Math in Modern Society
Mathematics of Voting
Extra Practice

1. There are four candidates in a local election. The preference schedule is given below.

Number of voters	**10**	**5**	**18**	**7**	**3**	**2**
First place	A	A	D	C	B	B
Second place	B	D	C	B	C	A
Third place	C	C	B	A	A	D
Fourth place	D	B	A	D	D	C

Total number of votes: **45**

(a) Determine the winner of the election under each of the four methods discussed in class.
Plurality: The winner is the candidate with the most first place votes. That would be **Candidate D**, with 18 first-place votes.
Borda count: The Borda points are as follows:

A	B	C	D
104	112	123	111

So the Borda winner is **Candidate C**.
Plurality-with-elimination: The first candidate eliminated is B, since B only has 5 first-place votes. This means the distribution of first-place votes becomes:

A	C	D
17	10	18

so C is the next candidate to be eliminated. Then we have

A	D
27	18

So the winner under this method is **Candidate A**.
Pairwise comparisons: The possible head-to-head comparisons are as follows, with the preferred candidate in bold: A vs. **B**, A vs. **C**, **A** vs. D, B vs. **C**, B vs. **D**, C vs. **D**, so the points are: $A = 1$, $B = 1$, $C = 2$, $D = 2$. So, C and D are tied, and there needs to be some sort of tie breaker.

(b) In this case, which methods violate which fairness criteria?
First of all, we should notice that we have no majority candidate and no Condorcet candidate, so these two fairness criteria are not violated - in fact, they do not quite apply. We can see from the plurality-with-elimination method that IIA is violated as well - when candidate pull out of an election (or are eliminated), this changes who the winner is, under the plurality method. This means that the **plurality** method violates IIA, as does plurality with elimination. The monotonicity criterion does not apply.

2. Consider the election with four candidates given by the following preference schedule.

Number of voters	**7**	**4**	**2**
First choice	A	B	D
Second choice	B	D	A
Third choice	C	C	C
Fourth choice	D	A	B

(a) What is the total number of voters? **13**

(b) How many votes are needed for the majority? **7**

(c) Does anyone have the majority? If so, who? And under which methods does this candidate win (you should be able to answer this without applying any of the methods)?
Candidate A. This candidate will win under the plurality, plurality-with-elimination, and pairwise comparisons.

(d) Find the Condorcet candidate in this election.
The head-to-head comparisons are: **A** vs B, **A** vs. C, **A** vs. D, **B** vs. C, **B** vs. D, **C** vs. D. Since A wins all the head-to-head comparisons, A is the Condorcet candidate.

(e) Find the winner under the Borda Count method.
The Borda counts are

A	B	C	D
38	39	26	27

so **Candidate B** is the Borda winner.

(f) Suppose that Candidate C drops out of the race. Who among the remaining candidates wins the election under the Borda count method?
The new Borda points are

A	B	D
29	28	21

The new Borda winner is **Candidate A**.

(g) Based on your work above, which of the four fairness criteria are violated in this election? Explain. (That is, mention all four criteria and explain how there were or were not violated, under the Borda Count method.)
Under the Borda count method, both the majority and Condorcet criteria were violated, since both of those criteria stated that the winner should be candidate A, but under the Borda count method, it was candidate B. Also, the IIA criterion was violated, since when C dropped out, it was completely irrelevant to the standings of the other candidates, but the winner changed under the Borda count method. The monotonicity criterion is not violated.

3. Consider the election determined by the Borda Count method, given by the following preference schedule:

Number of voters	**7**	**5**	**3**
First choice	A	B	C
Second choice	B	A	A
Third choice	C	C	B

(a) Find the winner under the Borda Count method.
The Borda counts are

A	B	C
37	32	21

So the winner is **Candidate A**.

(b) Does this election have a Condorcet candidate? Explain.
The head-to-head comparisons are as follows: **A** vs. B, **A** vs. C, **B** vs. C. Since Candidate A wins all their head-to-head comparisons, the Condorcet candidate is **Candidate A**.

(c) Does this election have a majority candidate? Explain.
Since there are a total of 15 ballots in this election, a majority candidate would need to get at least 8 first place votes. Since this does not happen, this election does not have a majority candidate.

(d) Does this election violate the Condorcet criterion? Explain.
No, since the winner of the election is the same as the Condorcet candidate.

(e) Does this election violate the majority criterion? Explain.
It does not, since there is no majority candidate.