

Math 105 - Math in Modern Society

Chapter 4 Solutions

Sept. 17, 2010

5. (a) Since the standard quota for each state is the exact fractional quota of each state, the sum of the all the standard quotas should produce the total number of seats in the Tropicana legislature.

$$40.50 + 29.70 + 23.65 + 14.60 + 10.55 = 119 \text{ seats.}$$

- (b) The standard divisor is

$$\frac{\text{total population}}{\text{total number of seats}} = \frac{23.8 \text{ million}}{119} = 0.2 \text{ million, or } 200,000.$$

- (c) Since we know that

$$\text{standard quota} = \frac{\text{state population}}{\text{standard divisor}},$$

we can set up the equation for state A:

$$40.50 = \frac{\text{state A population}}{200,000}, \quad \text{state A population} = 8.1 \text{ million.}$$

We get:

State	A	B	C	D	E
State population (in millions)	8.1	5.94	4.73	2.92	2.11

15. Under Hamilton's method, we need to first calculate the standard quotas - but this has already been done for us.

State	A	B	C	D	E
Standard quota	40.50	29.70	23.65	14.60	10.55

Then, we assign each state its **lower quota**:

State	A	B	C	D	E
Lower quota	40	29	23	14	10

There are only 116 seats assigned. We still need to assign the last 3 seats. We can see that the **residues** are as follows, and can assign the seats based on which states have the highest residues:

State	A	B	C	D	E
Residues	0.50	0.70	0.65	0.60	0.55
Leftover seats		+1	+1	+1	

So, we have the final Hamiltonian apportionment:

State	A	B	C	D	E
Apportionment	40	30	24	15	10

22. (a) Since we know that Katie's standard quota is 6.53 and that Hamilton's method obeys the quota rule, we know that Katie cannot get more than 7 pieces of candy or less than 6 pieces of candy. Therefore, there is no way for Katie to get only 5 pieces of candy.

- (b) Similarly, since we are using Hamilton's method, which obeys the quota rule, we see that Katie can't get more than 7 pieces, so it is impossible for her to get 9 pieces.
- (c) The only way for Lilly to get 9 pieces of candy is if Katie gets only 6 and Jaime gets none. However, consider the standard quotas in that situation: even if Jaime's standard quota is 0, Lilly's standard quota is **at most** $15 - 6.53 = 8.47$. So, we apportion the lower quotas:

Daughter	Katie	Lilly	Jaime
Lower quota	6	8	0

we have 1 candy left over, but that must go to the person with the highest residue, which is Katie. So there is no way that Lilly can get 9 pieces of candy.

27. We already know that taking the lower quota from the standard divisor (0.2 million) gives us the following apportionment, which is 3 seats short.

State	A	B	C	D	E
Lower quota	40	29	23	14	10

So we know that we need to lower the divisor slightly. We know the state populations as well:

State	A	B	C	D	E
State population (in millions)	8.1	5.94	4.73	2.92	2.11

Let us try the divisor $D = 0.19$ million.

State	A	B	C	D	E
Lower quota	42	31			

We can already see that that divisor is too small, since we have already added 4 seats, when we only had 3 leftover. Let us try $D = 0.195$ million.

State	A	B	C	D	E
Lower quota	41	30	24	14	10

This adds up to 119, so this is the correct apportionment.

47. Under Webster's method, we round the quotas with respect to the geometric mean. So given the standard quotas:

State	A	B	C	D	E
Standard quota	40.50	29.70	23.65	14.60	10.55

we can round them as follows

State	A	B	C	D	E
Rounded quota	41	30	24	15	11

However, this adds up to 121, which is two seats too many. So we need to raise the divisor. Let us try $D = 0.205$ million. We know the state populations, so we get

State	A	B	C	D	E
Rounded quota	40	29	23	14	10

This adds up to 116. So we need to lower the divisor a little. Let us try $D = 0.202$.

State	A	B	C	D	E
Rounded quota	40	30	24	15	10

This adds up to 119, so this apportionment is correct.

12. (a) Geometric mean of 4 and 5 is $\sqrt{4 \cdot 5} = 4.472$. Since $4.46 < 4.472$, we round this down to 4.
- (b) Geometric mean of 4 and 5 is $\sqrt{4 \cdot 5} = 4.472$. Since $4.48 > 4.472$, we round this up to 5.
- (c) Geometric mean of 50 and 51 is $\sqrt{50 \cdot 51} = 50.4975$. Since $50.498 > 50.4975$, we round this up to 51.
- (d) We have $\sqrt{12 \cdot 01} = 3.46554$. Geometric mean of 3 and 4 is $\sqrt{3 \cdot 4} = 3.461$. Since $3.46554 > 3.461$, we round this up to 4.