Problem set 7

January 16, 2007

1. Simon gives a proof of the Trotter product formula for one parameter subgroups. See if you can write up this proof in a way that makes sense to you. This can be found on page 131.

2. On page 167 of Simon's book he claims that if $X_1, X_2, \ldots, X_n$ are the generators of the Lie Algebra of a torus with the further property that

$$\exp \sum_j a_j X_j = e,$$

if and only if $a_j \in \mathbf{Z}$, then *it is easy to see that* the integer powers of

$$\exp \sum_j \alpha_j X_j$$

will be dense in the torus provided the α_j are independent over the rationals. This seems quite plausible to me but I don't see how to prove it. I would appreciate it if you can show me how to prove this.

3. Suppose that C is a collection of linear transformations on a finite dimensional vector space V which commute with one another. If every transformation in C is diagonalizable show that there is a basis for V which consists of vectors e_j that are simultaneous eigenvectors for each linear transformation in C.