Nonlinear optics in photonic structures

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Binational Consortium of Optics, “Nonlinear and multiscale photonics” April 2, 2005

Introduction

We take advantage of periodic structures and nonlinear effects to propose new stable and robust systems relevant to optical systems.

- Periodic structure - material with a periodically varying index of refraction ("grating")

- Nonlinearity - dependence of the refractive index on the intensity of the electric field (Kerr nonlinearity)

The effects we seek:

- existence of stable localized solutions – solitary waves, solitons
- short formation lengths of these stable pulses
- possibility to control the pulses – speed, direction (2D, 3D)

Prospective applications:

- rerouting of pulses
- optical memory
- low-loss bending of light

Outline

I. 1D periodic structures –
photonic crystal fibers
fiber gratings
  - Gap solitons (Theory: A.A, S. Wabnitz, Exp: Eggleton, Slusher (Bell Labs))
  - Fiber gratings with defects – stopping of light (Goodman, Slusher, Weinstein 2002)

II. 2D periodic structures – waveguide gratings (T. Dohnal, A.A)
  - *x*- uniform medium with a *z*- grating
  - *z*- grating, guiding in *x*
    2D soliton-like bullets
  - Interactions of 2 bullets
  - waveguide gratings with defects

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Nonlinear optics in waveguides

- Starting point : Maxwell's equations
- Separation of variables, transverse mode satisfies eigenvalue problem and envelope satisfies nonlinear wave equation derived from asymptotic theory

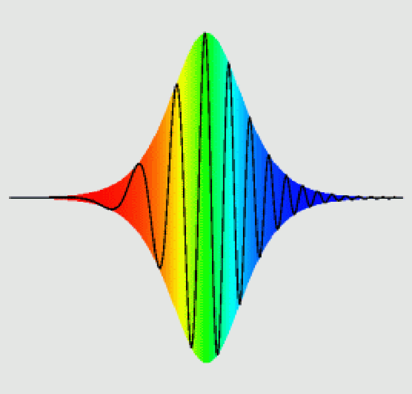
$$E(x,y,z,t) = A(\varepsilon z, \varepsilon t)U(x,y)e^{i(kz-\omega t)} + cc.$$

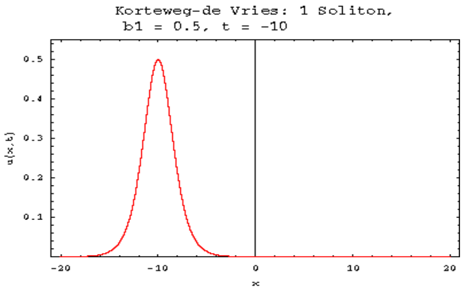
$$\nabla^2 U + n^2(x,y)\left(\frac{\omega^2}{c^2}\right)U = \beta^2 U$$

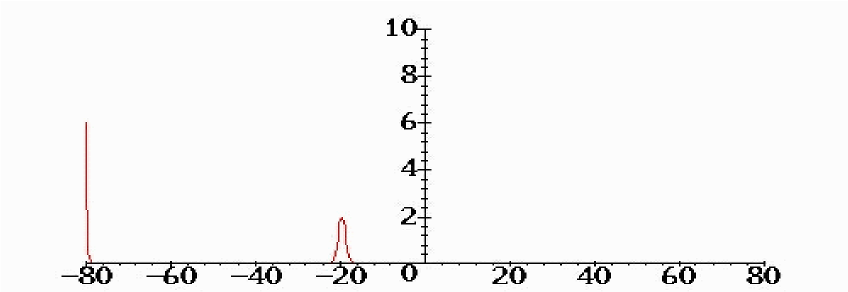
$$i\partial_Z A + \partial^2_{T^2} A + \Gamma |A|^2 A = 0$$

This is the nonlinear Schroedinger equation known to describe optical pulses in fibers and light beams in waveguides

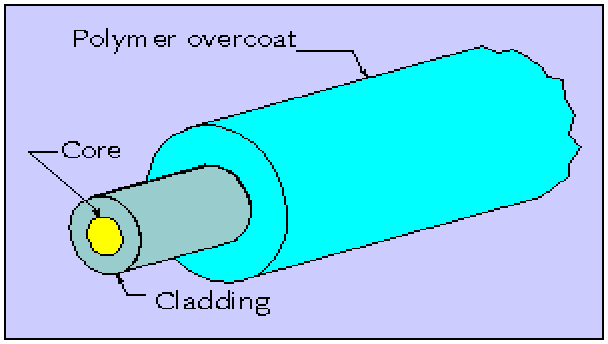
Solitons in homogeneous systems





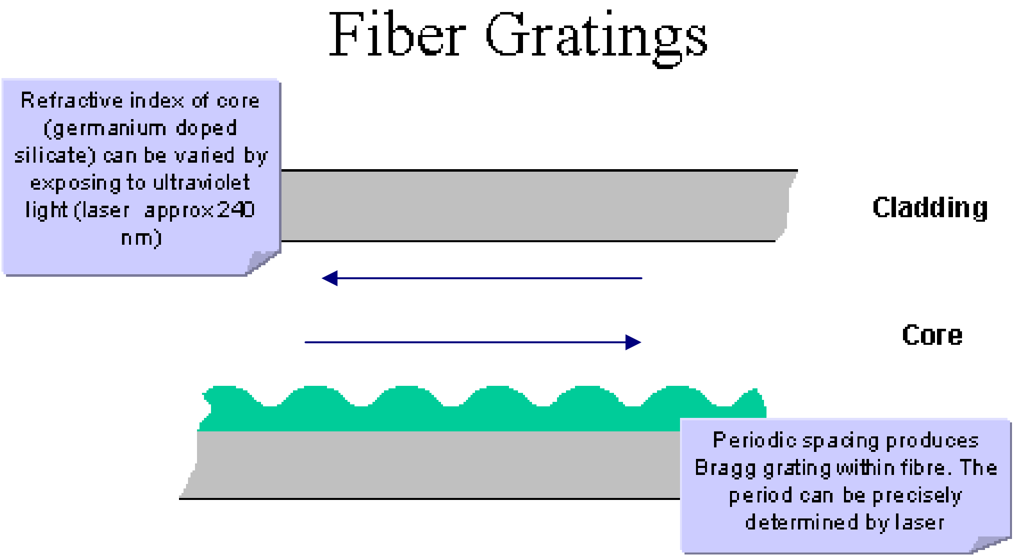


Elastic soliton interaction in the
Nonlinear Schroedinger Eq.



optical fiber

Optical fiber gratings



$E_f(Z,T)$ (forwardmovingenvelope)

$E_b(Z,T)$ (backwardmovingenvelope)

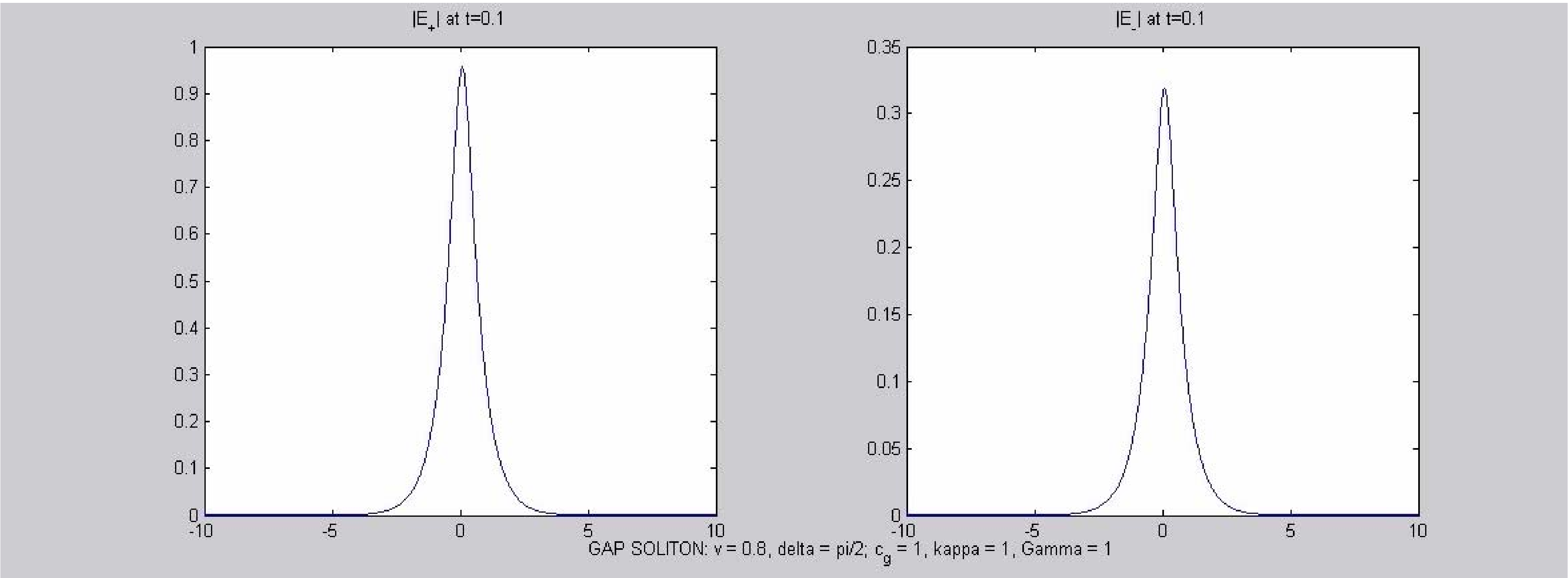
1-dim Coupled Mode Equations

$$\partial_t E_+ = -c_g \partial_z E_+ + i\kappa E_- + i\Gamma(|E_+|^2 + 2|E_-|^2)E_+$$

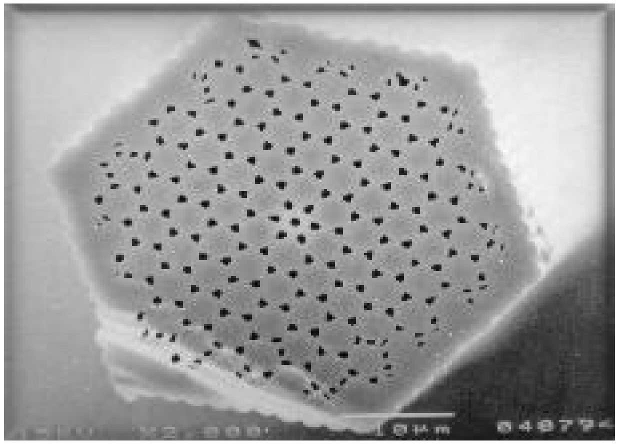
$$\partial_t E_- = c_g \partial_z E_- + i\kappa E_+ + i\Gamma(|E_-|^2 + 2|E_+|^2)E_-$$

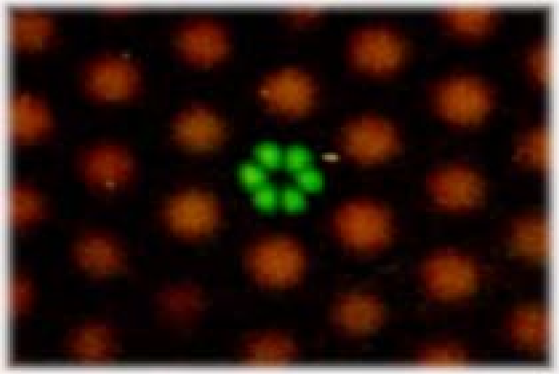
Gap solitons exist. Velocity proportional to amplitude mismatch
Between forward and backward envelopes.

1D Gap Soliton



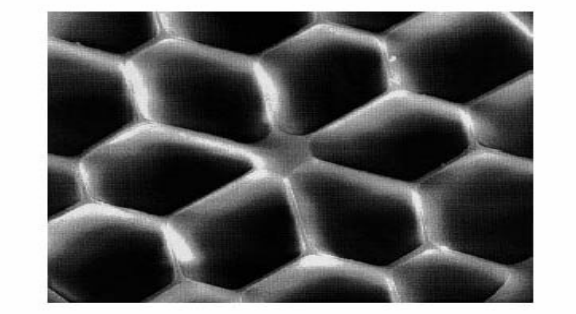
Photonic crystal fibers

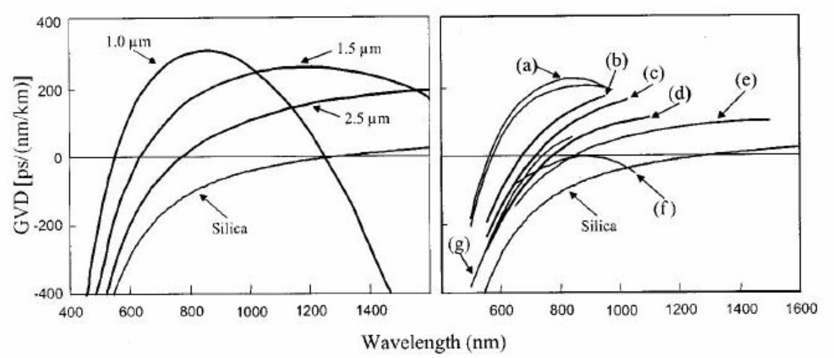


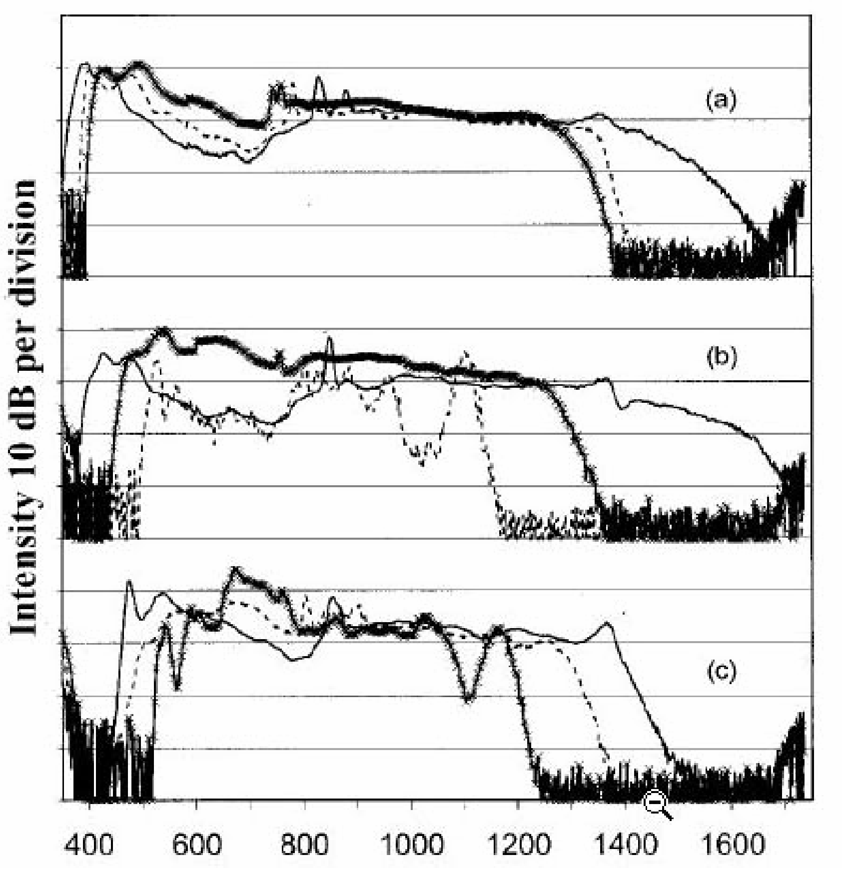


From P. Russell’s group in Bath

Experimental results



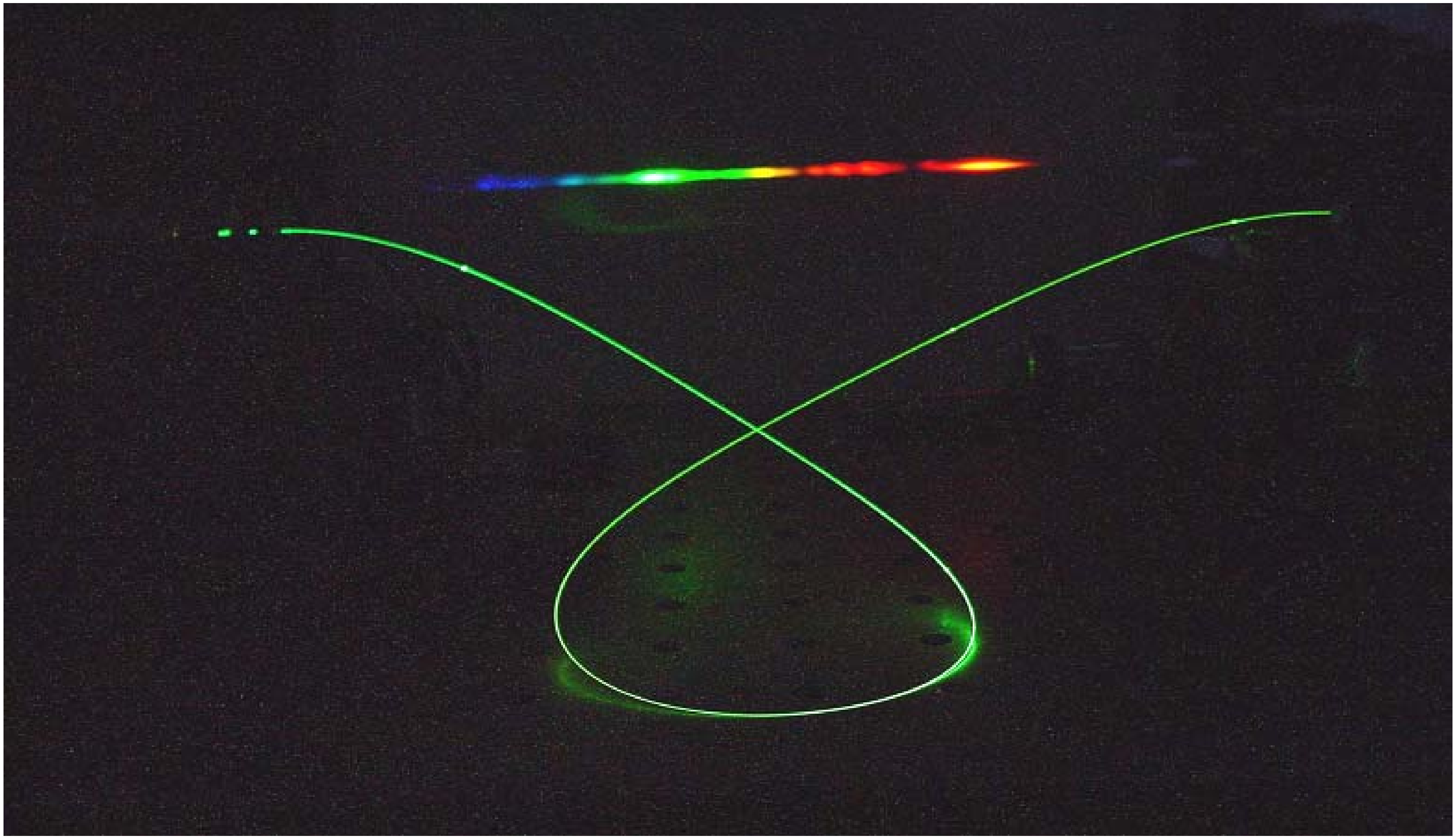




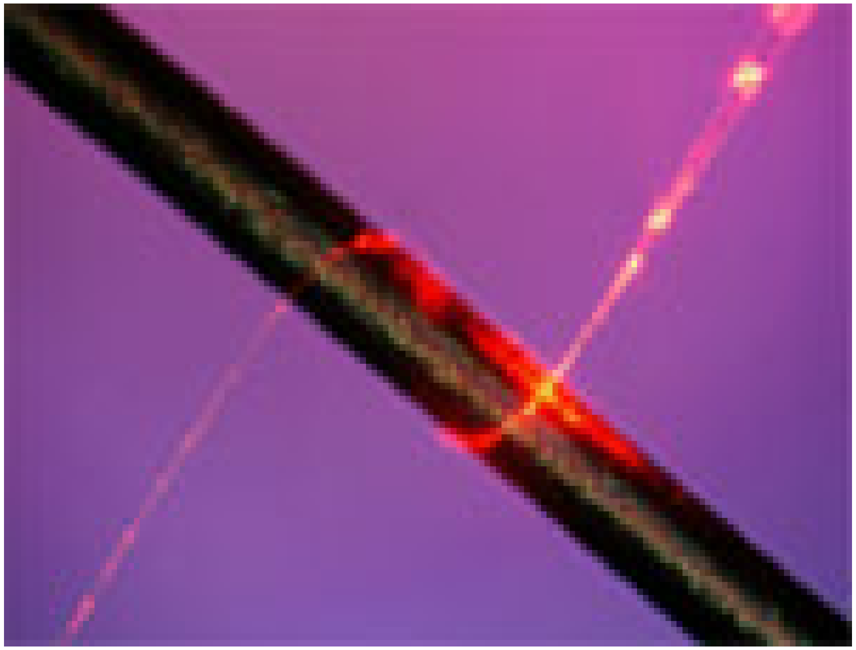
Wadsworth et al.,"Supercontinuum generation in photonic crystal fibers and optical fiber tapers: a novel light source", JOSA B, 19, 9, pp. 2148-2155, Sep. 2002.

Supercontinuum picture

F. Omeneto’s experiments LANL

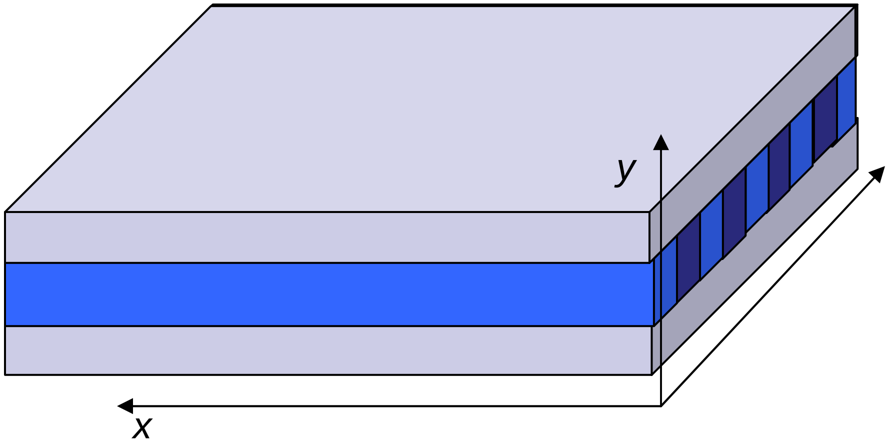


Optical nanowires



E. Mazur’s group at Harvard

II. Waveguide gratings – 2D in collaboration with Tomas Dohnal (soon to move to ETH)



Assumptions:

- dynamics in *y* arrested by a fixed *n*(*y*) profile
- *xy*-normal incidence of pulses
- characteristic length scales of coupling, nonlinearity and diffraction are in balance

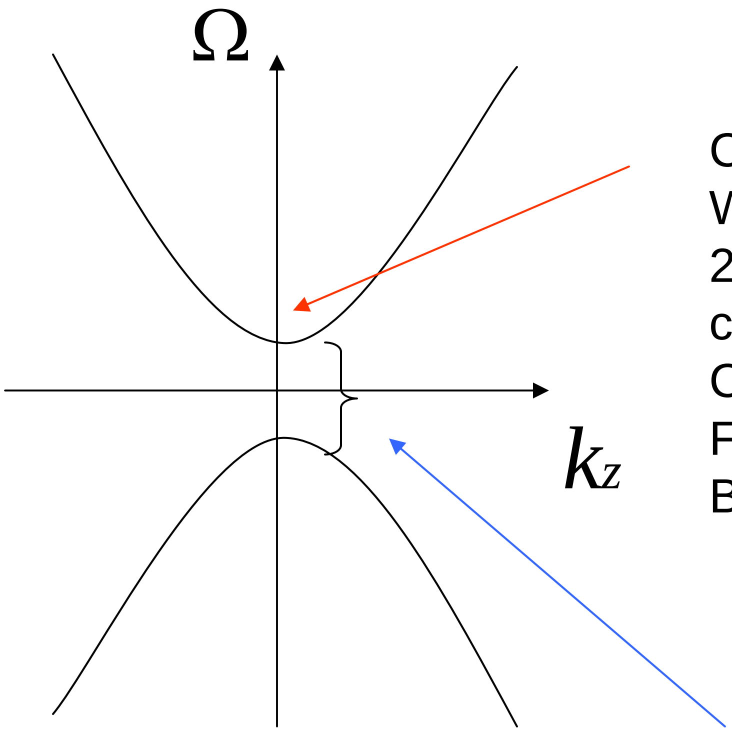
BARE 2D WAVEGUIDE

- 2D NL Schrödinger equation
- collapse phenomena: point blow-up

WAVEGUIDE GRATING

- 2D CME
- no collapse
- possibility of localization

Dispersion relation for coupled mode equations



Close, but outside the gap
Well approximated by the
2D NLSE + higher order
corrections.
Collapse arrest shown by
Fibich, Ilan, A.A (2002).
But dynamics is unstable

Frequency gap region.
We will study dynamics in this
regime.

Governing equations : 2D Coupled Mode Equations

$$\partial_t E_+ = -c_g \partial_z E_+ + id\partial_{x^2} E_+ + i\kappa E_- + i\Gamma(|E_+|^2 + 2|E_-|^2)E_+$$

$$\partial_t E_- = \quad c_g \partial_z E_- + id\partial_{x^2} E_- + i\kappa E_+ + i\Gamma(|E_-|^2 + 2|E_+|^2)E_-$$

advection

diffraction

coupling

non-linearity

$$c_g, d, \kappa, \Gamma \geq 0, \quad E_\pm : [-L_x, L_x] \times [-L_z, L_z] \times [0, \infty) \to C.$$

Stationary solutions via Newton’s iteration

If $E_\pm(x,z,t) = \mathcal{E}_\pm(x,z)\,\mathrm{e}^{-i\omega t}$ then

$$\begin{aligned} \omega\mathcal{E}_+ + ic_g\partial_z\mathcal{E}_+ + \partial_x^2\mathcal{E}_+ + \kappa\mathcal{E}_- + \Gamma(|\mathcal{E}_+|^2 + 2|\mathcal{E}_-|^2)\mathcal{E}_+ = 0, \\ \omega\mathcal{E}_- - ic_g\partial_z\mathcal{E}_- + \partial_x^2\mathcal{E}_- + \kappa\mathcal{E}_+ + \Gamma(|\mathcal{E}_-|^2 + 2|\mathcal{E}_+|^2)\mathcal{E}_- = 0. \end{aligned} \tag{1}$$

Solve (1) as a **NL eigenvalue problem** for $\left(\omega, \begin{pmatrix}\mathcal{E}_+ \\ \mathcal{E}_-\end{pmatrix}\right)$ via Newton’s iteration.

Need one more equation:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\mathcal{E}_+|^2 + |\mathcal{E}_-|^2 dz dx = N.$$

Initial guess: $\left(\omega^{(0)}, \mathcal{E}_\pm^{(0)}(x,z)\right)$

separable waveform $\mathcal{E}_\pm^{(0)}(x,z) = \mathcal{F}_\pm(z)G(x)$, $\omega^{(0)} = \kappa\cos(\delta)$

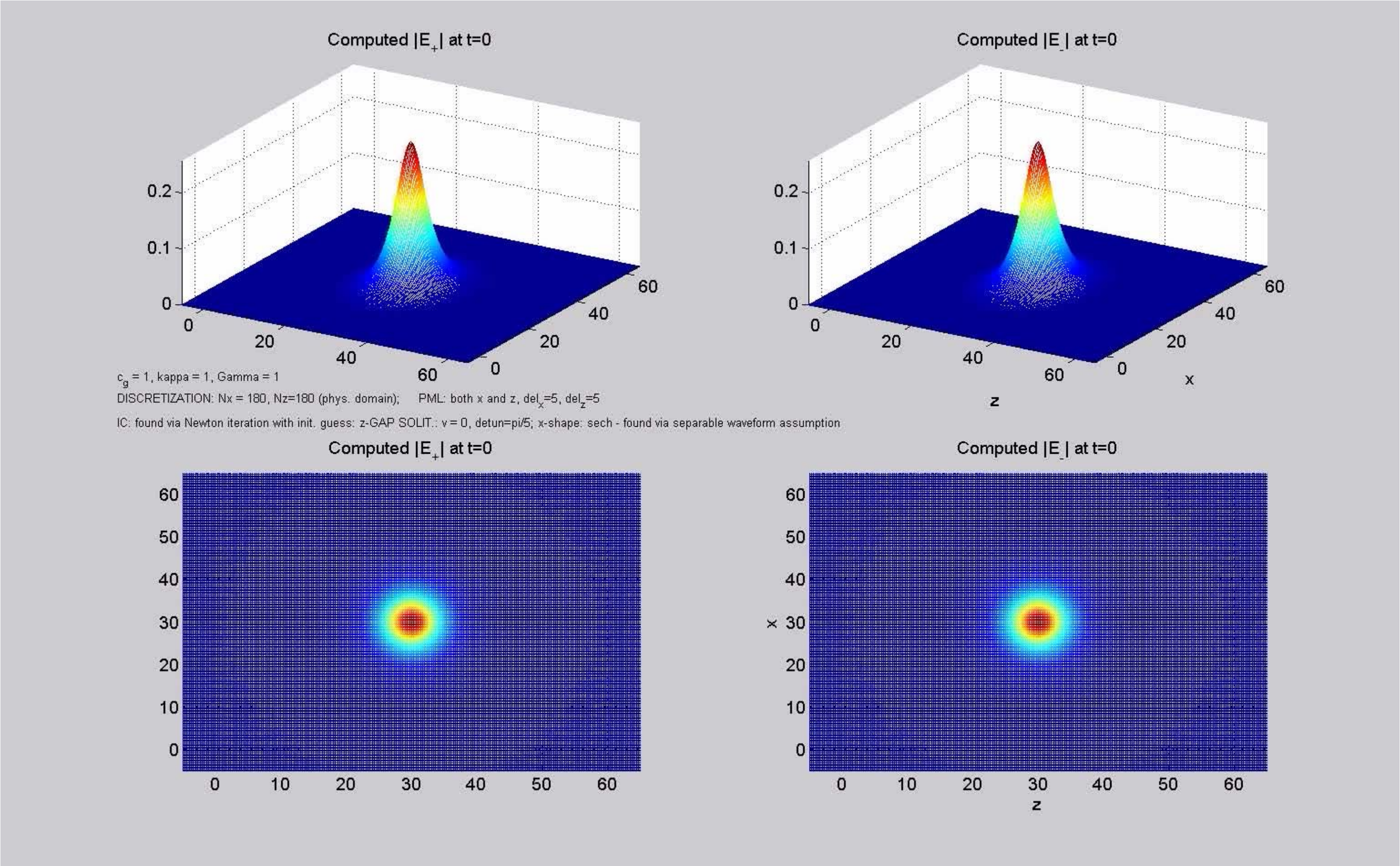
where $\mathcal{F}_\pm(z)\,\mathrm{e}^{-i\omega^{(0)}t}$ is the 1D gap soliton with $v = 0$ (free parameter $\delta \in (0,\pi)$)

Substitute and integrate in z:

$$G'' + b(G^3 - G) = 0, \qquad b = 2\frac{\kappa}{\delta}(\sin(\delta) - \delta\cos(\delta)), \qquad \delta \in (0, \pi/2)$$

$$\Rightarrow G(x) = \sqrt{2}\,\mathrm{sech}(\sqrt{b}x)$$

Stationary case I



Computed $|E_+|$ at t=0

Computed $|E_-|$ at t=0

c_g = 1, kappa = 1, Gamma = 1

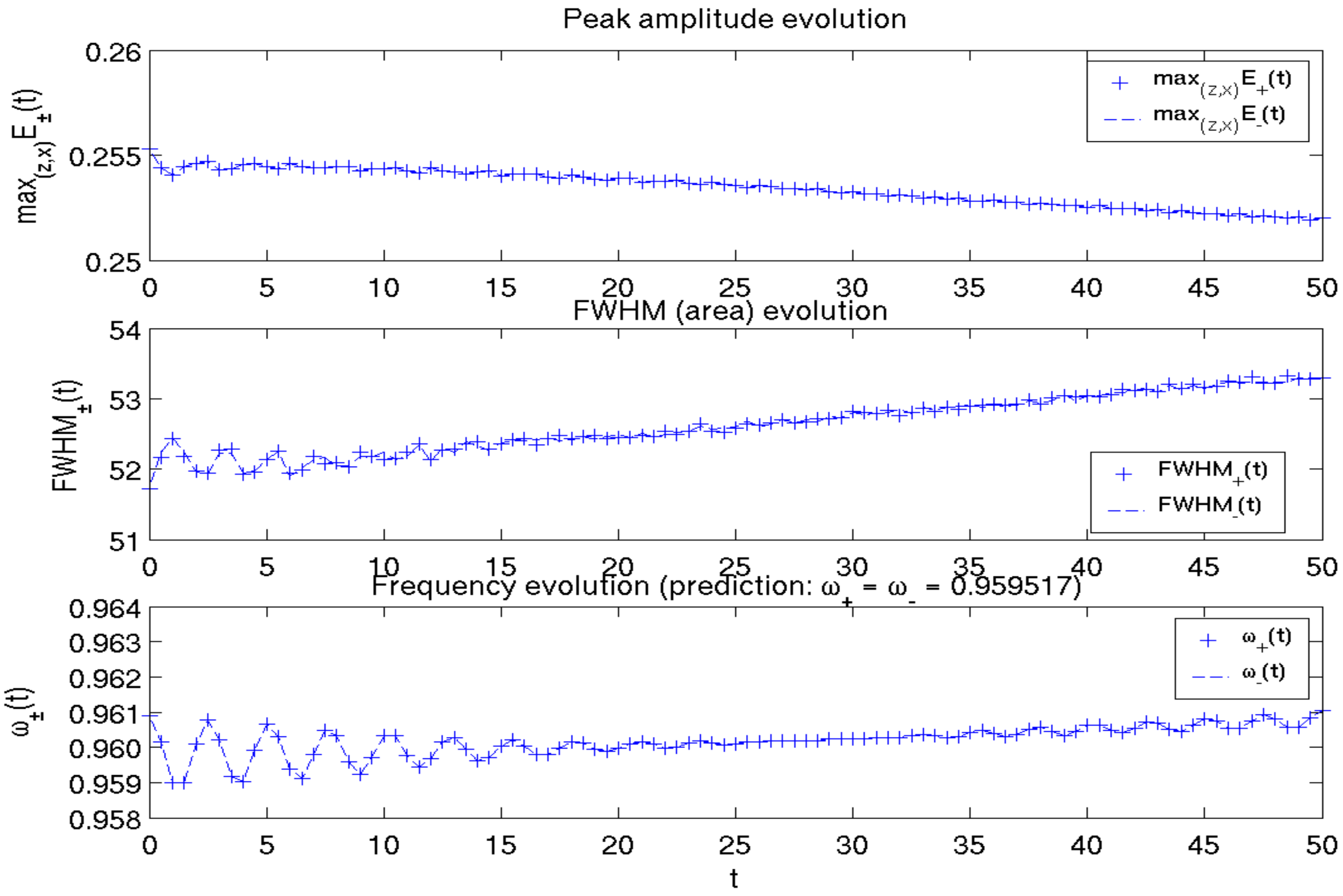
DISCRETIZATION: Nx = 180, Nz=180 (phys. domain); PML: both x and z, del_x=5, del_z=5

IC: found via Newton iteration with init. guess: z-GAP SOLIT.: v = 0, detun=pi/5; x-shape: sech - found via separable waveform assumption

Computed $|E_+|$ at t=0

Computed $|E_-|$ at t=0

Stationary case I



Nonexistence of stable bound states

CME are Hamiltonian: $\partial_t E_\pm = -i\,\frac{\delta H}{\delta E_\pm^*}, \quad \partial_t E_\pm^* = i\,\frac{\delta H}{\delta E_\pm},$

$$\begin{aligned} H = \int_{\mathbb{R}}\int_{\mathbb{R}} \quad & i c_g \left(E_-^* \partial_z E_- - E_+^* \partial_z E_+\right) - \kappa \left(E_- E_+^* + E_-^* E_+\right) + \\ & |\partial_x E_+|^2 + |\partial_x E_-|^2 - \Gamma \left(\tfrac{1}{2}|E_+|^4 + 2|E_-|^2|E_+|^2 + \tfrac{1}{2}|E_-|^4\right) dx\,dz \end{aligned}$$

Assume there is a **stable bound state** $\begin{pmatrix} \mathcal{E}_+(x,z) \\ \mathcal{E}_-(x,z) \end{pmatrix} \mathrm{e}^{-i\omega t}$ with $\int |\mathcal{E}_+|^2 + |\mathcal{E}_-|^2 dx\,dz = N.$

$(\mathcal{E}_+, \mathcal{E}_-)$ must be a **local minimizer** of H within S.

($S =$ all functions with a finite H and the sum of squares of the L^2 norms equal to N.)

Fact: $(\mathcal{E}_+, \mathcal{E}_-)$ must also minimize H within any subset of S.

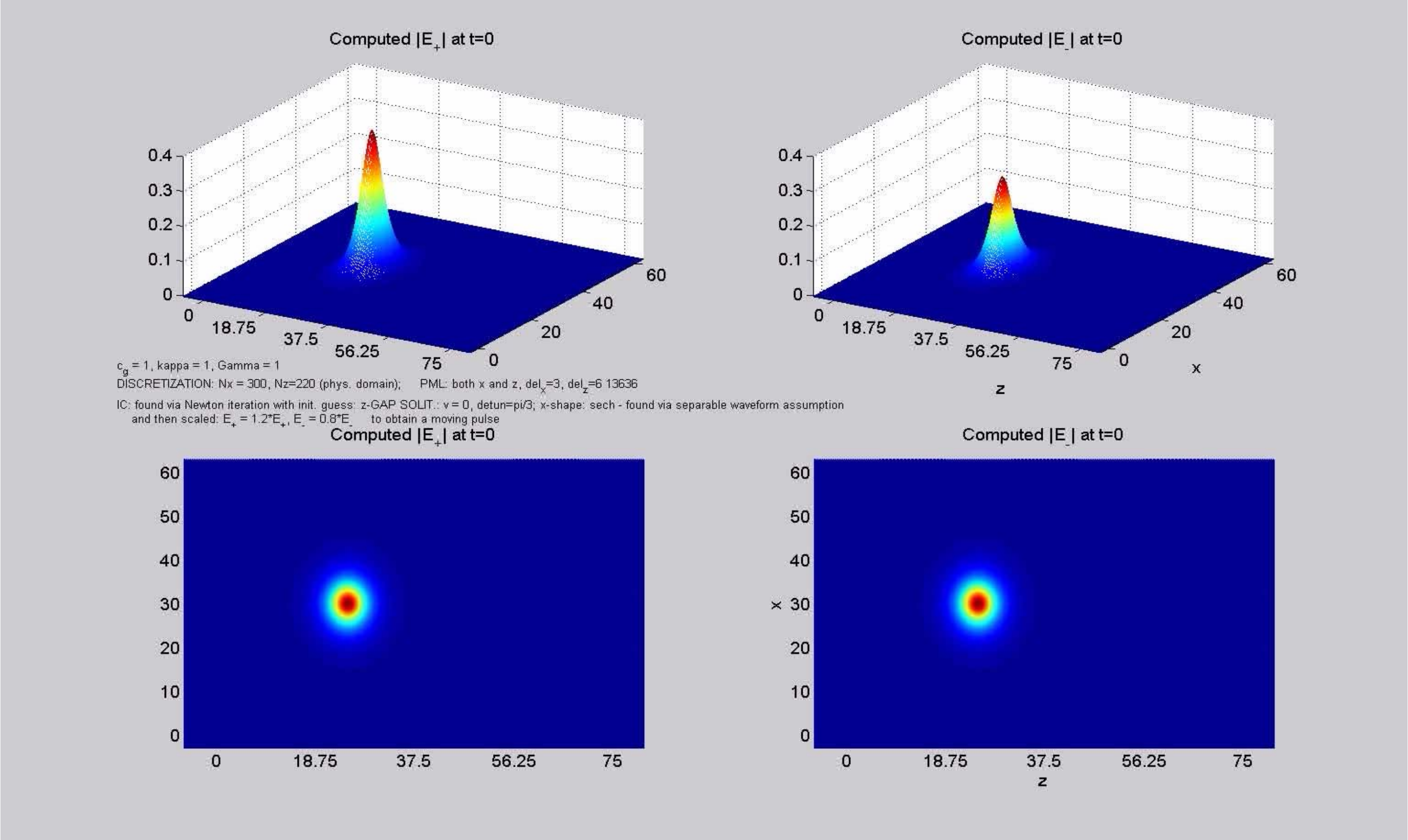
Consider $S_1 := \left\{ \alpha \begin{pmatrix} \mathcal{E}_+(x/\mu, z/\nu) \\ \mathcal{E}_-(x/\mu, z/\nu) \end{pmatrix} : \ \alpha^2 \mu\nu = 1 \right\}$ with $\alpha, \mu, \nu > 0$ $\qquad (S_1 \subset S)$

Within S_1 Hamiltonian H reduces to (employing the constraint $\alpha^2\mu\nu = 1$)

$$H = H_r(\alpha, \nu) = \frac{1}{\nu} A_1 - A_2 + \alpha^2 \nu^2 A_3 - \alpha^2 A_4, \qquad A_i \ldots \text{ integrals of the fixed } \mathcal{E}_\pm$$

H_r has only a **saddle** critical point $\Rightarrow$ contradiction with $(\mathcal{E}_+, \mathcal{E}_-)$ minimizing H.

Moving breather

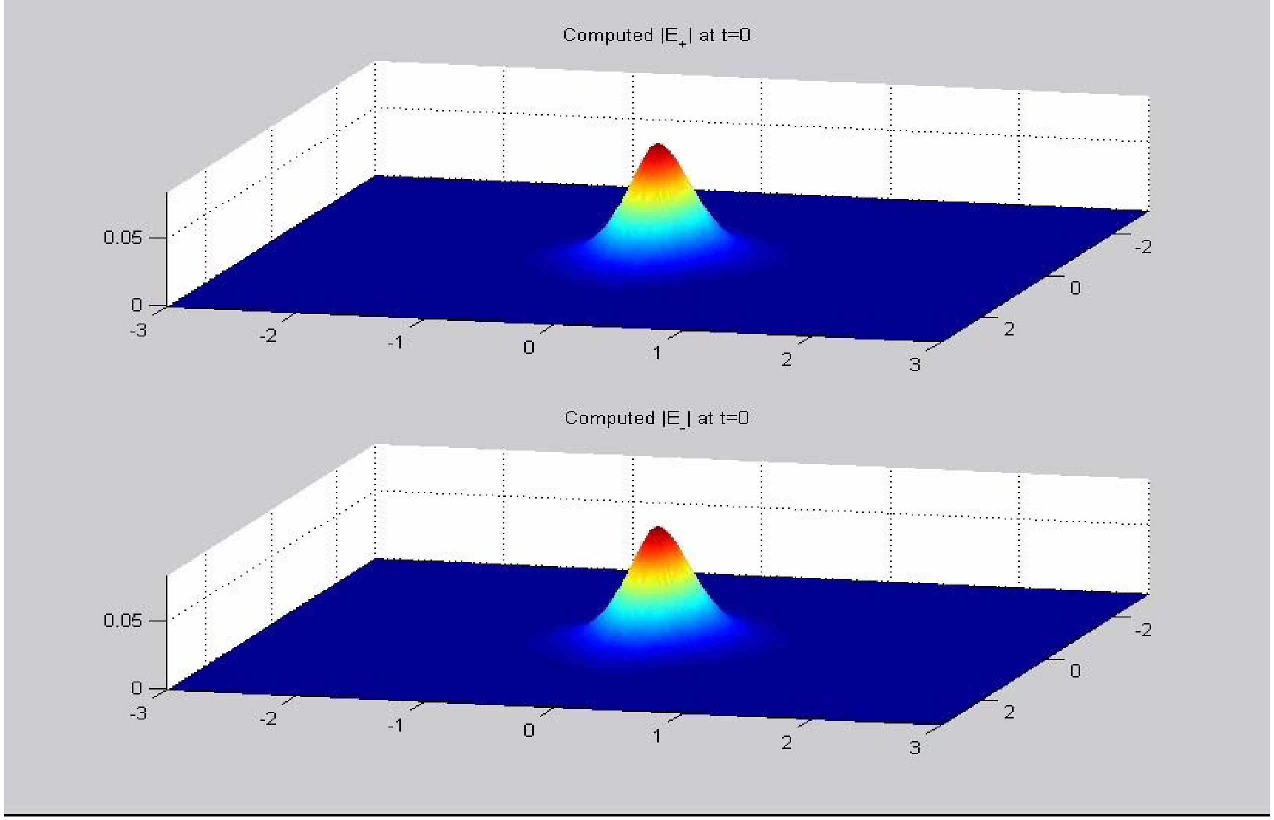


c_g = 1, kappa = 1, Gamma = 1

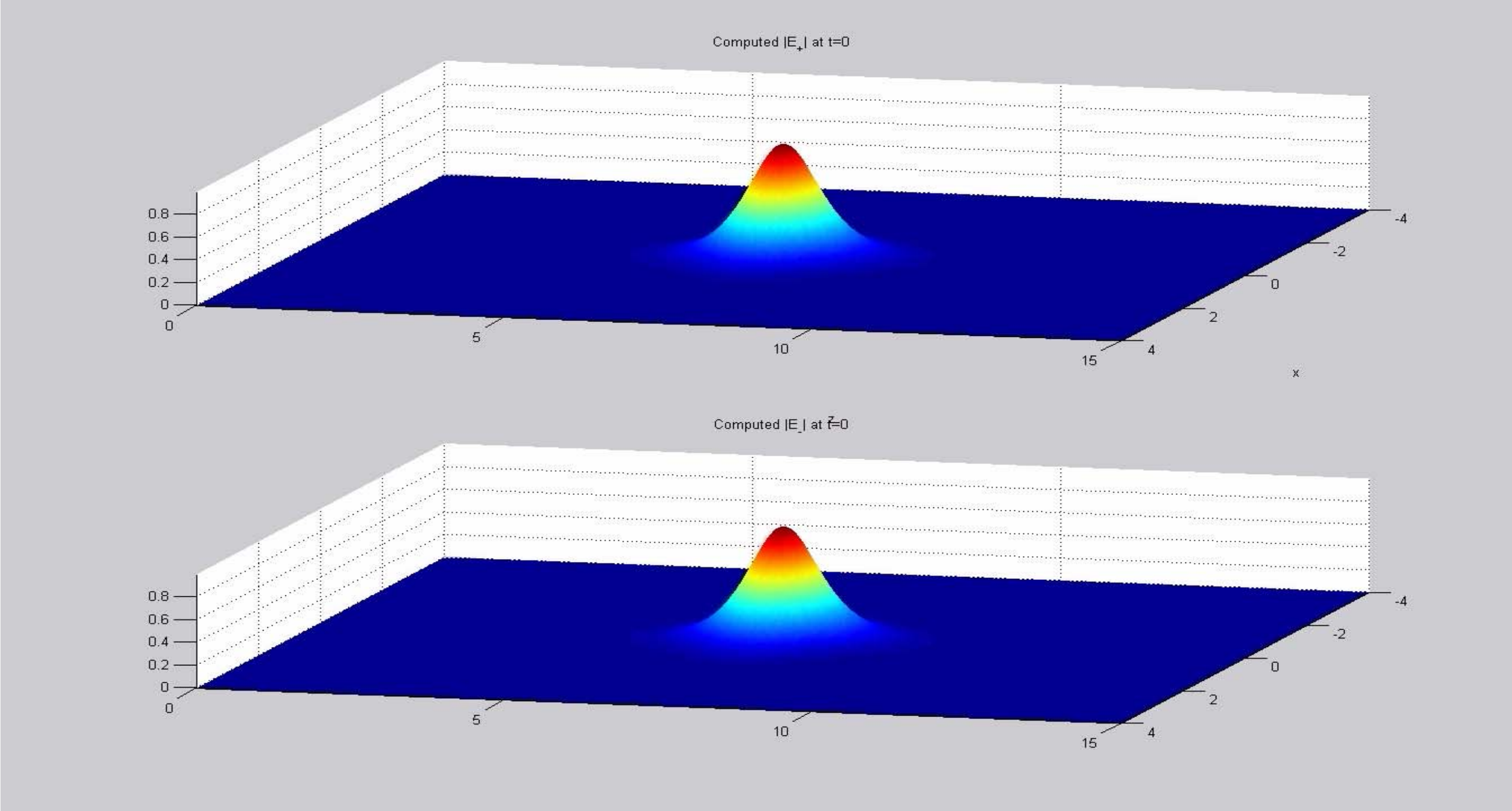
DISCRETIZATION: Nx = 300, Nz=220 (phys. domain); PML: both x and z, del_x=3, del_z=6 13636

IC: found via Newton iteration with init. guess: z-GAP SOLIT.: v = 0, detun=pi/3; x-shape: sech - found via separable waveform assumption and then scaled: $E_+ = 1.2^*E_+$, $E_- = 0.8^*E_-$ to obtain a moving pulse

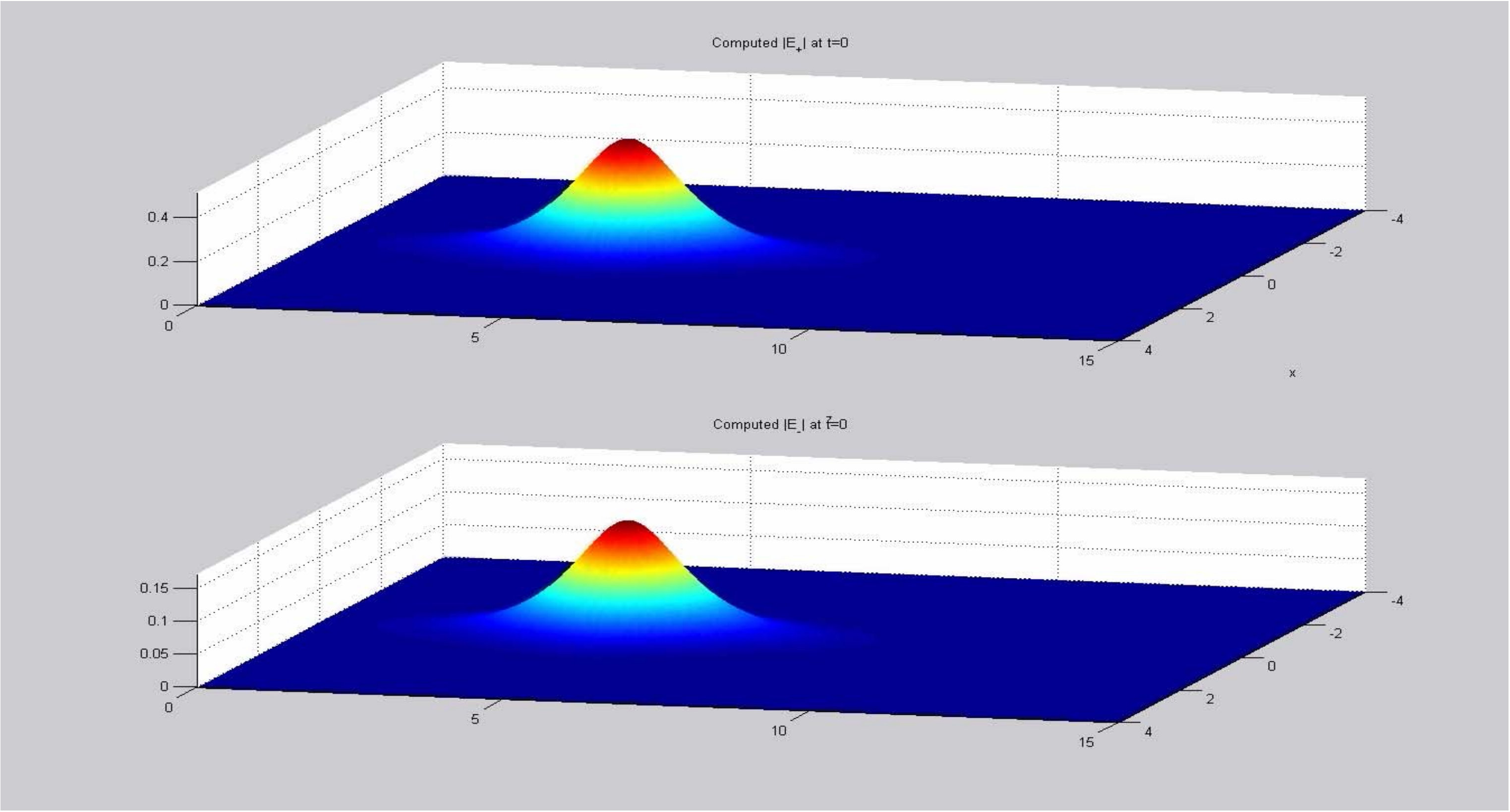
Gaussian initial condition, “large potential”



Sech initial condition

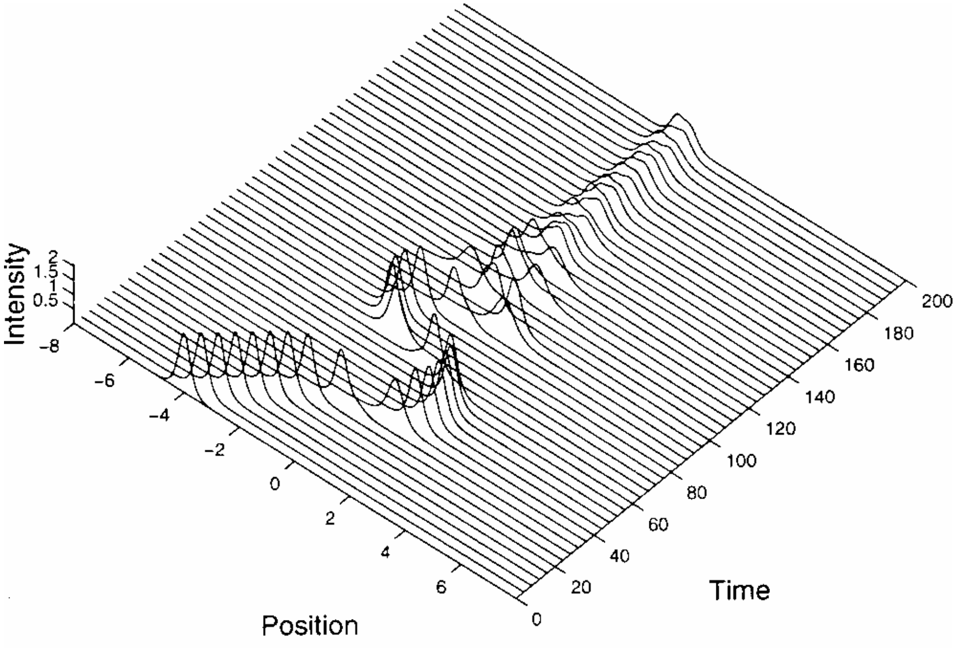


Gap soliton initial condition



Adding defects in 2-dim periodic structures

- Trapping of a gap soliton bullet in a defect. (this concept has been theoretically demonstrated by Goodman, Weinstein, Slusher for the 1-dim (fiber bragg grating with a defect) case).



Ref: R. Goodman et.al., JOSA B 19, 1635 (July 2002)

Linear Defect Modes (no *x*-grating)

$$\partial_t E_+ = -c_g \partial_z E_+ + i\partial_{x^2} E_+ + i\kappa(z)E_- + iV_1(x)E_+ + iV_2(z)E_+$$

$$\partial_t E_- = \ c_g \partial_z E_- + i\partial_{x^2} E_- + i\kappa(z)E_+ + iV_1(x)E_- + iV_2(z)E_-$$

If $V_1 = 2\beta^2\operatorname{sech}^2(\beta x)$, $V_2 = \dfrac{c_g}{2}\dfrac{k^2(\omega+\beta^2)\operatorname{sech}^2(k z)}{(\omega+\beta^2)^2+k^2\tanh^2(k z)}$,

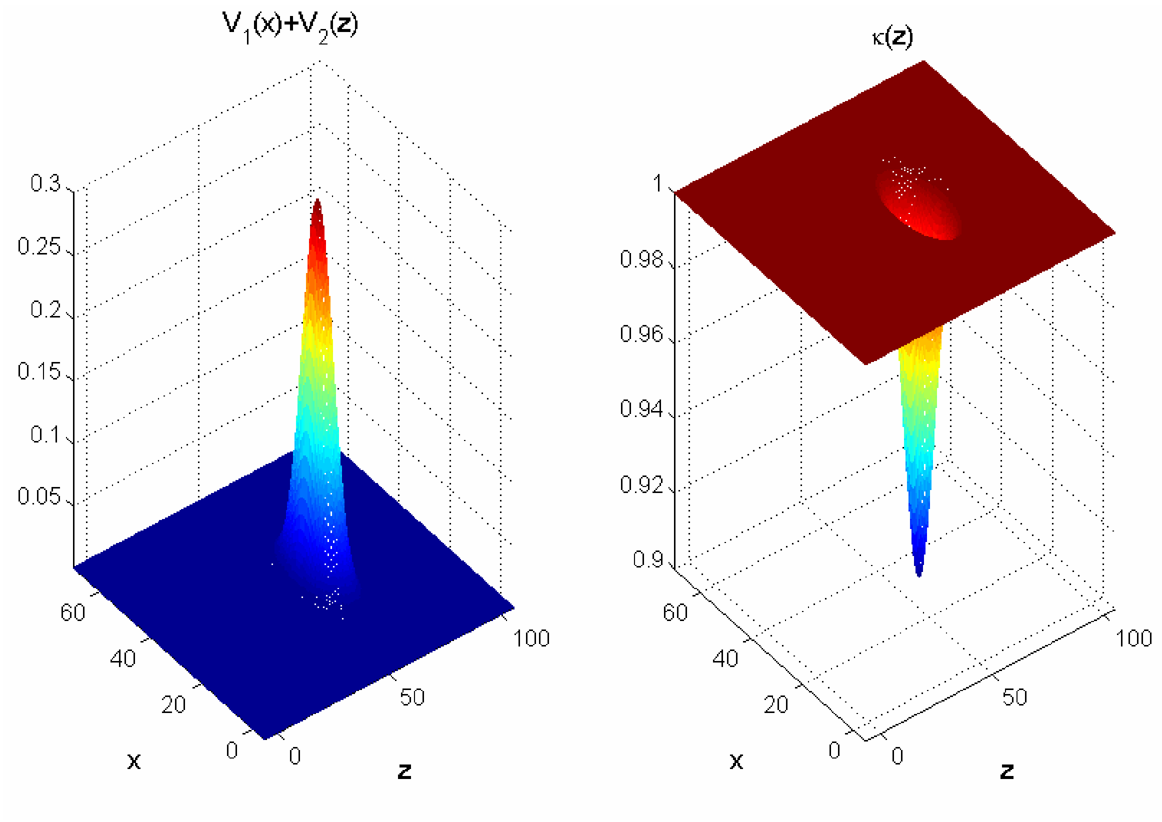
All space

$\kappa(z) = e^{i\alpha}\left[(\omega+\beta^2)^2+k^2\tanh^2(k z)\right]^{1/2}$ with $\alpha\in\{0,\pi\}$, $\beta,k\in R$, then

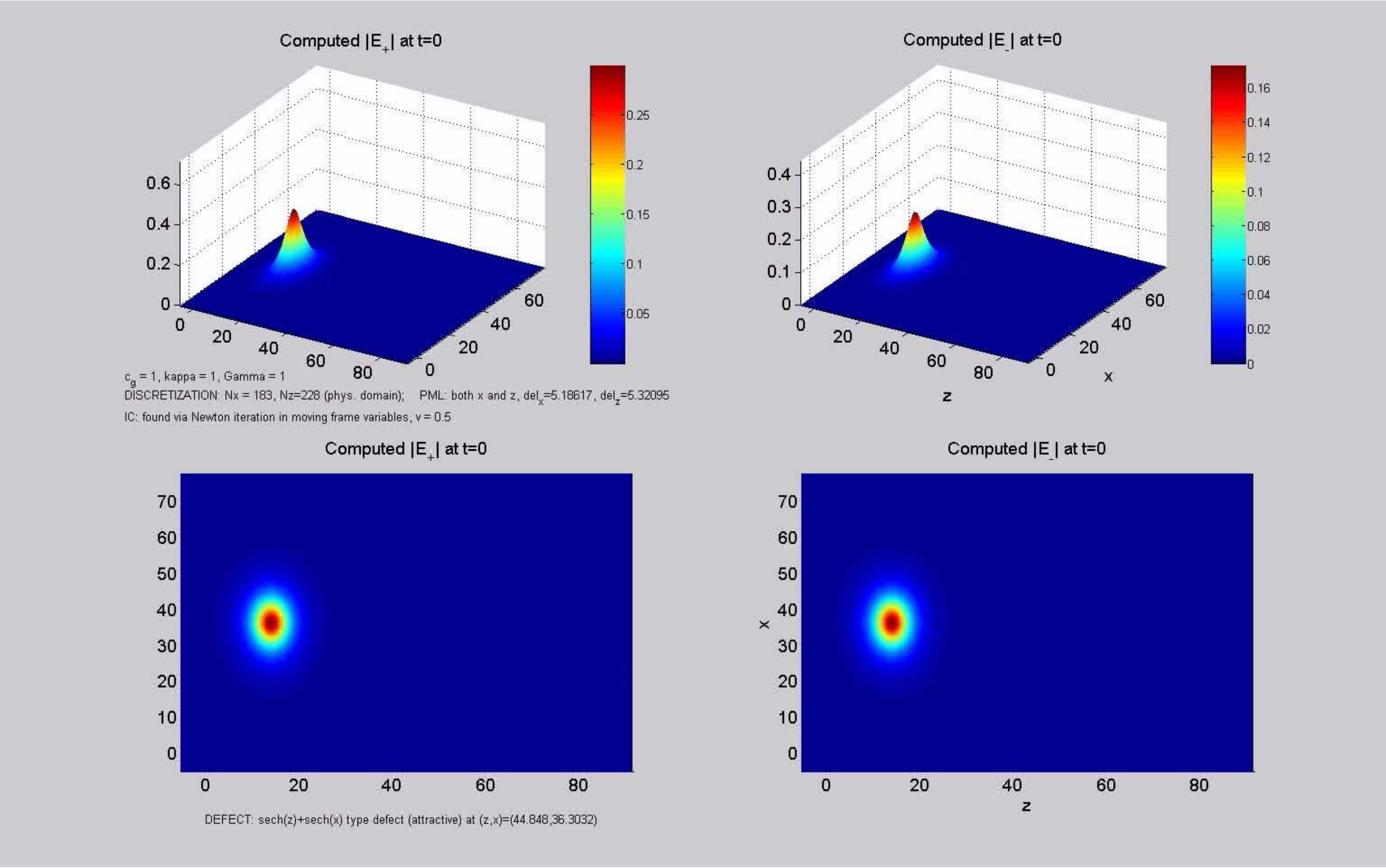
$$\begin{pmatrix} E_+ \\ E_- \end{pmatrix} = \begin{pmatrix} \exp\left(\dfrac{i}{2c_g}\arctan\left(\dfrac{k\tanh(k z)}{\omega+\beta^2}\right)\right) \\ ie^{-i\phi}\exp\left(\dfrac{-i}{2c_g}\arctan\left(\dfrac{k\tanh(k z)}{\omega+\beta^2}\right)\right) \end{pmatrix} e^{-i\omega t}\operatorname{sech}(k z)\operatorname{sech}(\beta x)$$

is a solution for $\phi = \dfrac{3}{2}\pi+\alpha$ and any $\omega\in R.$

Profile of defects



Transmission through a defect



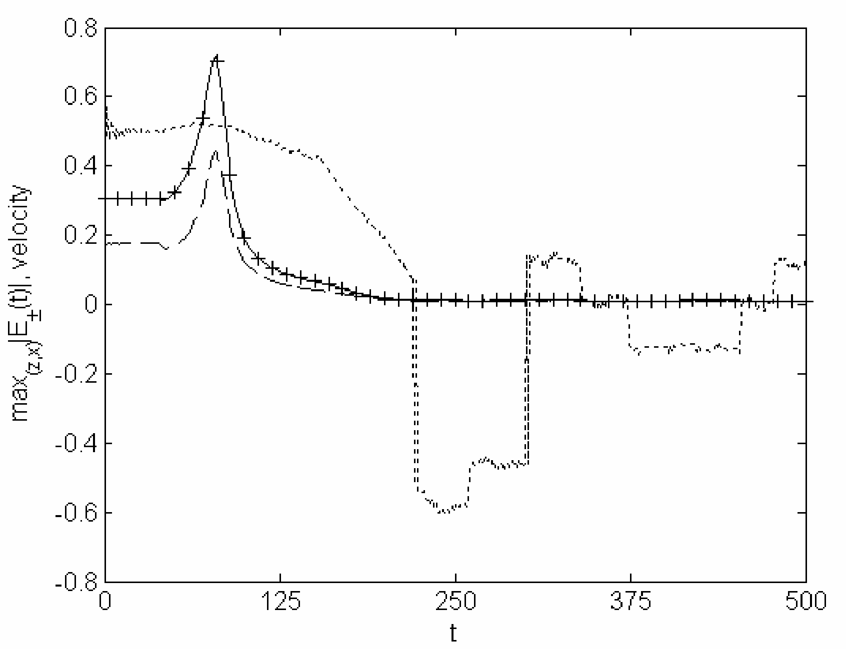
c_g = 1, kappa = 1, Gamma = 1

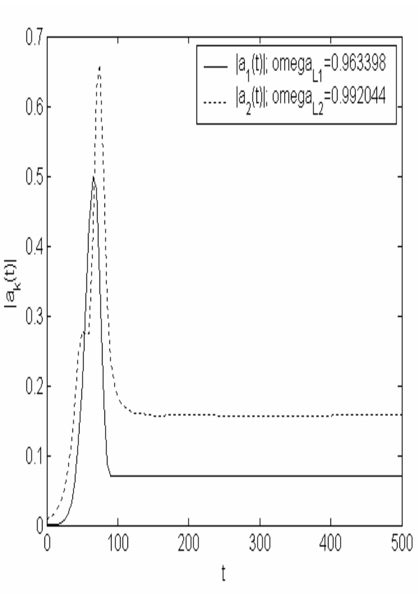
DISCRETIZATION: Nx = 183, Nz=228 (phys. domain); PML: both x and z, del_x=5.18617, del_z=5.32095

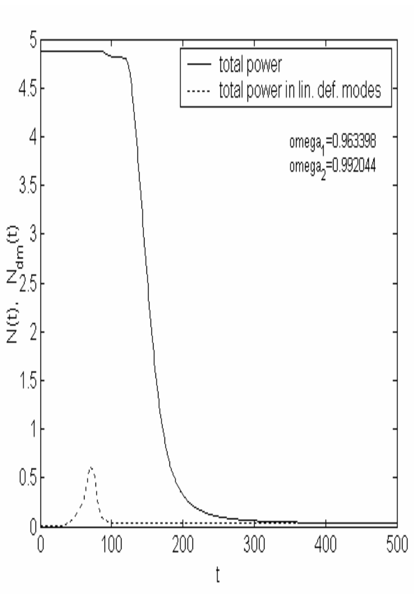
IC: found via Newton iteration in moving frame variables, v = 0.5

DEFECT: sech(z)+sech(x) type defect (attractive) at (z,x)=(44.848,36.3032)

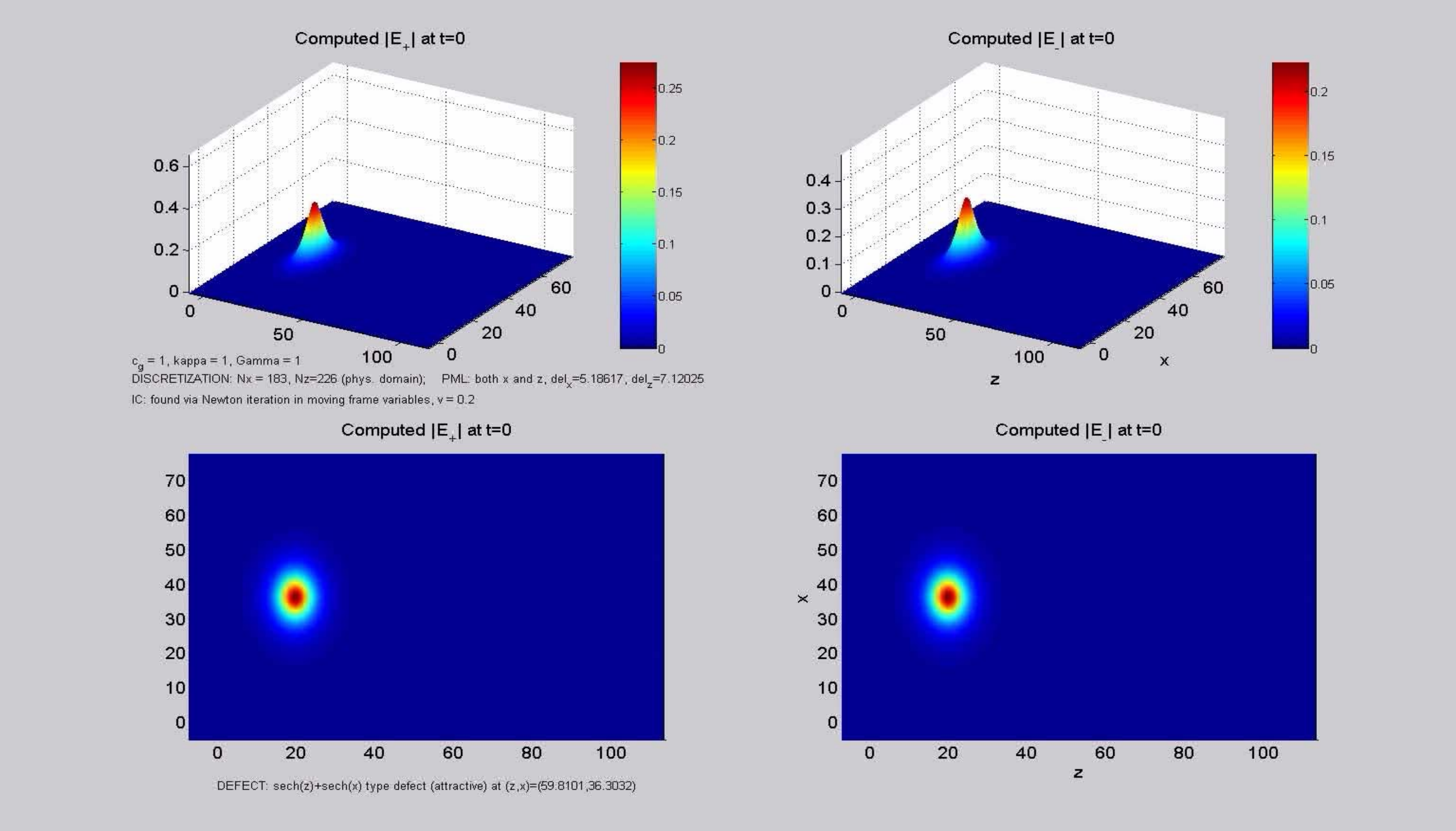
Energy, velocity and mode coupling







Trapping into 2 defect modes



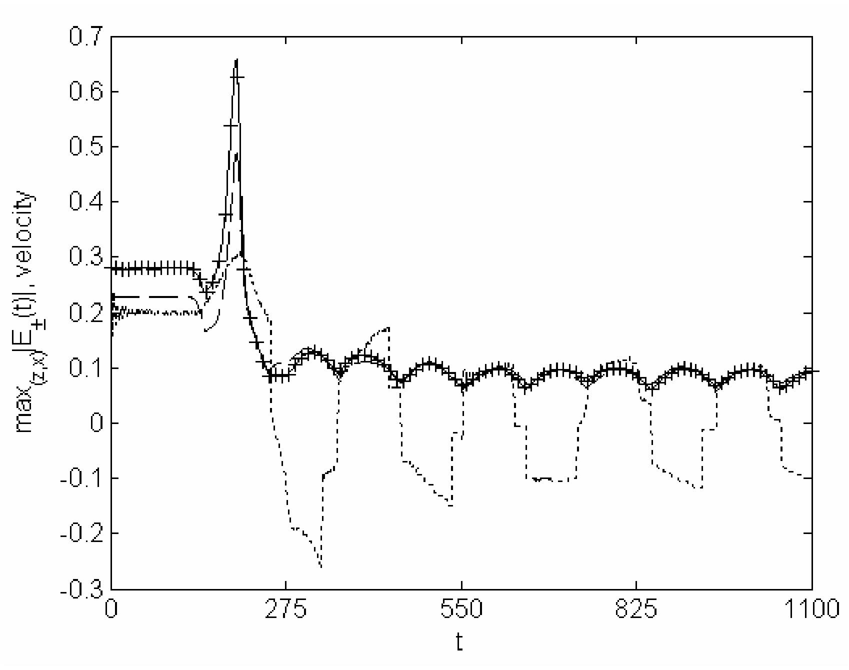
c_g = 1, kappa = 1, Gamma = 1

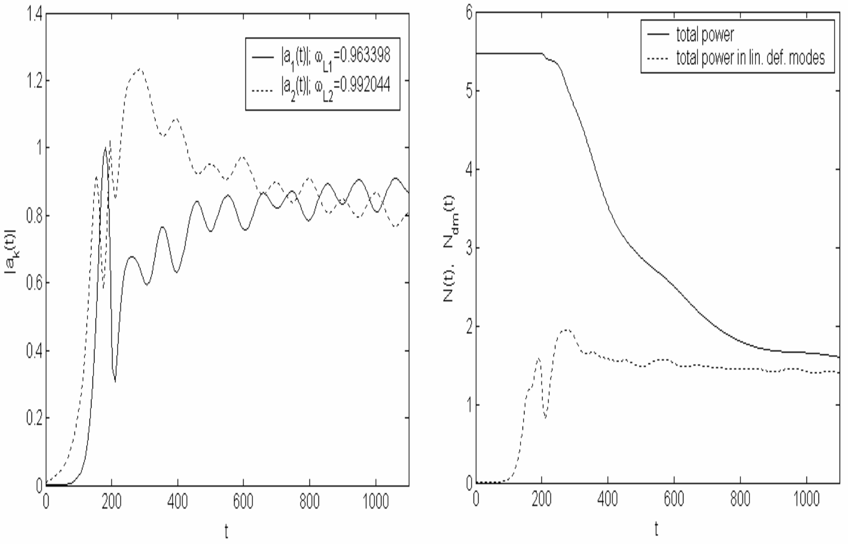
DISCRETIZATION: Nx = 183, Nz=226 (phys. domain); PML: both x and z, del_x=5.18617, del_z=7.12025

IC: found via Newton iteration in moving frame variables, v = 0.2

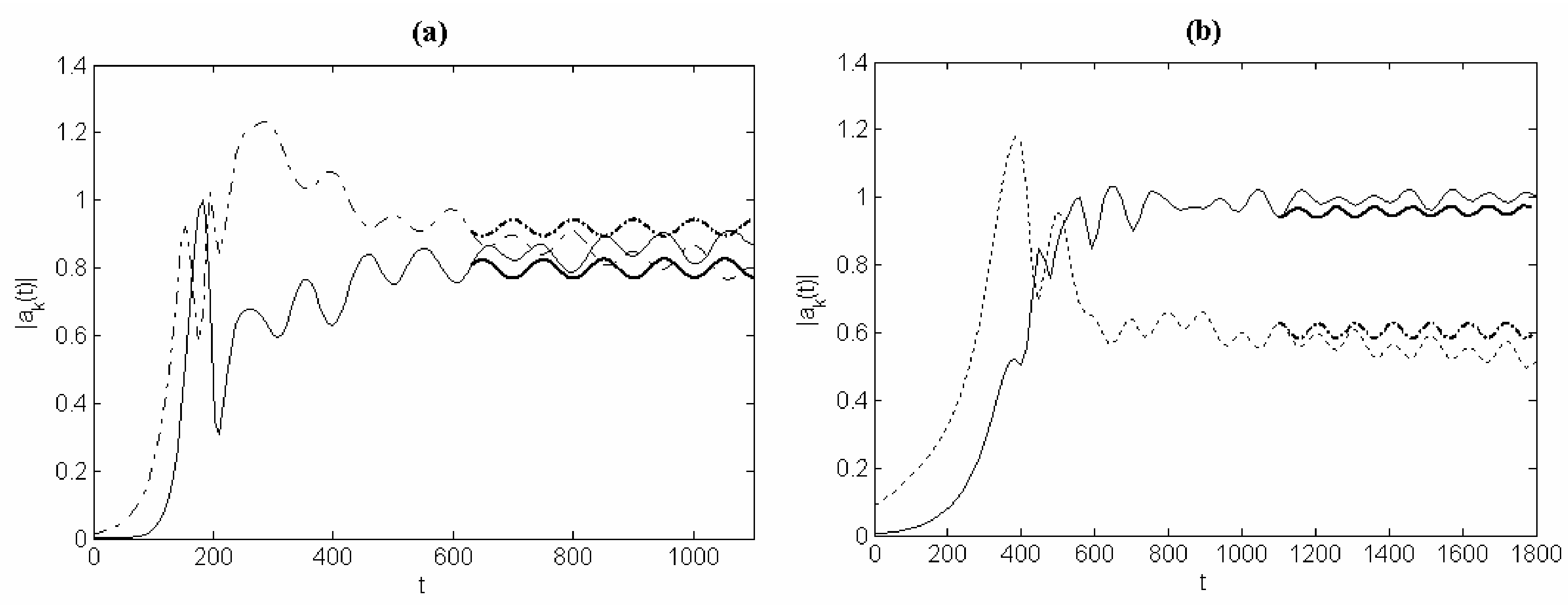
DEFECT: sech(z)+sech(x) type defect (attractive) at (z,x)=(59.8101,36.3032)

Energy, velocity, mode coupling

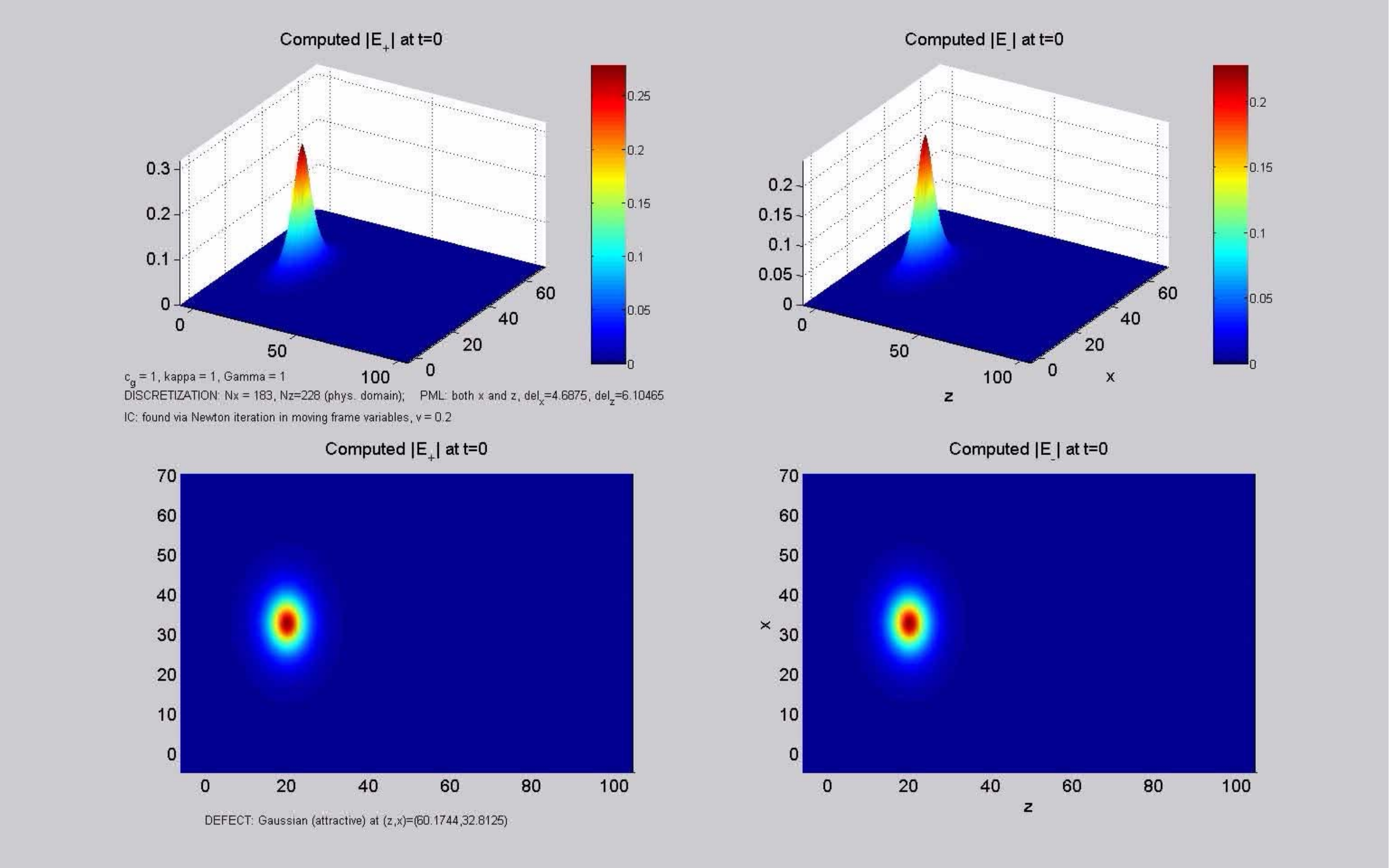




Comparison with truncated mode dynamics



Trapping into several defect modes



Computed $|E_+|$ at t=0

Computed $|E_-|$ at t=0

c_g = 1, kappa = 1, Gamma = 1

DISCRETIZATION: Nx = 183, Nz=228 (phys. domain); PML: both x and z, del_x=4.6875, del_z=6.10465

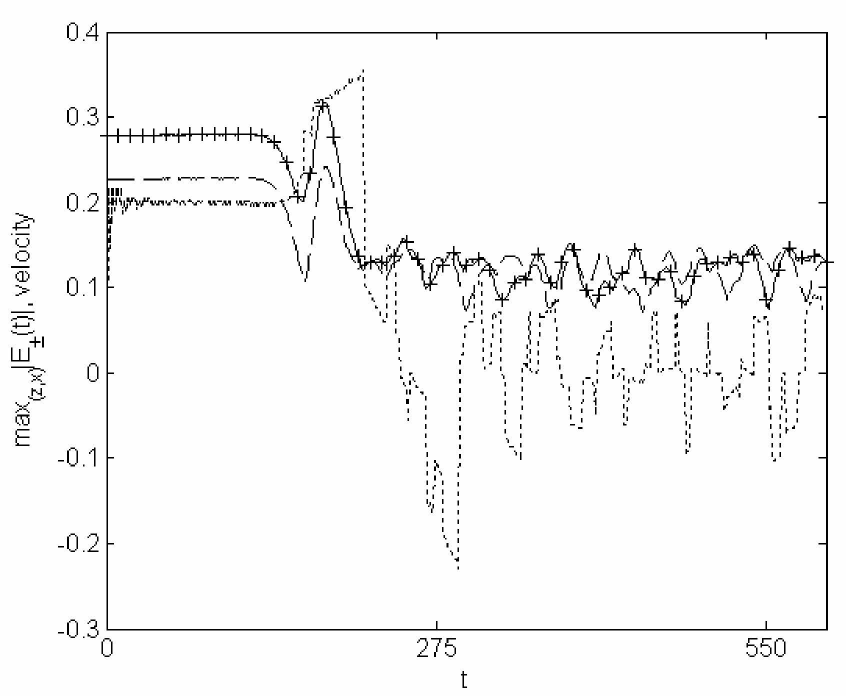
IC: found via Newton iteration in moving frame variables, v = 0.2

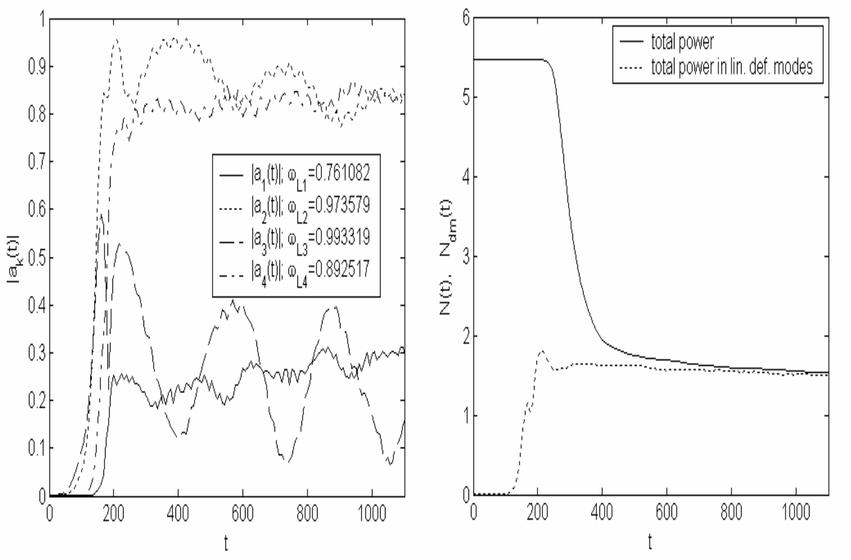
Computed $|E_+|$ at t=0

Computed $|E_-|$ at t=0

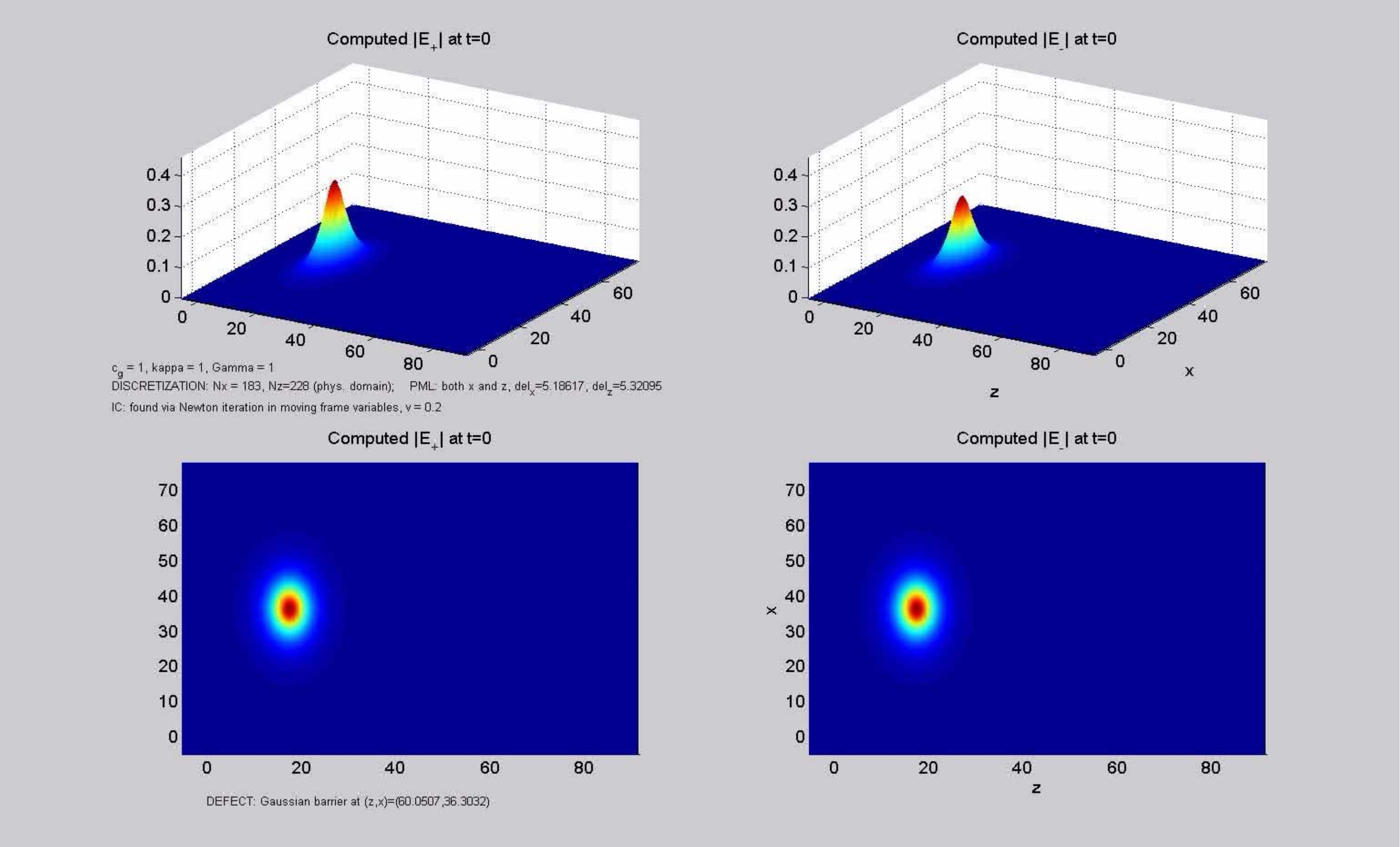
DEFECT: Gaussian (attractive) at (z,x)=(60.1744,32.8125)

Energy, velocity and mode coupling





Reflection from a “repelling” defect potential



Computed $|E_+|$ at t=0

Computed $|E_-|$ at t=0

c_g = 1, kappa = 1, Gamma = 1

DISCRETIZATION: Nx = 183, Nz=228 (phys. domain);   PML: both x and z, del_x=5.18617, del_z=5.32095

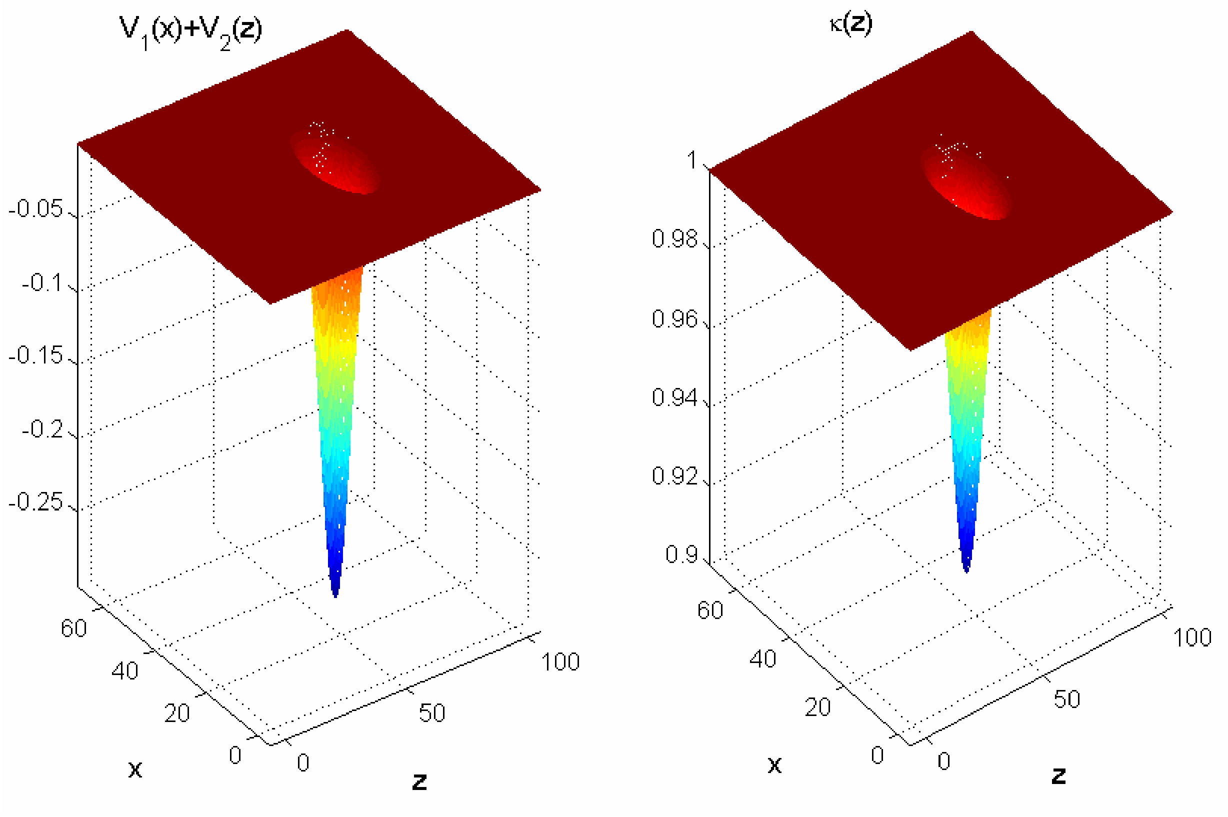
IC: found via Newton iteration in moving frame variables, v = 0.2

Computed $|E_+|$ at t=0

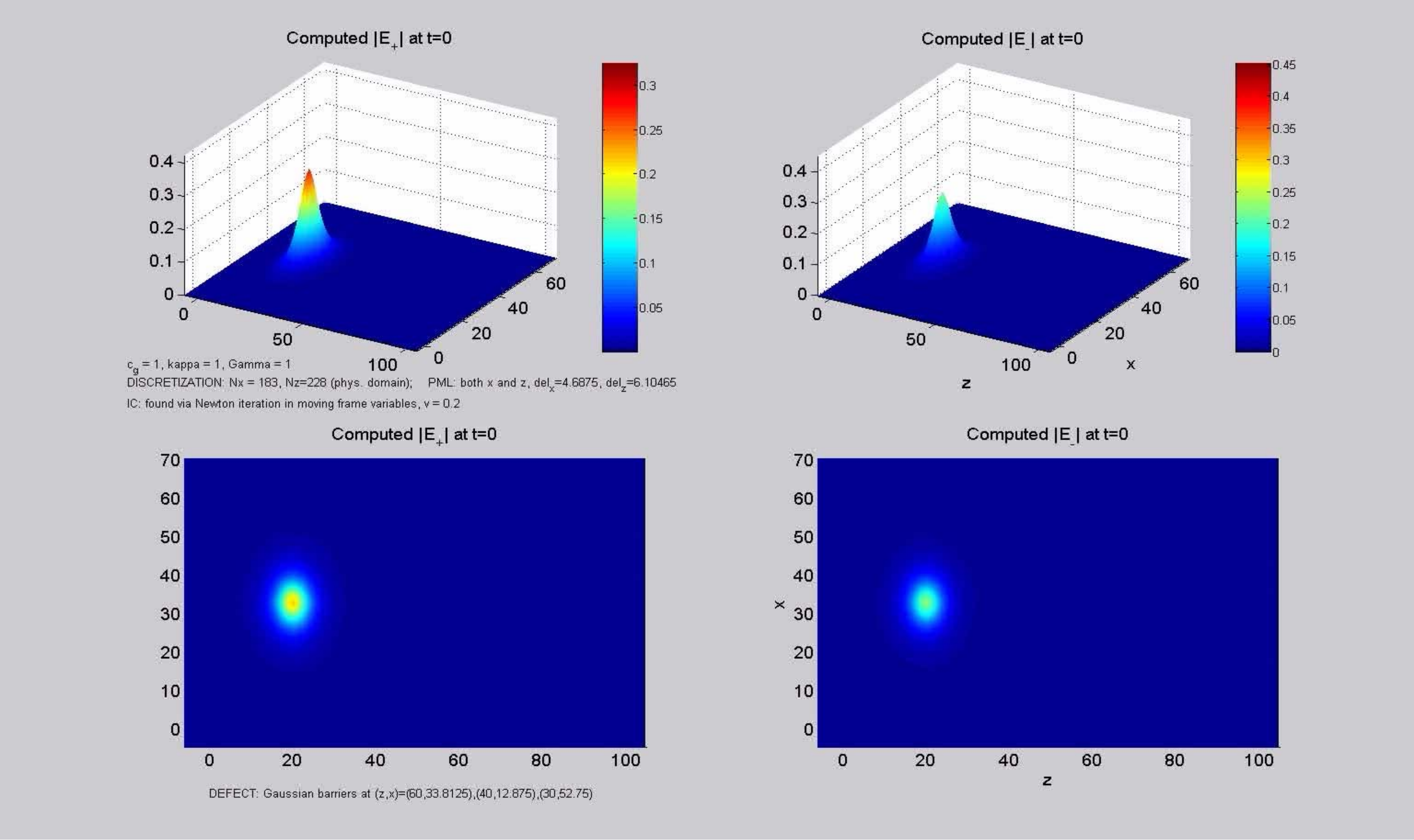
Computed $|E_-|$ at t=0

DEFECT: Gaussian barrier at (z,x)=(60.0507,36.3032)

Defect potential and grating



Reflection with a tilt, “bouncing” beam


Computed $|E_+|$ at t=0

Computed $|E_-|$ at t=0

c_g = 1, kappa = 1, Gamma = 1

DISCRETIZATION: Nx = 183, Nz=228 (phys. domain); PML: both x and z, del_x=4.6875, del_z=6.10465

IC: found via Newton iteration in moving frame variables, v = 0.2

Computed $|E_+|$ at t=0

Computed $|E_-|$ at t=0

DEFECT: Gaussian barriers at (z,x)=(60,33.8125),(40,12.875),(30,52.75)

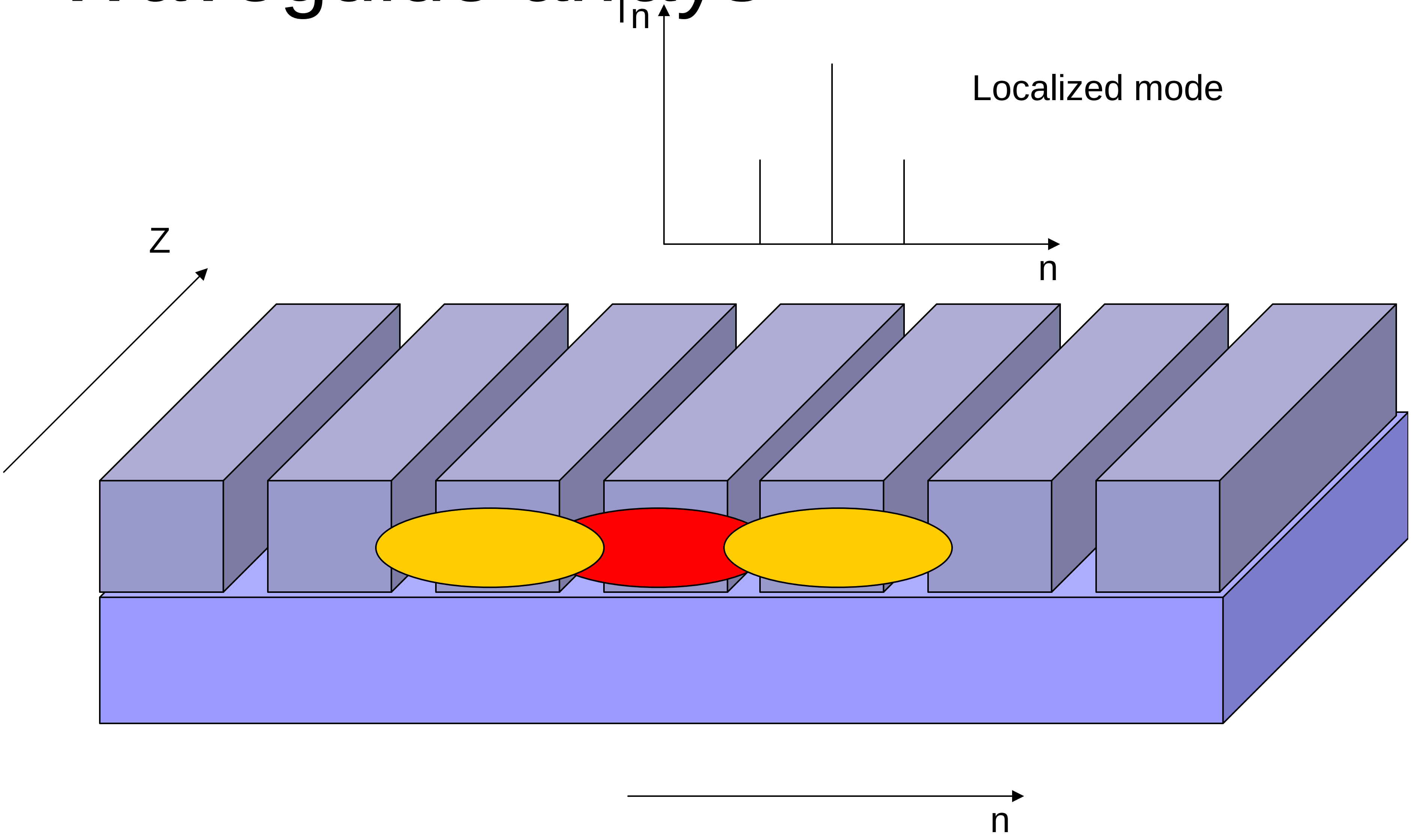
Conclusions

- 2d nonlinear photonic structures show promising properties for quasi-stable propagation of “slow” light-bullets
- With the addition of defects, we are currently studying if trapping can be achieved
- Research in line with other interesting schemes to slow down light for eventually having all optical logic devices (eg. buffers)

Conclusions (cont.)

- Examples of other interesting research on periodic structures to perform at slow group velocities include the addition of resonators (Melloni et al, Sipe, Boyd et al)
- Equivalent to the addition of defects, is having alterative layers of dielectrics and two-level atoms (Aitchison et al)

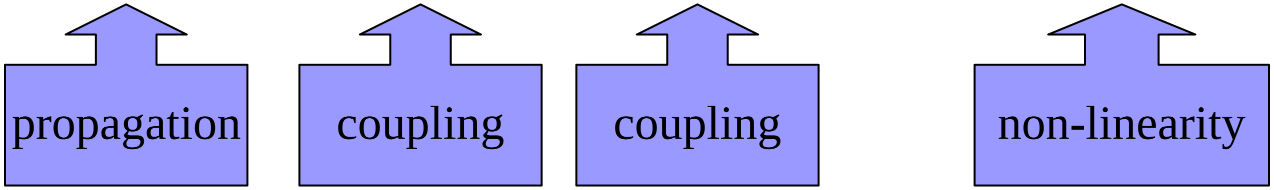
Waveguide arrays



Governing equations (coupled DNLSE)

- At each waveguide the field is represented by an envelope E_n(z,t)

$$\partial_z E_n = i\kappa(E_{n-1} + E_{n+1}) + i\Gamma |E_n|^2 E_n$$



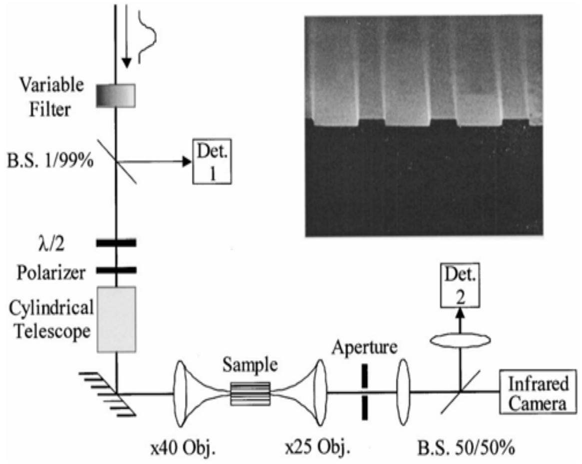
propagation

coupling

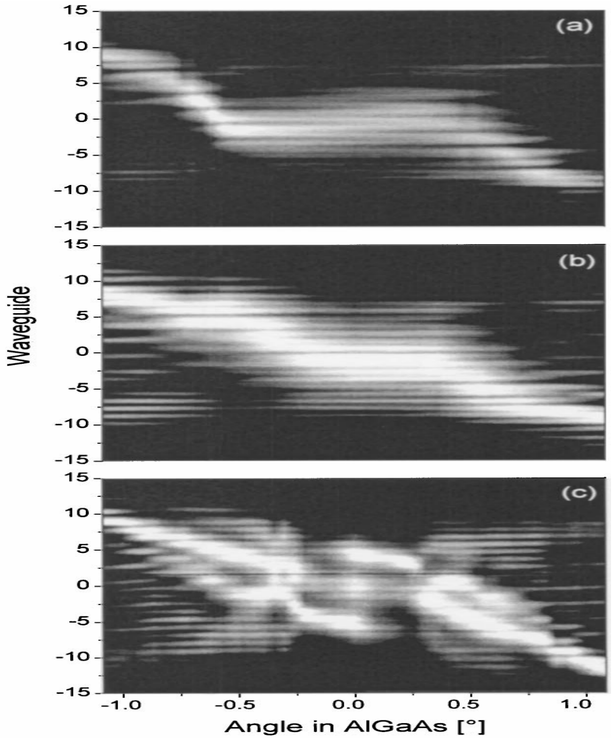
coupling

non-linearity

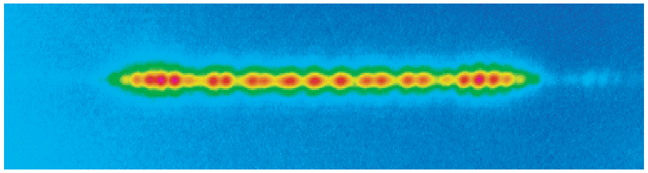
Experimental results



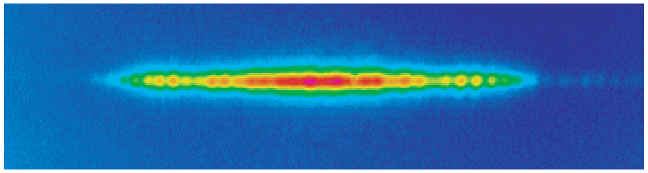
Experimental setup



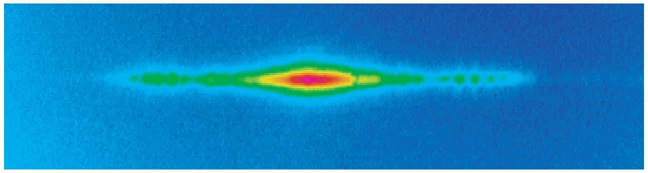
Soliton steering, breakup



Low power, linear regime



Intermediate power



Discrete soliton

Refs: Peschel et. Al., JOSA B 19, 2637 (11/2002)
Lederer and Siberberg, OPN, Feb. 2002