find three values of x not in the interval such that $\sin x = 1$.

8. Use identities to find the exact value of $\tan 105°$.

9. If $\sin y = -0.34$, find an approximate value for $\cos(2y)$.

10. Angles E and F are in standard position. Determine whether each statement is true or false.

    (a) T/F If E and F are coterminal, then $\sin E = \sin F$.

    (b) T/F If $\sin E = \sin F$, then E and F are coterminal.

    (c) T/F If E and F have the same reference angle, then $\sin E = \sin F$.

    (d) T/F If E terminates in Quadrant III, then $\tan E$ is negative.

11. Use the cofunction identity for sine to solve the following for x:

$$\sin(4x) = \cos(20 - 3x)$$

Solutions

Basics

1. 33°

2. 62°

3. 68°

4. The angle supplementary to $c°$ is $(180 - c)°$.

5. Use Pythagorean Theorem: $10^2 = (\sqrt{5})^2 + y^2$, so $y^2 = 100 - 5 = 95$, so $y = \sqrt{95}$.

6. $\frac{2}{3} = \frac{5+2}{x}$, so we have $2x = 21$. So, $x = 10.5$.

7. $\sin W = 2\sqrt{10}/7$, $\cos W = 3/7$, $\tan W = 2\sqrt{10}/3$

8. The legs have lengths $10\sin(40°) = 6.43$ cm and $10\cos(40°) = 7.66$ cm.

9. $\cos(43°) = a/25$ so $a = 25\cos(43°) = 18.28$ and $\sin(43°) = b/25$ so $b = 25\sin(43°) = 17.05$.

10. $\sin(58°) = x/h = 12/h$, so $h = 12/\sin(58°) = 14.15$ cm, and $\tan(58°) = x/y = 12/y$, so $y = 12/\tan(58°) = 7.50$ cm. Also, $Y = 32°$.

11. $A = 30°$ and $B = 60°$.

12. $\sin^2 D + \cos^2 D = 1$, so $(0.87)^2 + \cos^2 D = 1$. Then $1 - 0.7569 = 0.2431 = \cos^2 D$, so $\cos D = 0.49$. Note that since D is acute, we take the positive square root of $\cos^2 D$. Then $\tan D = \sin D/\cos D = (0.87)/(0.49) = 1.78$. And, $\csc D = 1/\sin D = 1/0.87 = 1.15$, $\sec D = 1/\cos D = 1/0.49 = 2.04$, $\cot D = 1/\tan D = 1/1.78 = 0.56$.

13. Pythagorean Identity: $\sin^2 B + \cos^2 B = 1$, and we can substitute $\sin^2 B + (-0.31)^2 = 1$, so $\sin^2 B = 1 - (-0.31)^2 = 0.9039$, so $\sin B = \pm 0.95$.

14. $\sin 30° = 6/h$, so $h = 6/\sin 30° = 6/(1/2) = 12$. $\tan 30° = 6/y$, so $y = 6/\tan 30° = 6/(1/\sqrt{3}) = 6\sqrt{3}$.

15. $60°$

16. $\sin A = -1/\sqrt{101}$, $\cos A = 10/\sqrt{101}$, $\tan A = -1/10$.

17. $x = -10$, $x = 4$, $x = 11$, $x = 18$

18. $\cos(15°) = \cos(60° - 45°) = \cos(60°)\cos(45°) + \sin(60°)\sin(45°)$

$$= (1/2)(1/\sqrt{2}) + (\sqrt{3}/2)(1\sqrt{2}) = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

19. $\sin(105°) = \sin(60° + 45°) = \sin(60°)\cos(45°) + \sin(45°)\cos(60°)$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Problems

1.

2. Since $\angle P$ and $\angle R$ are the other two angles in a right triangle, they are clearly complementary, since their sum is $90°$. So $R = 90° - P$. So when we say that $\sin P = \cos(90° - P)$, we are really saying $\sin P = \cos R$. From the sketch, we can see that $\sin P = p/q$ and $\cos R = p/q$. So we have $\sin P = \cos R = \cos(90° - P)$.

3. Pythagorean Theorem: $x^2 + y^2 = h^2$

$$\begin{aligned} \frac{x^2+y^2}{y^2} &= \frac{h^2}{y^2} \\ \frac{x^2}{y^2} + 1 &= \frac{h^2}{y^2} \\ (\tan X)^2 + 1 &= (\sec X)^2 \\ \tan^2 X + 1 &= \sec^2 X \end{aligned}$$

4. $a = 5\sqrt{3}$, $b = 5$, $c = 5\sqrt{3}$, $d = 5\sqrt{6}$

5. In one second, the beam completes 1/12 of a rotation, so $1/12 \cdot 360° = 30°$. It takes the beam 12 seconds to complete one rotation, so it completes four rotations in one minute (60 seconds). Therefore, it rotates through $1440°$ in one minute. Similarly, it rotates through $86400°$ in one hour.

6. (a) True. (b) If x is positive, then $x > 1$. (c) False. Think about 1/2. It is certainly positive, but it is not greater than one.

7. (a) The graph of $\sin(x)$:
The function has period $360°$. (b) $x = 90°$, $x = 450°$. (c) $x = -270°$, $x = 810°$, $x = 1170°$

8. $\tan(105°) = \tan(45° + 60°) =$

$$\frac{\sin(45° + 60°)}{\cos(45° + 60°)} = \frac{\sin(45°)\cos(60°) + \sin(60°)\cos(45°)}{\cos(45°)\cos(60°) - \sin(45°)\sin(60°)} = \frac{(1/\sqrt{2})(1/2) + (\sqrt{3}/2)(1/\sqrt{2})}{(1/\sqrt{2})(1/2) - (1/\sqrt{2})(\sqrt{3}/2)} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

9. We know that $\cos(2y) = \cos^2 y - \sin^2 y$. Since we know $\sin y = -0.34$ we only need to find $\cos^2 y$, so we can use the Pythagorean Identity: $\sin^2 y + \cos^2 y = 1$.

$$\begin{aligned} (-0.34)^2 + \cos^2 y &= 1 \\ 0.1156 + \cos^2 y &= 1 \\ \cos^2 y &= 1 - 0.1156 = 0.8844 \end{aligned}$$

So, $\cos(2y) = \cos^2 y - \sin^2 y = 0.8844 - (-0.34)^2 = 0.7688$.

10. T/F Statements:

 (a) T: If E and F are coterminal, they not only have the same reference angle, but they terminate in the same quadrant. So $\sin E$ and $\sin F$ have the same sign (they're either both postive or both negative). This in addition to having the same reference angle means $\sin E = \sin F$.

 (b) F: We can have $\sin E = \sin F$, for example, $\sin(45°) = 1/\sqrt{2} = \sin(135°)$, but $45°$ and $135°$ are clearly NOT coterminal.

 (c) F: Both $30°$ and $330°$ have reference angles of $30°$, but $\sin(30°) = \sqrt{3}/2$ and $\sin(330°) = -\sqrt{3}/2$.

 (d) F: In Quadrant III, both $\sin E$ and $\cos E$ are negative. Since $\tan E = \sin E / \cos E$, which is a negative over a negative, $\tan E$ is positive if E is in Quadrant III.

11. The cofunction identity for sine is $\sin B = \cos(90 - B)$. So, $B = 4x$ and $90 - B = 20 - 3x$. Using substitution, this gives us

$$\begin{aligned} 90 - 4x &= 20 - 3x \\ 70 &= x. \end{aligned}$$