

# Practice Final Math 362

April 28, 2007

1. A woman has 10 keys just one of which opens her front door. If she tries keys at random, but sets aside keys that do not work, what is the probability that she opens the door on the first, second, or third try? (this is one question with just one answer)
2. Suppose that an urn contains 5 red and 7 blue balls. Suppose that 8 balls are drawn from the urn without replacement.
  - (a) What is the probability that the sample contains 3 or fewer red balls? What is the probability that the sample contains 1 or more red balls?
  - (b) What is the probability that a red ball is drawn on the 8<sup>th</sup> draw?
3. Suppose a plant ships batteries in boxes that contain 6 batteries. Before shipping 2 batteries in each box are randomly selected and tested and the box is shipped only if both batteries are good. Suppose the batteries produced by the plant are independently good with probability .99.
  - (a) What is the probability that a box ships. In a sample of 1000 boxes coming off the line what is the expected number of boxes that ship?
  - (b) What is the probability that a box coming off the line contains at least one defective battery. Given that a box ships (i.e., passes the test) what is the probability that it still contains at least one defective battery?
4. Suppose that the number of bit errors in a data transmission of length  $N$  bits is (approximately) a Poisson random variable with parameter  $\lambda = 10^{-4}N$ .
  - (a) What is the probability that a transmission of 100 bits has at least one error? What is the probability that such a transmission has 2 or more errors?
  - (b) Suppose that data is being sent in packets of 100 bits. If there is an error in the transmission of a bit in the packet the packet is resent. What is the expected number of times such a packet is resent? You can assume that the successive transmissions are independent. Hint: Determine the probability distribution for the random variable,  $X$ , which counts the number of times the packet is resent. What sort of random variable is  $X$ ?

5. Suppose that two clerks A and B are serving customers who wait to be served in a single line. Suppose that the time,  $T_A$ , it takes A to start and finish serving a single customer is an exponential random variable with mean 4 minutes. Suppose the time,  $T_B$ , it takes B to serve a customer is an exponential random variable with mean 7 minutes. Assume that  $T_A$  and  $T_B$  are independent.

(a) If three customers all get in line at  $t = 0$  and A and B immediately start serving the first two what is the expected time that the third customer will wait in line before one of the clerks A or B starts to serve her. Hint: the time she waits is  $\min\{T_A, T_B\}$ .

(a) What is the probability that she is served by A? Hint consider the event  $T_A < T_B$ .

6. Suppose that an exit poll of 2000 randomly selected voters indicates that candidate A has won the election with 51% of the vote. In the actually tally candidate A loses the election with just 48% of the vote. Suppose that the exit poll represents a binomial sample from a population with  $p = .48$  being the probability of randomly selecting a person who votes for A. Use the normal approximation to estimate the probability that the exit poll result will deviate by more than 0.03 from the mean. Note that if  $X$  is the number of candidates in the sample who said they will vote for A then the percentage estimate is  $X/2000$  (not  $X$  itself!)

7. Suppose that A and B both arrive at an aquarium between in the one hour period between 12:00 noon and 1:00 pm. Assume that the arrival times for A and B are independent uniform random variables on the interval  $[0,1]$ ,  $T_A$  and  $T_B$  (so if  $T_A = \frac{1}{2}$  then A arrives at 12:30). Suppose that each waits 10 minutes for the other and then leaves.

(a) What is the probability that A and B meet?

(b) If A arrives at 12:00 what is the probability that she will meet B? If A arrives a 12:30 what is the probability that she will meet B?