Math 559 Problem set 3

September 14, 2006

1. Use the character table for the symmetric group, S_3, to determine the multiplicities for the decomposition of the natural action of S_3on $\mathbf{C}^n \otimes \mathbf{C}^n \otimes \mathbf{C}^n$into irreducibles.

2. Suppose that χ_U is the character of a representation for a finite group G. Show that χ_U is the character of an irreducible representation if and only if:

$$\frac{1}{o(G)} \sum_{g \in G} |\chi_U(g)|^2 = 1$$

3. Find the character of the representation of the symmetric group S_4 acting on the orthogonal complement of $[1, 1, 1, 1]^\tau$. Using (2) above to show that this representation is irreducible.

4. The group, S_4, acts by conjugation on the conjugacy class K which is the orbit of the permutation with cycle decomposition (1,2)(3,4). Thus S_4acts on the vector space of functions $f : K \to \mathbf{C}$ by,

$$U(g)f(x) = f(g^{-1}xg).$$

Find the character of this representation and its decomposition into irreducibles. In particular you should find the character of a two dimensional irreducible representation this way.

5. Use the results of 3 and 4 above to complete the character table for S_4.