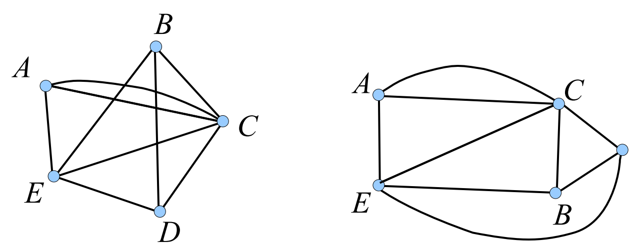
Math 105 - Math in Modern Society

Chapters 5 and 6 Solutions

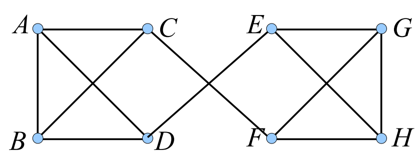
Dec. 6, 2010

Chapter 5

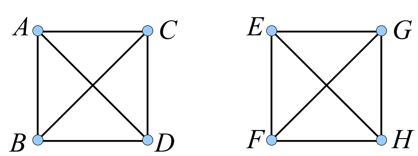
6. Consider the graph with $\mathcal{V} = \{A, B, C, D, E\}$ and $\mathcal{E} = \{AC, AE, BD, BE, CA, CD, CE, DE\}$. Draw two different pictures of this graph.
   Solution. Here are two possible ways to draw this graph:



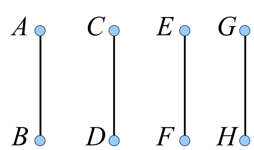
10. (a) Give an example of a connected graph with eight vertices such that each vertex has degree 3.
    Solution. This is one solution. Are there other possibilities?



   (b) Give an example of a disconnected graph with eight vertices such that each vertex has degree 3.
   Solution. This is one solution. Are there other possibilities?

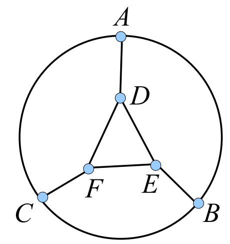


   (c) Give an example of a graph with eight vertices such that each vertex has degree 1.
   Solution. There is really only one possible graph that fits this description:



Chapter 6

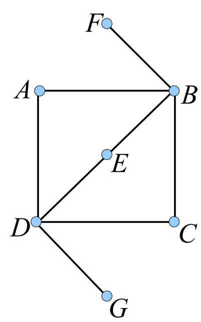
8. For the graph shown:



   (a) list all Hamilton circuits using A as the reference point.
   Solution. $ADEFCBA$, $ADFEBCA$, $ABEDFCA$, $ABCFEDA$, $ACFDEBA$, $ACBEFDA$

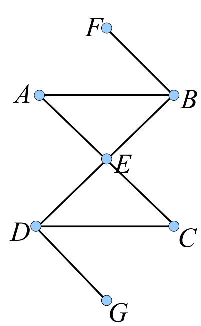
   (b) list all Hamilton circuits using D as the reference point.
   Solution. $DFCABED$, $DEBACFD$, $DFEBCAD$, $DEFCBAD$, $DACFEBA$, $DABEFCAD$, $DACBEFD$, $DABCFED$

9. Explain why the graph shown below has neither Hamilton circuits nor Hamilton paths.



   Solution. In order to have a Hamilton circuit, we need to be able to start and end in the same place. If we don't start at F, there is no way to reach F and leave it without passing through B more than once. If we do start at F, there is no way to reach G without passing through D more than once. So we cannot have a Hamilton circuit. Note that this reasoning also works for why we cannot have a Hamilton path.

10. Explain why the graph shown has no Hamilton circuit but does have a Hamilton path.



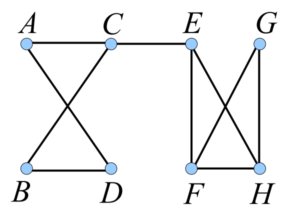
   Solution. Again, with a circuit, we have to be able to pass through points F and G, and there is no way to do this without passing through either B or D multiple times. However, if we start our Hamilton path at F, we can travel the following route:

$$FBAECDG.$$

   The mirror of this route starting at G is also a Hamilton path.

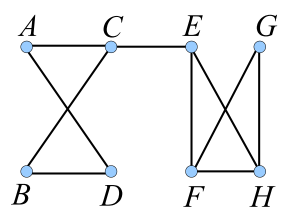
66. Explain why the cheapest edge in any graph is always part of the Hamilton circuit obtained by using the nearest-neighbor algorithm.
Solution. The nearest-neighbor algorithm works by always picking the cheapest edge that is connected to the vertex you're at. When you're at a vertex that is connected to the cheapest edge in the graph, it makes sense for you to travel it, as it will always be the "nearest neighbor" of that vertex.

69. (a) Explain why a graph that has a bridge cannot have a Hamiton circuit.
Solution. Consider the graph below:



The edge CE is a bridge, since without it, the graph would be disconnected. We can see that near the bridge, if we started with A, B, C or D, we would have to cross the bridge to get to the other vertices, but then we would have to cross it again to get back, meaning we pass through both C and E more than once. So we cannot have a Hamilton circuit.

(b) Give an example of a graph with bridges that has a Hamilton path.
Solution. However, the same graph has a Hamiton path.



A Hamilton path is $ADBCEHGF$.