

Practice Exam 3: Math 362

1. Suppose that the number of patients, N_t , arriving at a clinic in the time interval $[0, t]$ is a Poisson random variable with mean λt where the rate, λ , is 5 patients per hour.

(a) Let T_1 be the arrival time for the first patient. Find the expected value $E(T_1)$.

(b) Let T_2 denote the arrival time for the second patient. Find the cumulative distribution for T_2 . Hint: rewrite the event $T_2 > t$ in terms of N_t .

2. Suppose a bus is scheduled to arrive at the station at 12:00 noon. Let X denote the number of minutes by which the bus misses its scheduled arrival time. Thus $X = -1$ means the bus arrives 1 minute before 12:00 noon. Suppose that X is a normal random variable with mean 0 and standard deviation 1.5. If Frederika arrives at the bus terminal at 11:59 am and waits for the bus what is the probability that she will catch the bus? Assume that the bus always departs the moment that it arrives.

3. Suppose that two light bulbs have lifetimes L_1 and L_2 that are independent exponential random variables with parameters θ_1 and θ_2 . Suppose that both light bulbs are switched on at the same time. Find the distribution for L , the first time at which one of the light bulb burns out. Hint: $L = \min(L_1, L_2)$ and,

$$(\min(L_1, L_2) > t) = (L_1 > t) \cap (L_2 > t).$$

4. Suppose that a petri dish covers a disk of radius 1 in the plane with center at $(0,0)$. Suppose the dish is uniformly smeared with a bacterial culture and let (X, Y) denote the coordinates of the center of the first bacterial colony to appear. If A is a subset of the unit disk, suppose that the probability that $(X, Y) \in A$ is $\frac{\text{Area}(A)}{\pi}$.

(a) Let $r = \sqrt{X^2 + Y^2}$ and find $P(r < .5)$.

(b) Find the marginal density for Y .

(c) Find probability that $0 < X < .5$ given that $Y = .5$.

5. Suppose that X and Y are independent uniform random variables on the interval $[0, 1]$. Find,

$$P(Y \leq X^2).$$