Problem Set 5 Math 559

October 9, 2006

1. Use the theory of induced representations to classify the irreducible representations of $\mathbf{Z}_2 \times \mathbf{Z}_2 \otimes_\alpha S_3$(semi-direct product). Use the Frobenius character formula to determine the character table for this group. This will mostly turn out to be an exercise in figuring out the conjugacy classes in this group.

2. A Lie algebra is said to be abelian iff $[A, B] = 0$ for every pair A, B in the algebra. Show that a connected Lie group is abelian iff its Lie algebra is abelian. Show that if $G$is a connected abelian group the map,

$$\exp : T_e G \to G$$

is a surjective group homomorphism from the additive group $T_e G$ to G. Use this to show that every connected abelian Lie group is isomorphic to $\mathbf{T}^k \times \mathbf{R}^n$ where $\mathbf{T} \simeq S^1$is the circle group.

3. If G is a Lie group then a subgroup Γ of G is said to be discrete if there is a neighborhood U of $e \in G$ such that $U \cap \Gamma = \{e\}$. Show that if Γ is a discrete subgroup of a Lie group G then there exists a manifold structure on G/Γ so that the projection,

$$\pi : G \to G/\Gamma,$$

is a covering space projection with the covering transformations given by right translation by elements of Γ. If Γ is also a normal subgroup show that G/Γ is a Lie group and that the map π induces an isomorphism of Lie algebras $LG \to LG/\Gamma$.