

The Mathematics of Getting Around

Math 105

Section 003 - Prasad

November 19, 2010

Reading Quiz

1. Pick any four of the following terms and define them as completely as you **in your own words**.

routing problem	exhaustive route
unicursal tracing	edge set
multiple edges	vertex set
degree of a vertex	even and odd vertices
length of a path	Euler circuit
bridge	Euler path
isolated vertices	loop
eulerizing a graph	connected/disconnected

2. In your own words, explain what a **graph** is.

Graphs

The
Mathematics
of Getting
Around

Math 105

Reading Quiz

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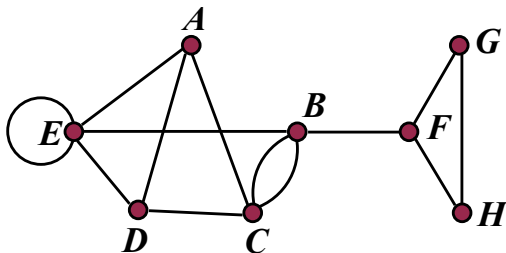
Euler's
Theorems

Flury's
Algorithm

Eulerization

Formally, a **graph** is a representation of a set of objects (the **vertex set**) and the relations between those objects (the **edge set**).

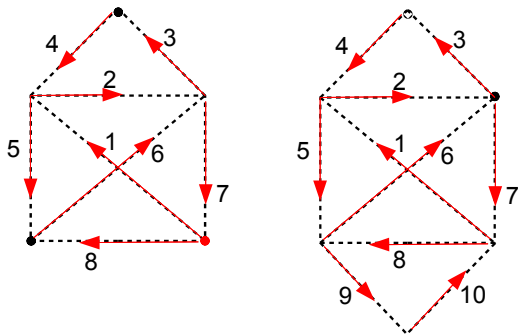
Graph properties



- 1 Identify the odd vertices and the even vertices on this graph.
- 2 Identify a **path** of length 4.
- 3 Identify a **circuit** of length 4.
- 4 Identify any bridges.

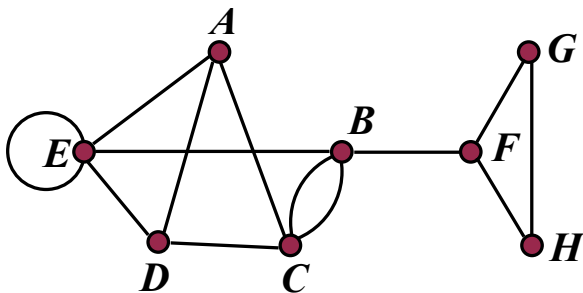
Euler paths and circuits

An **Euler path** in a connected graph travels through *all* the edges only once. An **Euler circuit** does the same thing, but it must end at the same vertex it started from. A graph cannot have both an Euler path and an Euler circuit at the same time. However, it is possible for a graph to have neither.



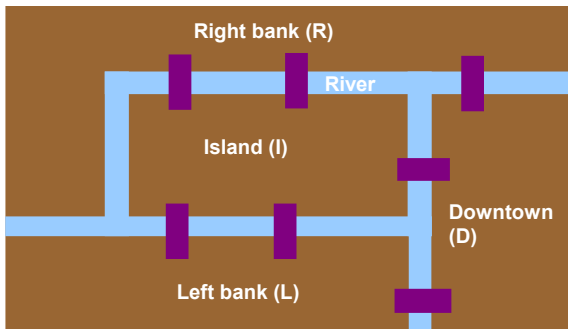
Euler paths and circuits

Does this graph have an Euler path or an Euler circuit?



Bridges of Koenigsberg

How do we make a graph that models the bridges of Koenigsberg problem? Remember that we want to find a path through the town that crosses every bridge only once.



Euler's Circuit Theorem

Theorem: If a graph is **connected** and every vertex is **even**, then it has an Euler circuit. If a graph has **any** odd vertices, then it does not have an Euler circuit.

Questions:

- 1 Why should the graph be connected?
- 2 Can you think of how having at least one odd vertex would make an Euler circuit impossible?

Euler's Path Theorem

The
Mathematics
of Getting
Around

Math 105

Reading Quiz

Graphs

Euler's
Theorems

Flurry's
Algorithm

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Theorem: If a graph is **connected** and has **exactly two odd vertices**, then it has an Euler path. Any such path must start at one of the odd vertices and end at the other one. If a graph has **more than two** odd vertices, then it cannot not have an Euler path.

Questions:

- 1 Can you think of how having more than two odd vertices would make an Euler circuit impossible?

Euler's Sum of Degrees Theorem

Theorem: The sum of the degrees of all the vertices of a graph equals twice the number of edges (and therefore is an even number). A graph always has an even number of odd vertices.

Questions:

- 1 Justify the first statement!
- 2 How does the second statement follow from the first?

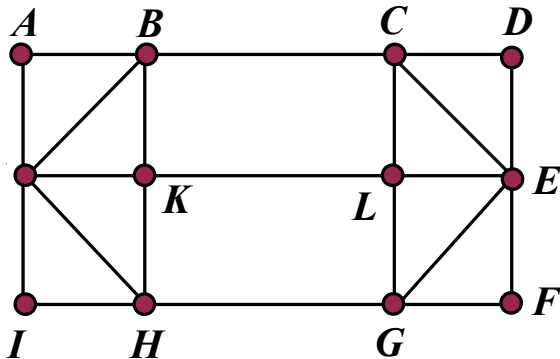
Fleury's Algorithm

Fleury's Algorithm is basically a way of **finding** an Euler circuit or path. In order to even implement this algorithm, you have to know that you even **can** find one or the other.

- 1 Pick a starting vertex.
- 2 If you have a choice, don't pick a bridge to a part of the graph you haven't been to yet.
 - 1 The best way to think about this choice is consider the parts of the graph you've already traveled to be "deleted"
- 3 Once you can't go any further, you'll be done.

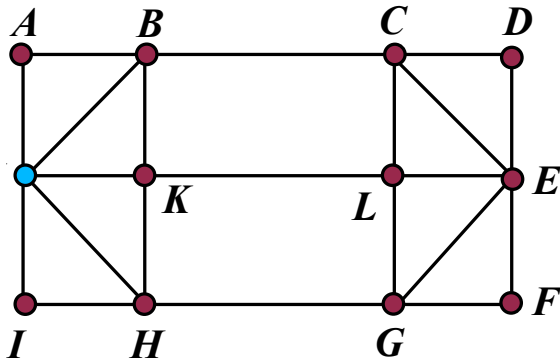
Fleury's algorithm

Find an Euler path or Euler circuit for the following graph:



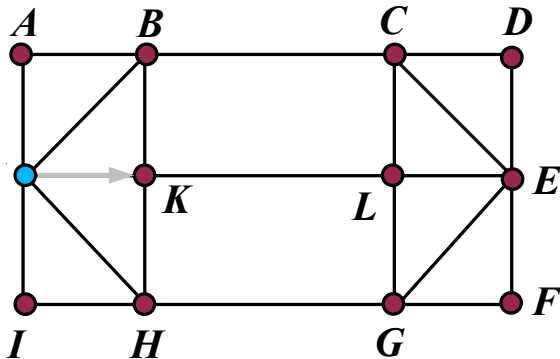
Fleury's algorithm

Step One:



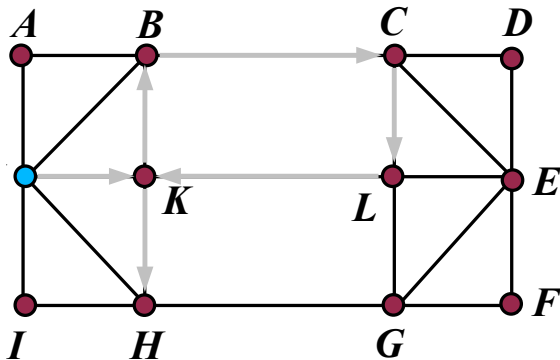
Fleury's algorithm

Making choices:



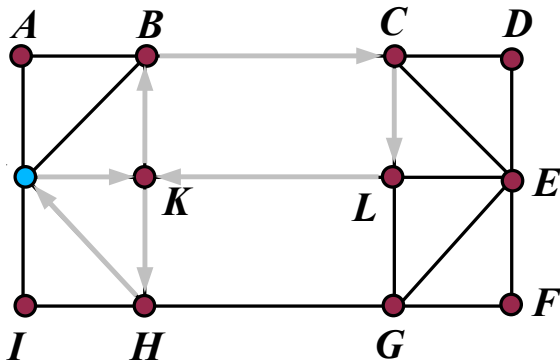
Fleury's algorithm

Now which way should we go?



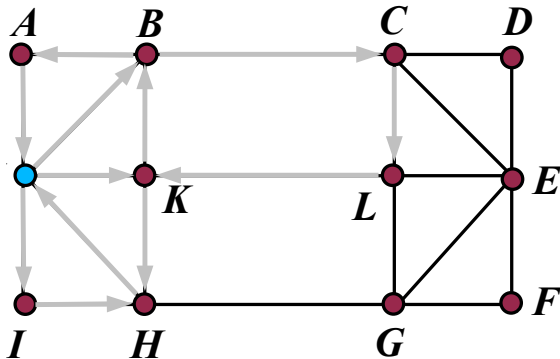
Fleury's algorithm

And now?



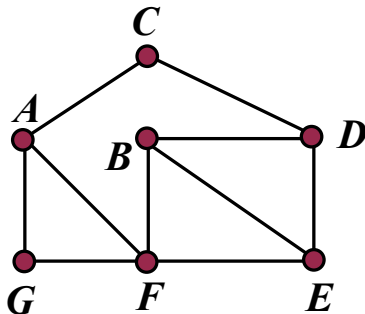
Fleury's algorithm

And now that we have no choice, we can travel along HG .



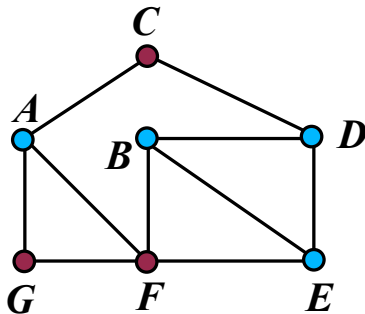
Eulerizing Graphs

What if we need to find an optimal exhaustive route through a graph that we cannot find an Euler path or Euler circuit for?



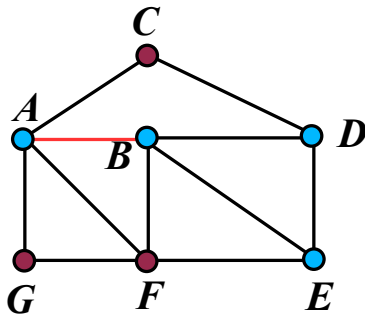
Eulerizing Graphs

There are **four** odd vertices:



Eulerizing Graphs

If we add an edge between two of the odd vertices, what happens?



Can we find an Euler path or Euler circuit now?

Eulerizing Graphs

The
Mathematics
of Getting
Around

Math 105

Reading Quiz

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Eulerization

The process of **eulerizing a graph** is the process of adding edges which turn odd vertices into even vertices, so that we can find an Euler circuit. **Semi-eulerization** is the process of adding edges so that we can find an Euler path.