Bane Vasic and Ivan Djordjevic

Thanks to Ildar Gabitov

Spring School in Nonlinear and Multiscale Photonics

Binational Consortium of Optics

Modulation Codes for Ghost Pulse Reduction in 40 Gb/s Optical Communications Systems

Recent Work

- In high-speed transmission (at 40 Gb/s and above) the major nonlinear penalties come from intrachannel interactions: IFWM and IXPM.
- A common approach – a proper modulation format.
- Recent work

<u>Novel modulation formats:</u>

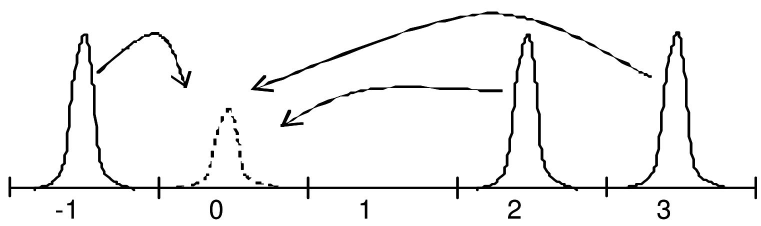
- Forzati, 2002, Alternate-phase RZ modulation format
- Liu, 2002, Inversion between adjacent marker blocks
- Xie, 2004, Alternate-polarization formats
- Djordjevic, 2004, Modified Duobinary RZ Modulation

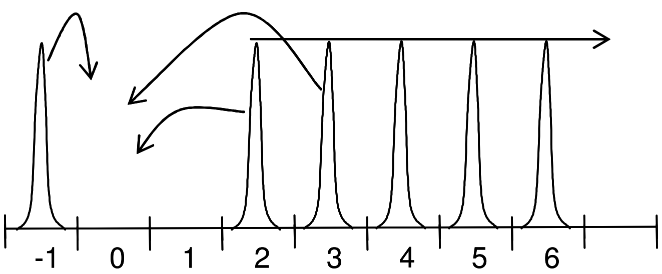
<u>Constrained (Modulation, Line) Coding:</u>

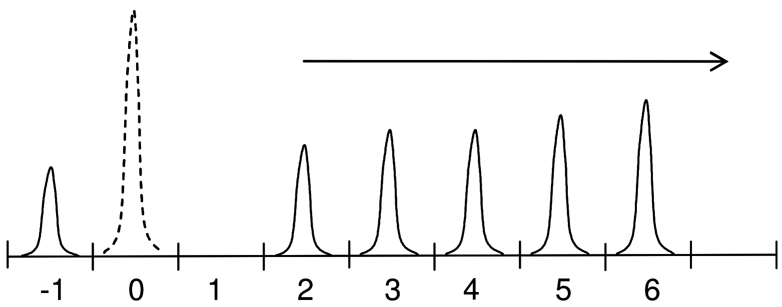
- Vasic, 2004, Rao 2004, Djordjevic 2004, constrained coding

Intra-Channel Nonlinear Effects

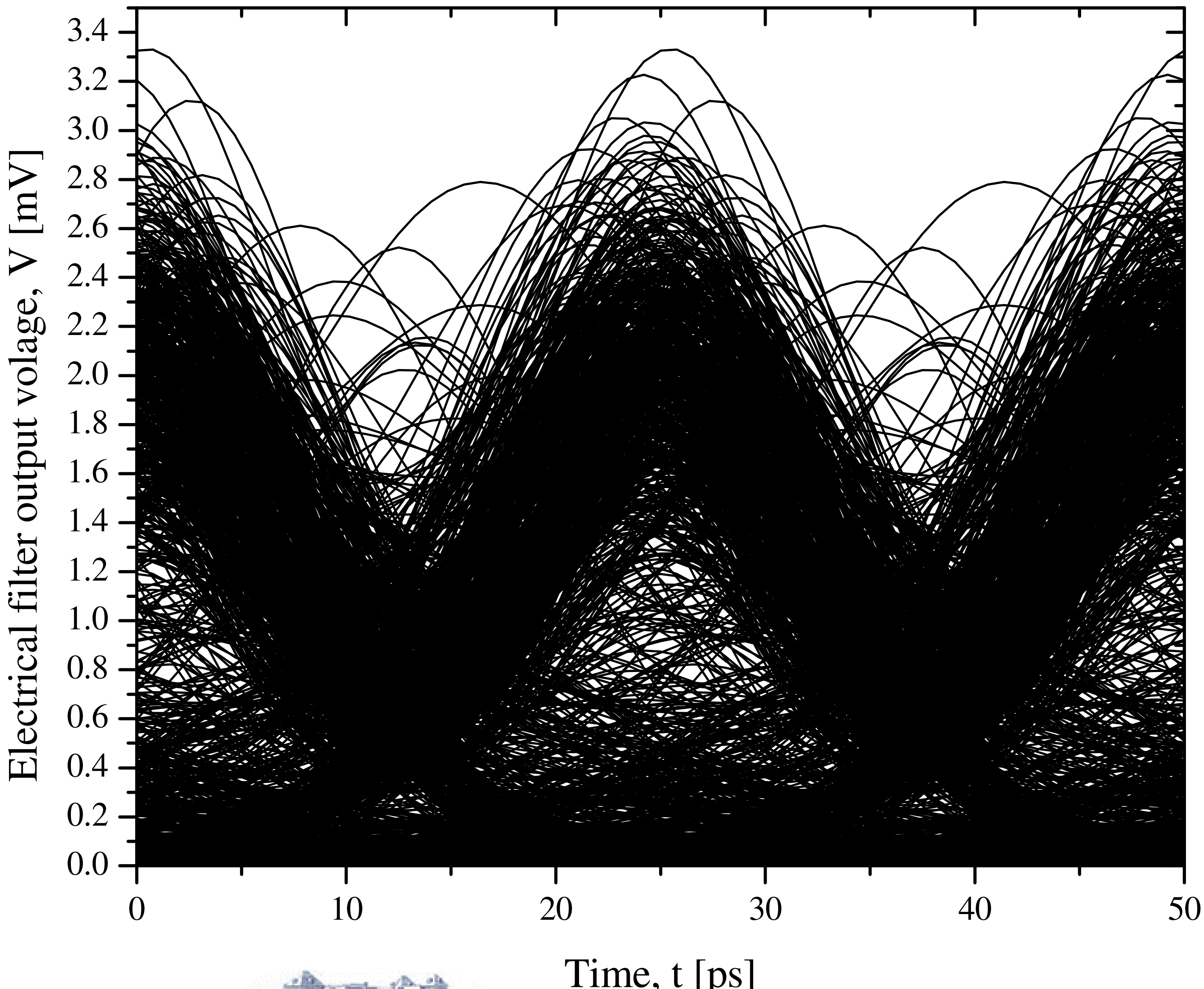
- A ghost pulse effect:







RZ, uncoded eye diagram, 80 spans



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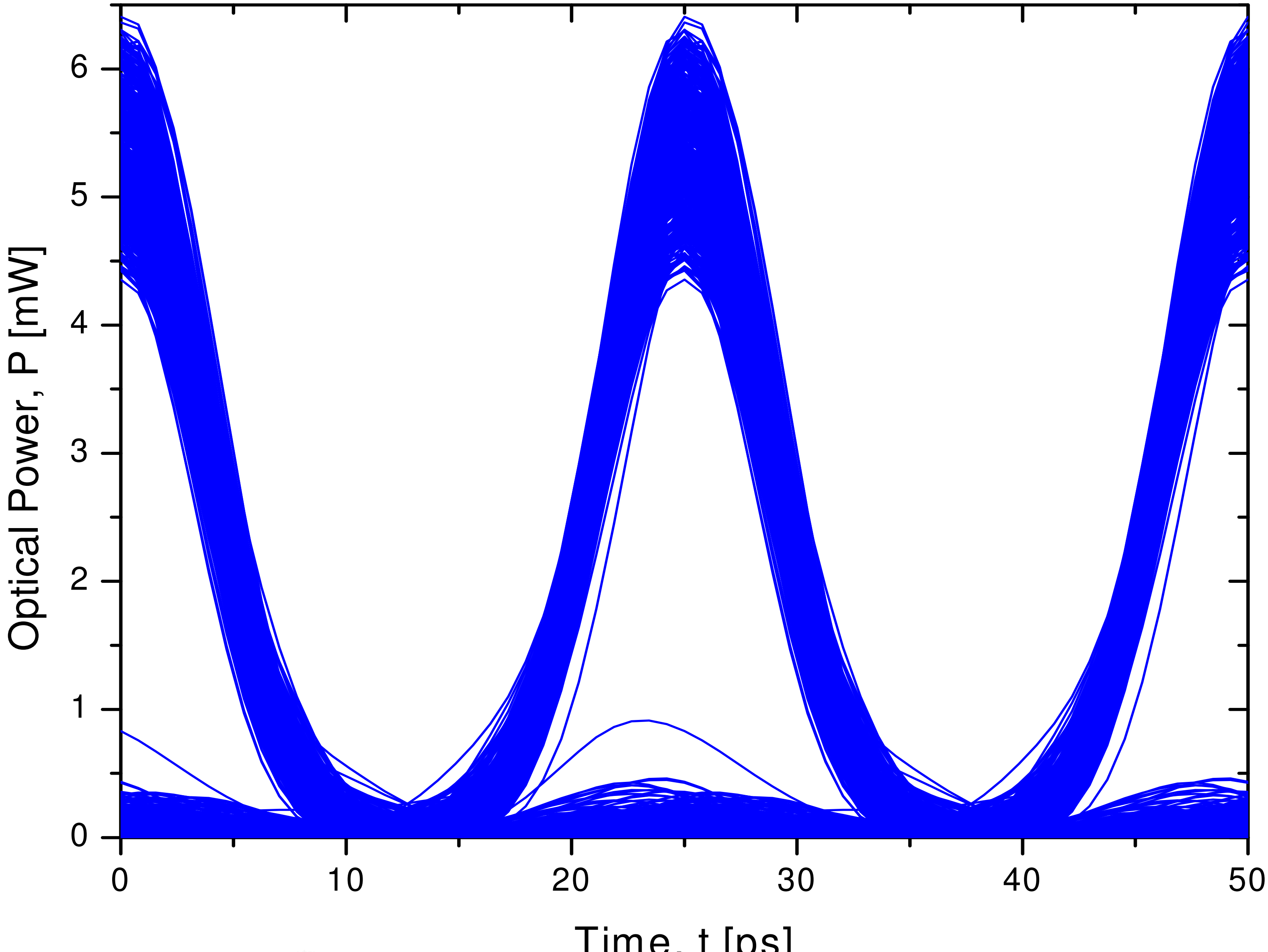
Pseudo-ternary code of rate 0.78, RZ, 60 spans



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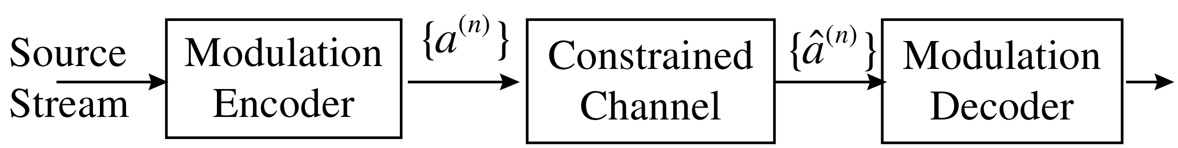
Constrained Codes

- The key idea: forbid the bit patterns that cause a ghost pulse effect.
- Translation from unconstrained bit stream to constrained bit stream done in the transmitter by a modulation code.
- Translation back in the receiver.



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Outline

- Constrained coding fundamentals.
- Codes to combat intra-channel nonlinearities.

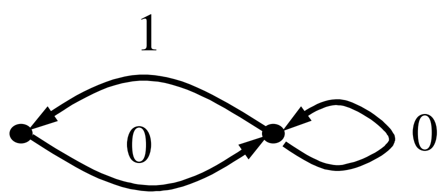
Constraint Classification

- Time-domain constraints
    - Runlength constraints
    - Special forbidden pattern
        - distance enhancing codes
        - self synchronized codes
- Spectral constraints
    - dc free constraint
    - spectral null constraint
    - higher order spectral zeros
    - discrete spectral components
- Composite constraints

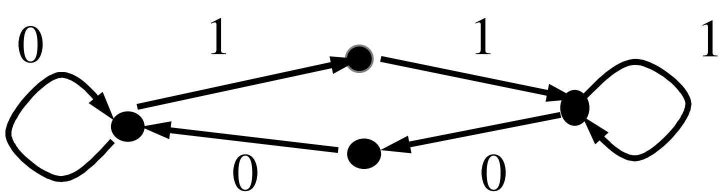
Examples of Constraints

Runlength constraints

(given by finite list *F* of forbidden words)

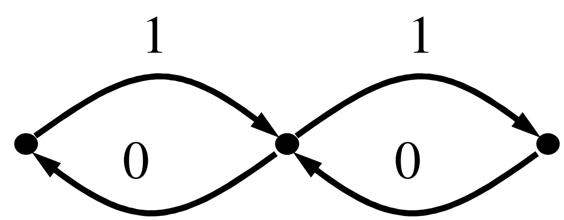


Forbidden word F={11}

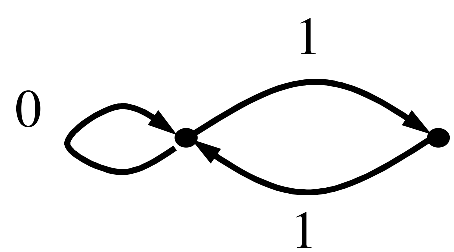


Forbidden words F={101, 010}

Spectral null constraints



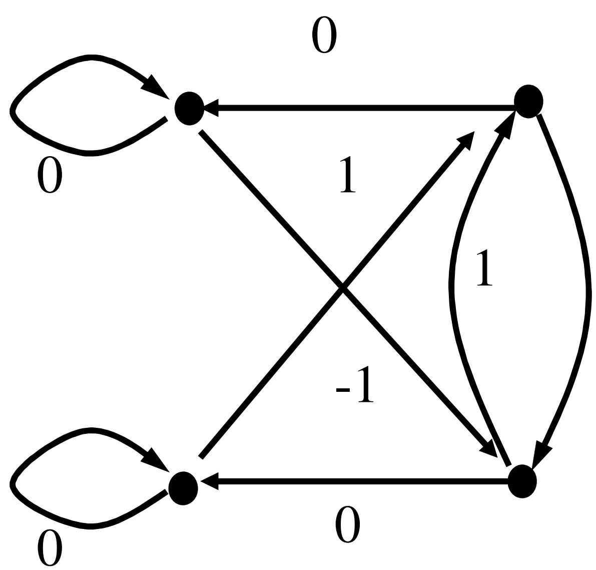
Biphase



Even

Alternating Mark Inversion (AMI)

- A known code used in optical communications



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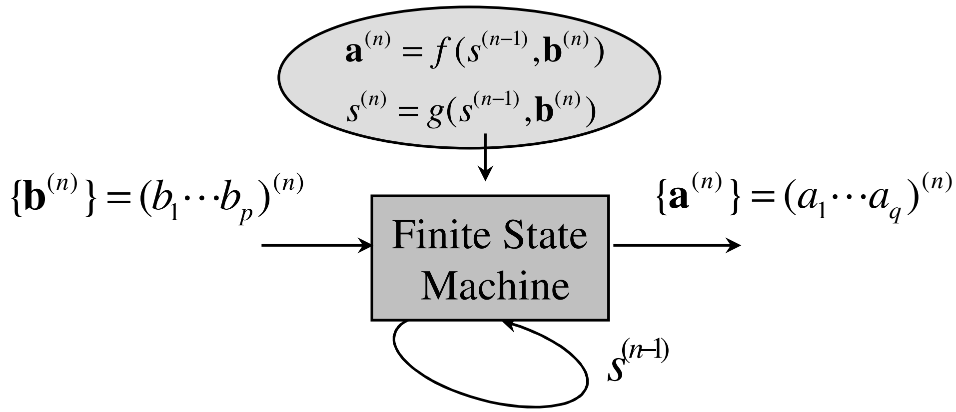
A *(d,k)* Runlength-Limited Constraint

- A (*d,k*) runlength-limited sequence - binary sequence such that:
- The number of consecutive zeros between two ones is at least *d* and at most *k*.
- For F={11}, $(d,k)=(1,\infty)$

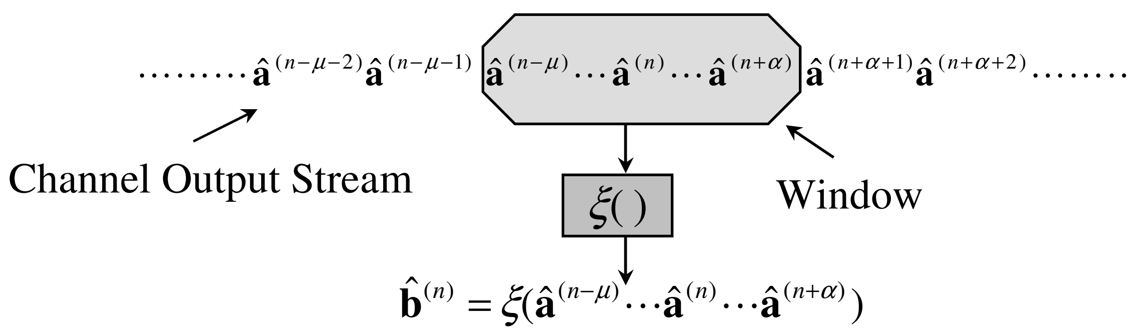
1 0 0 0 0 1 0 1 0 1 0 0 0 1 0 1 0 0 1 0 1 0 0 0 1 0

Encoder and Decoder

- Modulation encoder is a finite state machine with rate R=p/q



- Decoder should be state independent (preferably the sliding window type).



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Codes and Capacity

- What is the maximal code rate?
- Shannon defined the capacity of the constrained system*:*

$$C = \lim_{n \to \infty} \frac{1}{n} \log N(n)$$

N(n) - the number of sequences of length ***n*** satisfying the constraint.

- $N(n) \approx 2^{nC}$
- Theorem [Shannon,1948] : If there exists a constrained decodable code at rate $\boldsymbol{R = p/q}$, then $\boldsymbol{R \leq C}$.

- Theorem [Shannon,1948] : For any rate $\boldsymbol{R=p/q < C}$ there exists a block constrained code with rate ***mp:mq***, for some integer $\boldsymbol{m \geq 1}$.

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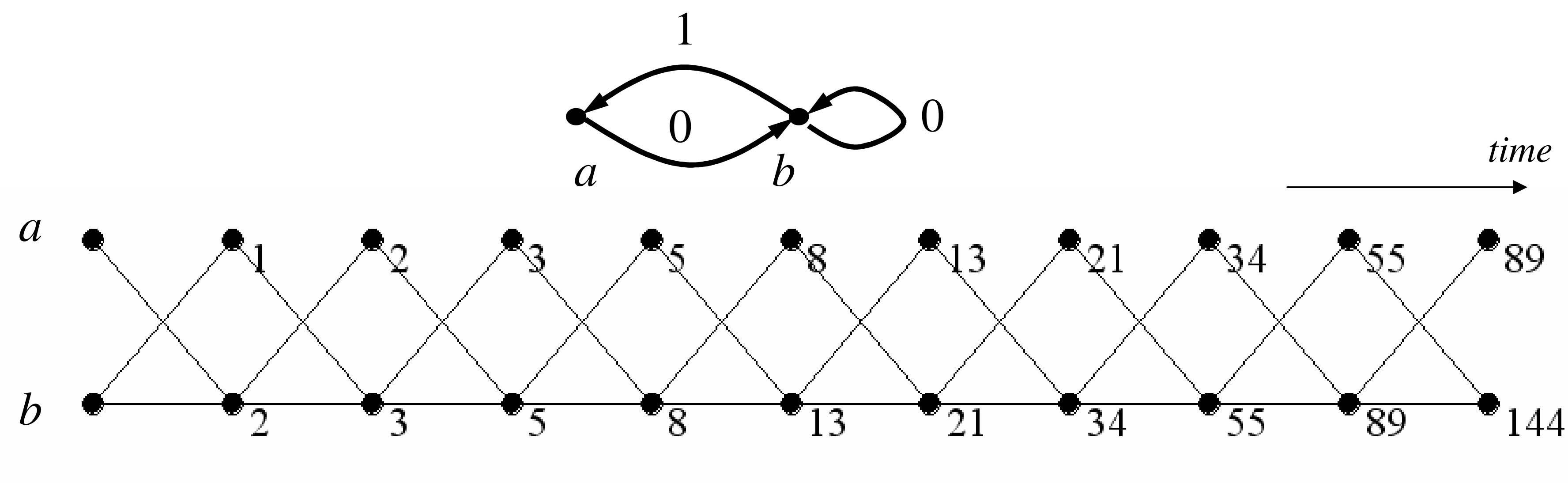
Counting Number of Sequences

- Transitions in one step determined by the graph adjacency matrix A.
- Transitions in n steps determined by A^n.



- Shannon showed that, for suitable representing graphs,

$$C = \log \rho\left(A\right)$$

$\rho(A)$ the spectral radius of the matrix A.

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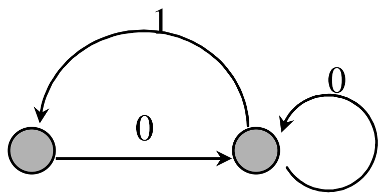
Computing Capacity

- Example $(d,k)=(1,\infty)$



$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\lambda^2 - \lambda - 1 = 0$$

$$\lambda_0 = (1 + \sqrt{5}) / 2$$

$$C = \log_2(\lambda_0) = 0.694\ldots$$

$$R = p / q = 2 / 3 < C \Rightarrow p = 2, q = 3$$

- Can be done for an arbitrary constraint.

Constrained Decoding Theorems

- Theorem[Adler-Coppersmith-Hassner, 1983]

  Consider a finite-type constrained system. If $\boldsymbol{p/q \leq C}$, then there exists a rate $\boldsymbol{p{:}q}$ sliding-block decodable, finite-state encoder.

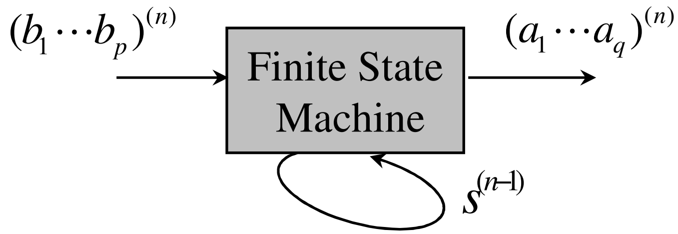
  (Proof is constructive: ACH or state-splitting algorithm.)

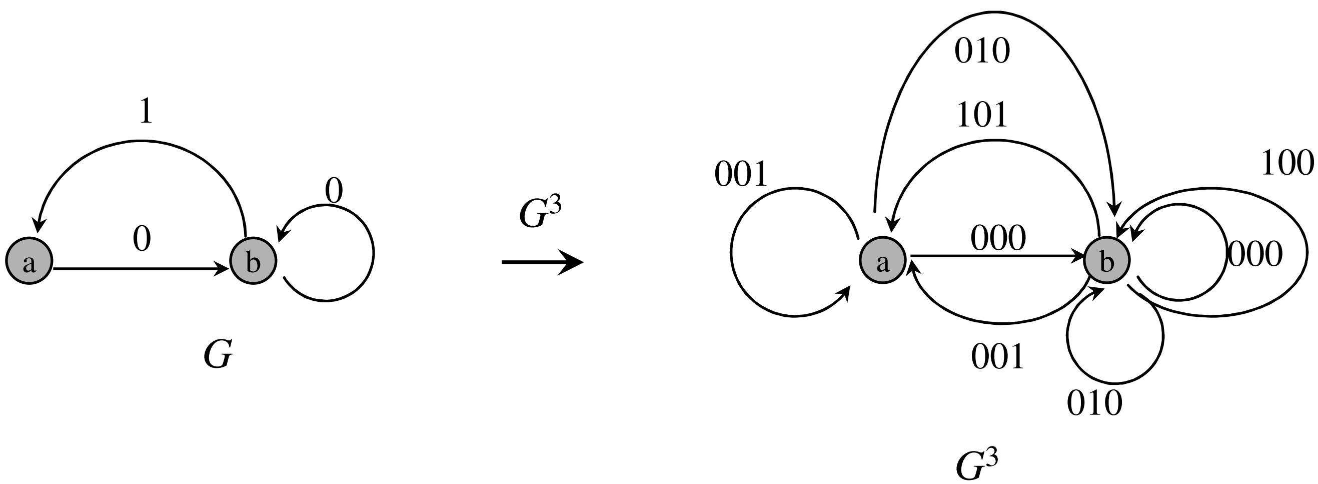
- Theorem [Karabed-Marcus, 1988]

  Similar statement for spectral null constraints.

Finite State Encoder

- Example $(d,k)=(1,\infty)$:
- $R<2/3$, $p=2$, $q=3$.





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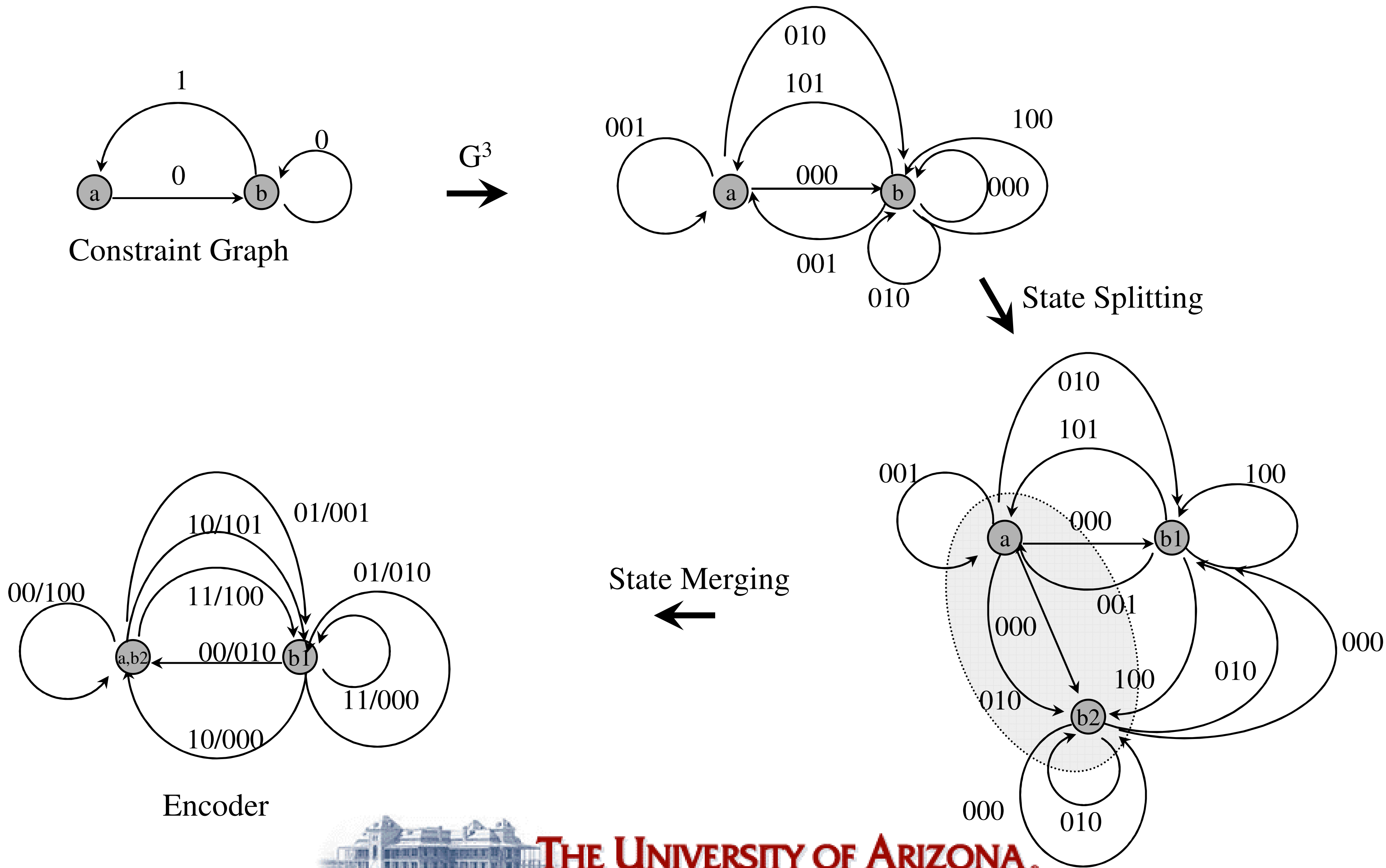
Code Construction Example: ACH Algorithm



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Application to 40 Gb/s Transmission

- In high-speed transmission (40 Gb/s and above) the major impairments are due to intrachannel nonlinearities:
  - intrachannel four-wave mixing (IFWM) and
  - intrachannel cross-phase modulation (IXPM).
- The strongest interaction occurs in the regime where the pulses partially overlap

Intra-Channel Nonlinear Effects

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A - \frac{j}{2}\beta_2 \frac{\partial^2 A}{\partial T^2} + \frac{1}{6}\beta_3 \frac{\partial^3 A}{\partial T^3} + j\gamma\left[|A|^2 A - T_R A \frac{\partial |A|^2}{\partial T} + \frac{j}{\omega_0}\frac{\partial}{\partial T}\left(|A|^2 A\right)\right]$$

$$A = A(z,T) \qquad T = t - z/v_g$$

$$\beta_2 = -\frac{\lambda^2}{2\pi c} D_c \qquad \beta_3 = -\frac{\lambda^3}{(2\pi c)^2}\left(D_c + \lambda \frac{dD_c}{d\lambda}\right) \qquad \gamma_0 = \frac{2\pi n_2}{\lambda A_{eff}} \qquad \gamma = \frac{\gamma_0 A}{1 + b_s |A|^2} \approx \gamma_0\left(1 - b_s |A|^2\right)$$

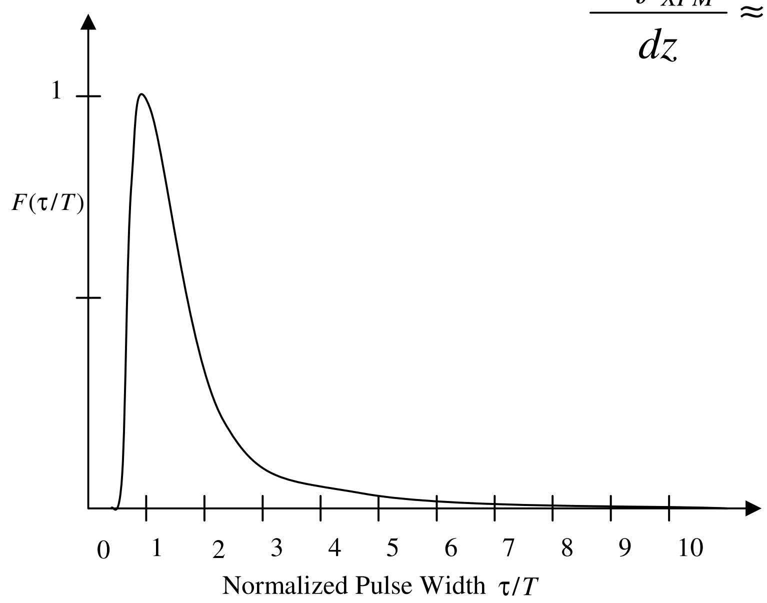
$$A = \sum_{l=1}^{L} A_l \Rightarrow \sum_{l=1}^{L}\left(\frac{\partial A_l}{\partial z} + \frac{\alpha}{2} A_l + \frac{i}{2}\beta_2 \frac{\partial^2 A_l}{\partial T^2} - \frac{\beta_3}{6}\frac{\partial^3 A_l}{\partial T^3}\right) = i\gamma \sum_{l,k,m=1}^{L} A_l A_k A_m^*$$

- SPM: $l=k=m$
- IFWM: $l\neq k\neq m$ or $l=k\neq m$
- IXPM: $l=m\neq k$ or $k=m\neq l$

Intrachannel Cross-Phase Modulation

- In dispersion managed systems, when neighboring pulses overlap, the time derivative of the power of one pulse edge causes a shift in frequency of the other pulse:

$$\frac{d\Delta f_{XPM}}{dz} \approx \pm 0.15 \frac{\gamma E_0}{T^2} F\left(\tau/T\right)$$

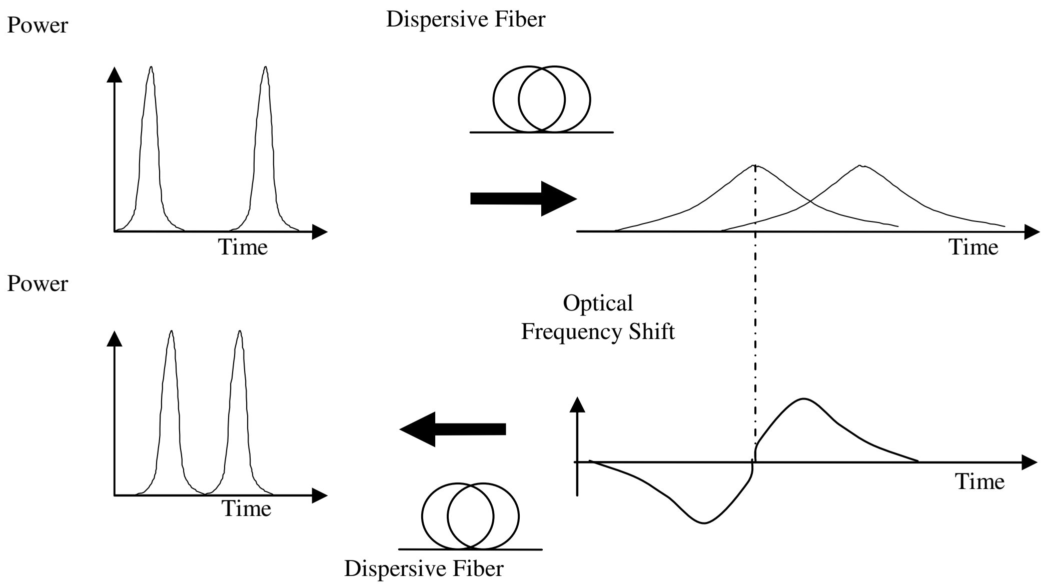


P. Bayvel and R. Killey, "Nonlinear optical effects in WDM transmission" *in Optical Fiber Telecommunications IVB* (I. Kaminov and T. Li ed.) Academic, 2002.

P. V. Mamyshev and N. A. Mamysheva, "Pulse-overlapped dispersion-managed data transmission and intrachannel four-wave mixing," *Opt. Lett*, vol. 24, pp. 1454 -1456, 1999.

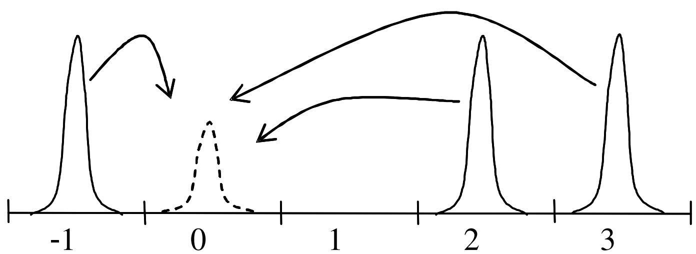
IXPM cont.

- The frequency shift is negligibly small when $t < 0.4\ T$ as the pulses do not overlap.
- The maximum distortion occurs when $\tau \cong T$, while for $\tau >> T$ the effect becomes negligible as the pulse power is low due to the large broadening.

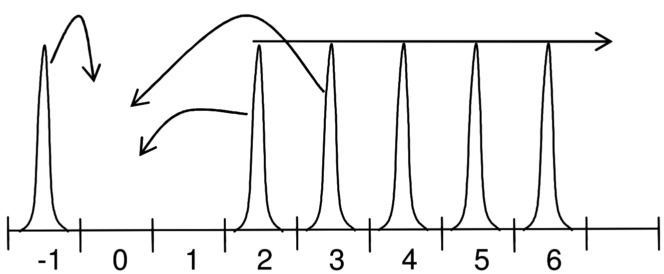


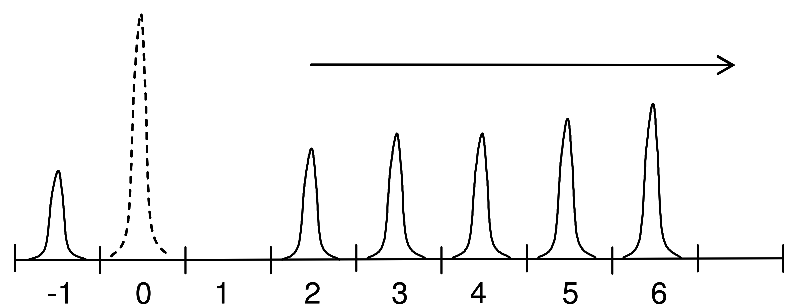
Intrachannel Four-Wave Mixing

- Dispersed pulses experiencing the nonlinearity see a portion of their filed shifted by a discrete frequency value due to FWM of spectral components within the same wavelength channel
- At sufficiently high dispersion the frequency shift is translated in a discrete time shift located near the middle of a neighboring pulse



- The location of interaction: $\boldsymbol{t_l + t_k - t_m}$ (approximately)

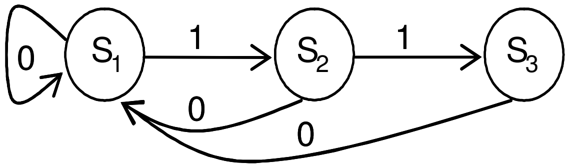




Constrained Codes

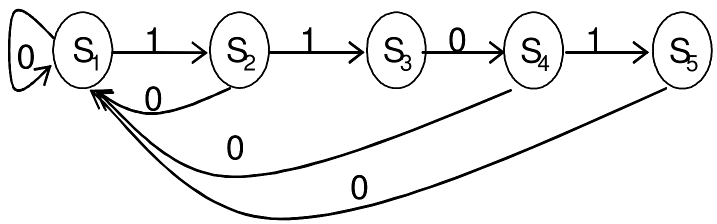
- The key idea: forbid the bit patterns that cause a ghost pulse effect.
- Possible Approaches:
  - the most troublesome sequences are identified and forbidden by constrained encoder,
  - the ‘zero’ symbol in so called resonant position is converted into ‘one’ symbol,
  - the constrained encoder is designed on such a way that different contributions to a ghost pulse creation cancel each other in resonant positions.

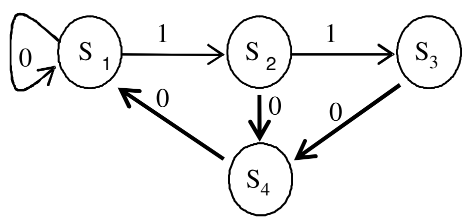
Binary Constrained Codes



$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

C=0.8791





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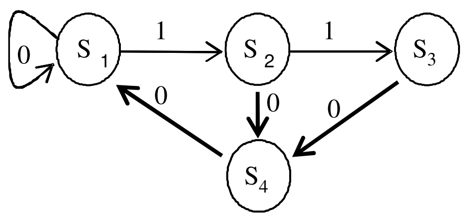
Binary Constrained Codes-cont.



Directed Graph for the 2/3 Rate Code

Previous State	Next State	Input Pattern	Branch Label
S_{11}	S_{11}	00	000
S_{11}	S_{121}	10	000
S_{11}	S_{122}	11	000
S_{11}	S_{3}	01	011
S_{121}	S_{21}	01	001
S_{121}	S_{22}	11	001
S_{121}	S_{11}	10	100
S_{121}	S_{122}	--	100
S_{121}	S_{121}	00	100
S_{122}	S_{41}	10	010
S_{122}	S_{42}	01	010
S_{122}	S_{41}	00	110
S_{122}	S_{42}	11	110

•**Sliding window decoder**:

If ABC = “011”

$I_1 = 0$

$I_2 = Y'$

For all other patterns of ABC

I_2 = C' (B' E F' + B D' F) + E' (A' B' C F' (I' + G')
+ D' (B C' H' (G I' + G' I) + A' B' C F))

and,

$I_1 = I_2$ (F' (E' H I' + C (H' I + G)) + A B) + C' (A' B' D F'
+ D' (A (B' F' I2' + E F) + A' (I2' (B (I' + G') + E' F) + E F')))

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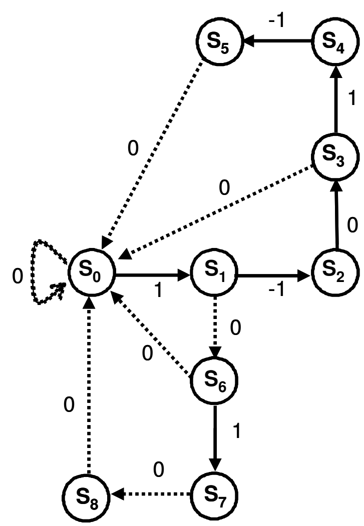
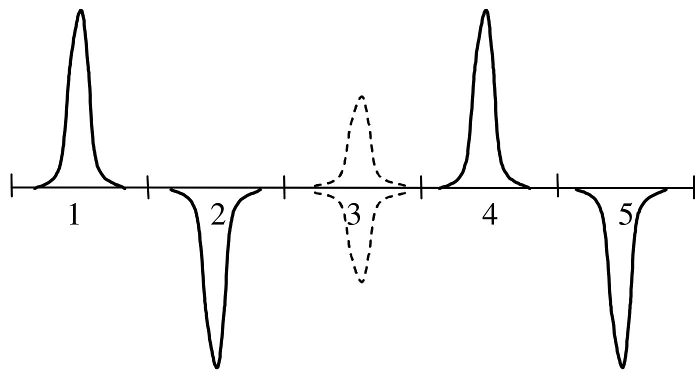
Pseudo-Ternary Codes

•The proper choice of initial pulse phases may result in complete cancellation of different ghost-pulse contributors at a zero-bit position in “resonance”.



C=0.78



1+4-2: 0+0-π
2+5-4: π+ π-0

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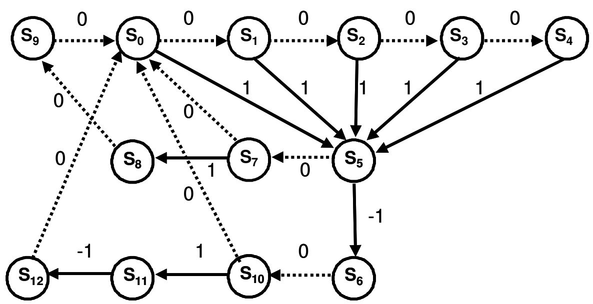
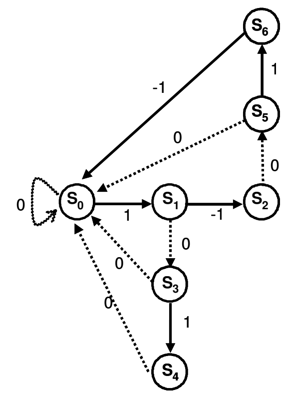
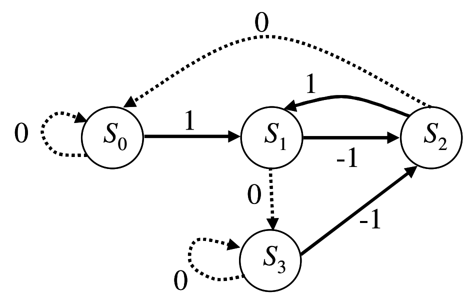
Pseudo-ternary codes cont.



C=0.76, RLL constraint 6



C=0.83



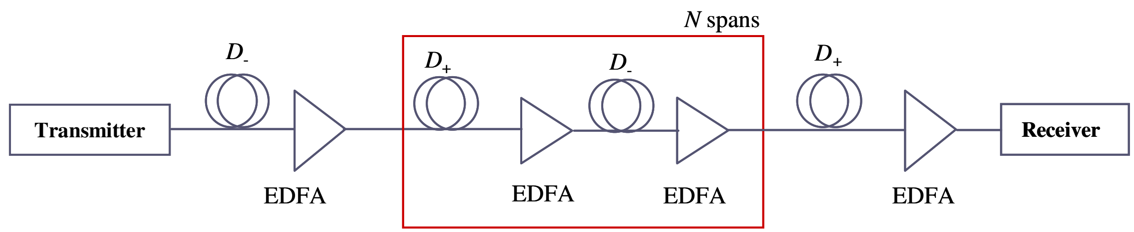
AMI

A rate 4/5 non-linear pseudo-ternary block code

•Any sequence of length 5: $c_i c_{i+1} c_{i+2} c_{i+3} c_{i+4}$, satisfies the following constraint: if for k, l, $m \in [i, i+4]$ and $k+l-m \in [i, i+4]$, k and l not necessarily distinct, $c_k = c_1 = c_{\mathrm{m}} = 1$ *or* $c_k = c_l = c_m = -1$, then $c_{k+l-m} \neq 0$

Input	*Codewords*			
0000	00000			
0001	00001	0000-1		
0010	00010	000-10		
0011	00100	00-100		
0100	01000	0-1000		
0101	10000	-10000		
0110	10001	-1000-1	1000-1	-10001
0111	10010	-100-10	100-10	-10010
1000	01001	0-100-1	0100-1	0-1001
1001	10-101	-1010-1		
1010	10-100	-10100		
1011	00-101	0010-1		
1100	00-110	001-10		
1101	01-100	0-1100		
1110	10-110	-101-10		
1111	01-101	0-110-1		

Dispersion Map Under Study



Simulator features

-All major impairments in long-haul optical transmission:

- •ASE noise.
- •Pulse distortion due to fiber nonlinearities (Kerr nonlinearities, SRS, …).
- •Chromatic dispersion (GVD, second-order GVD).
- •Crosstalk effects.
- •Intersymbol-interference (ISI), etc.

-Various modulation formats, detection and decoding schemes.

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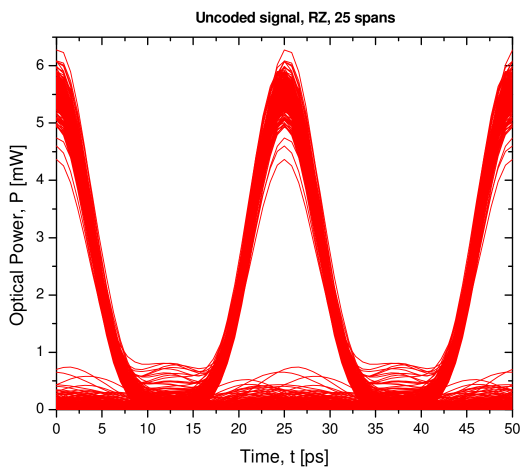
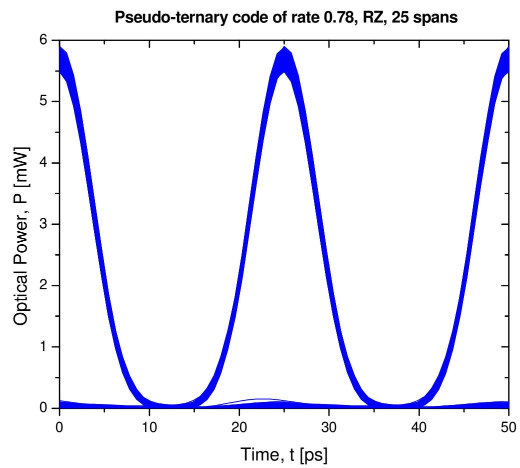
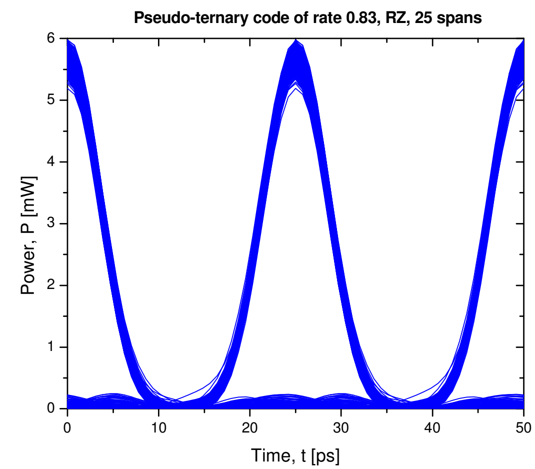
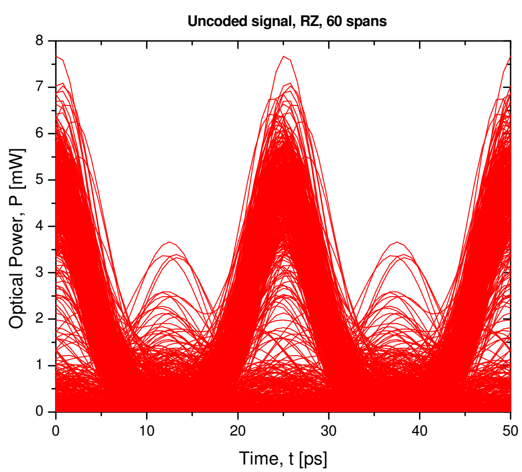
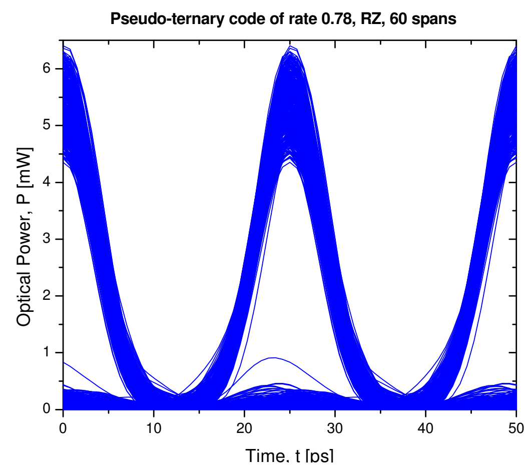
Eye Diagrams

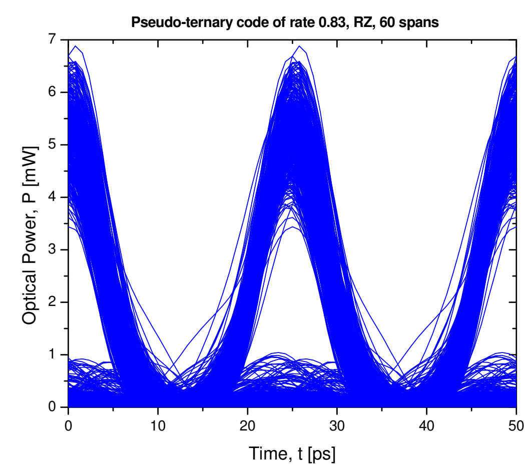




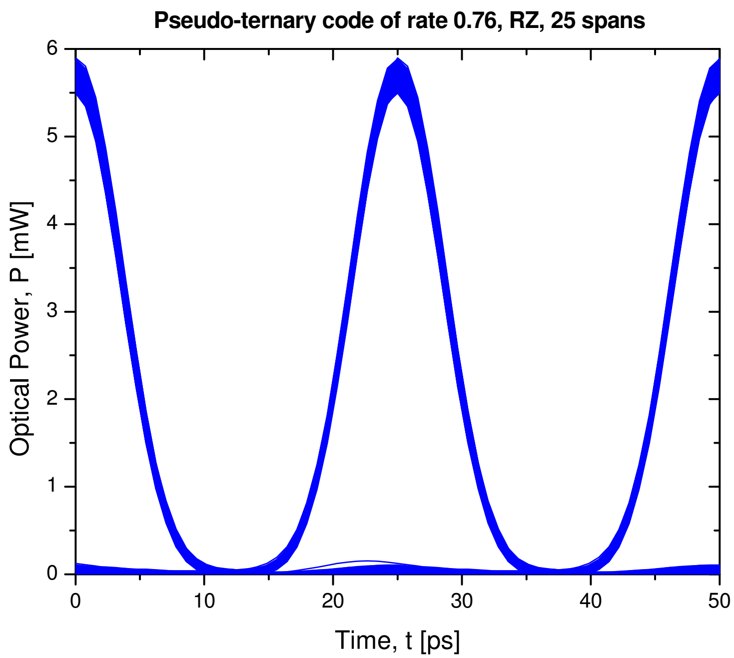


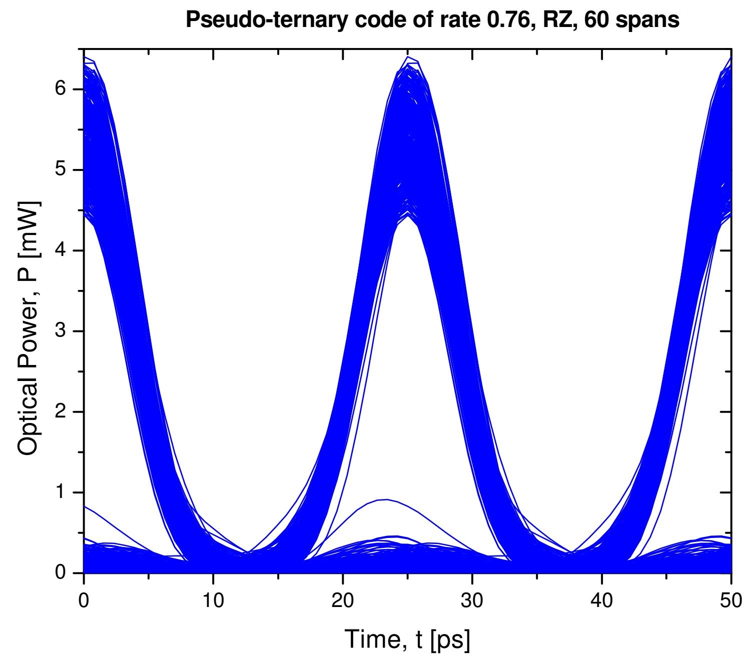




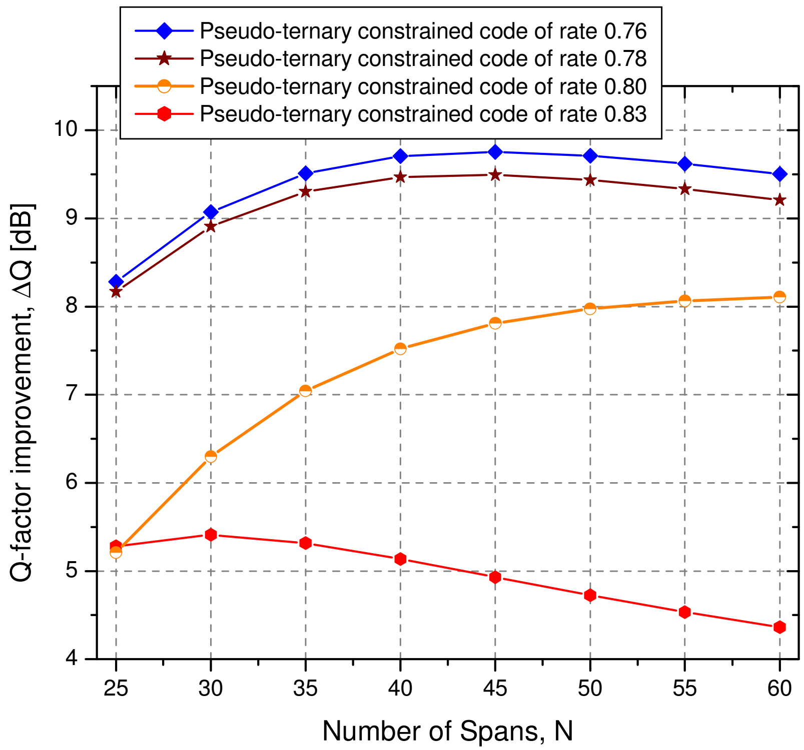


Eye diagrams-cont.

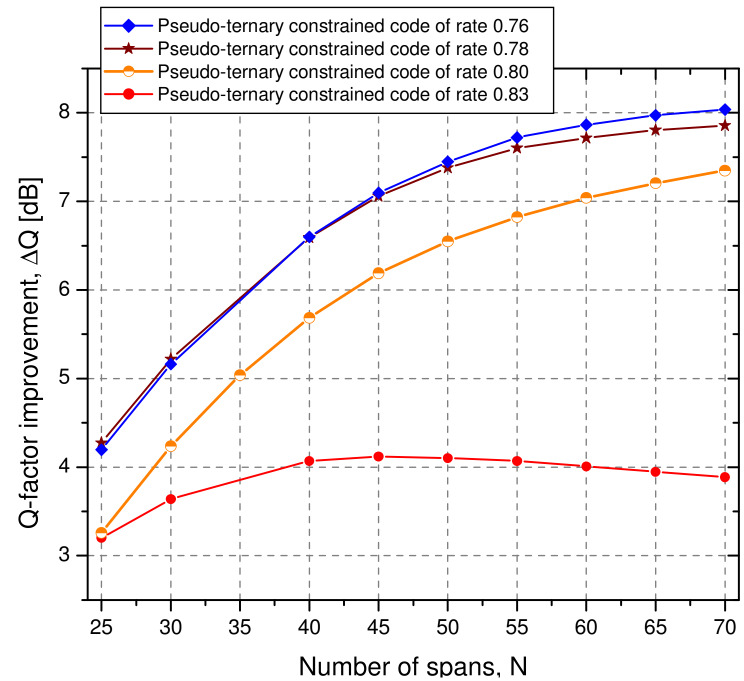




Q-factor Improvement



•In the absence of ASE noise



•In the presence of ASE noise

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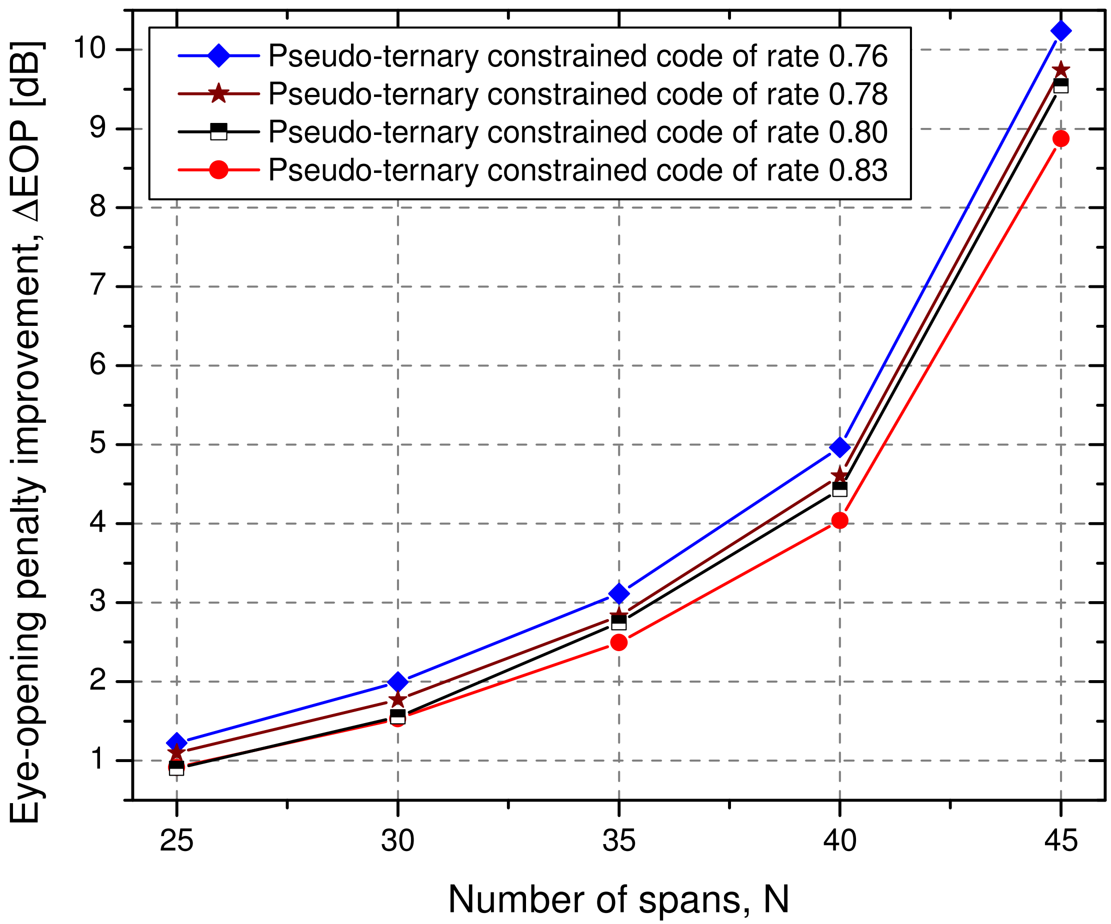
Eye Opening Penalty Improvement



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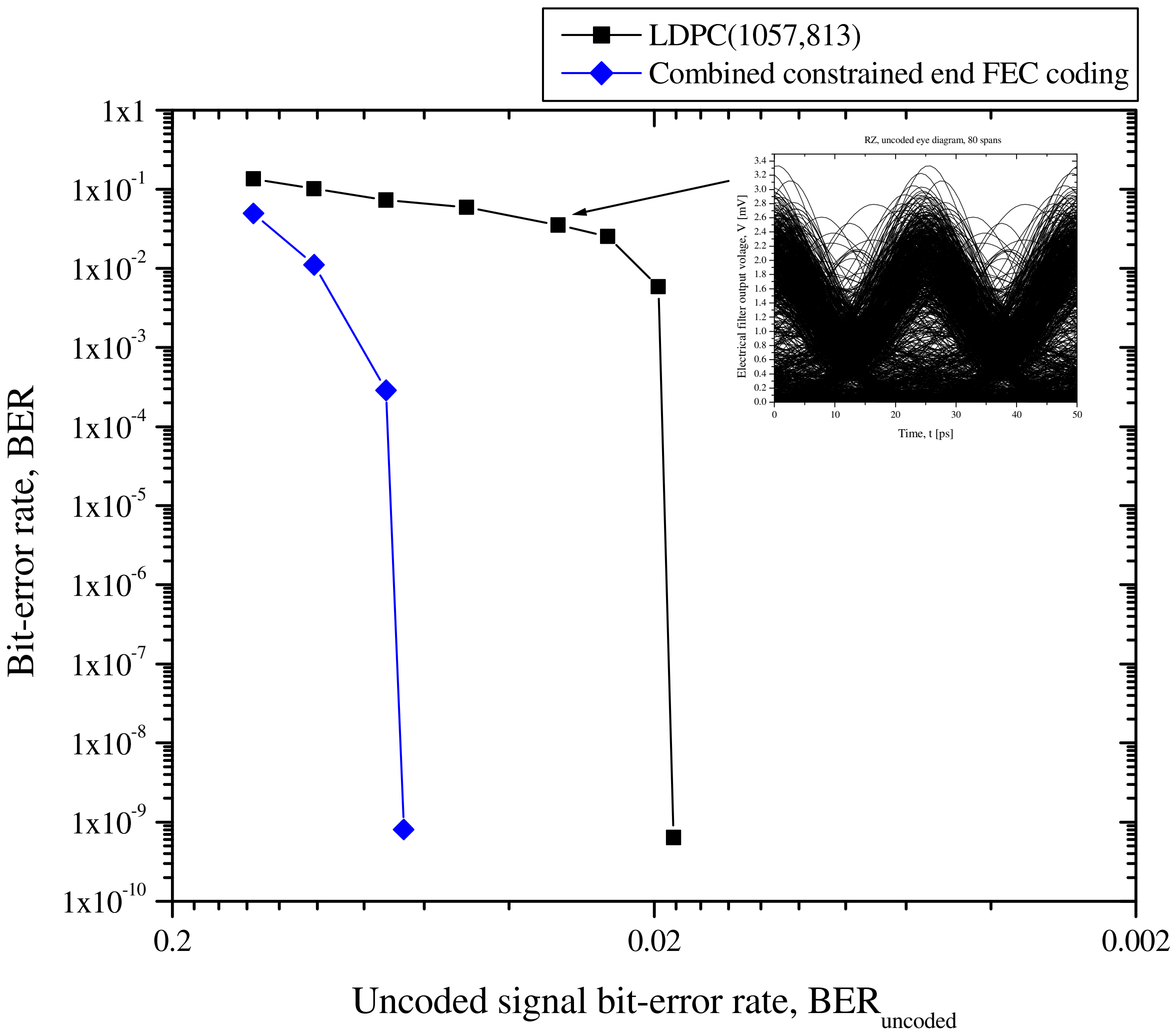
Combined Constrained and Error Control Coding



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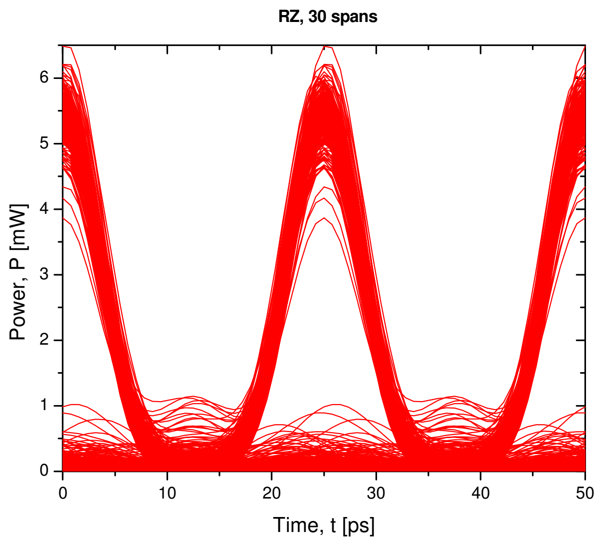
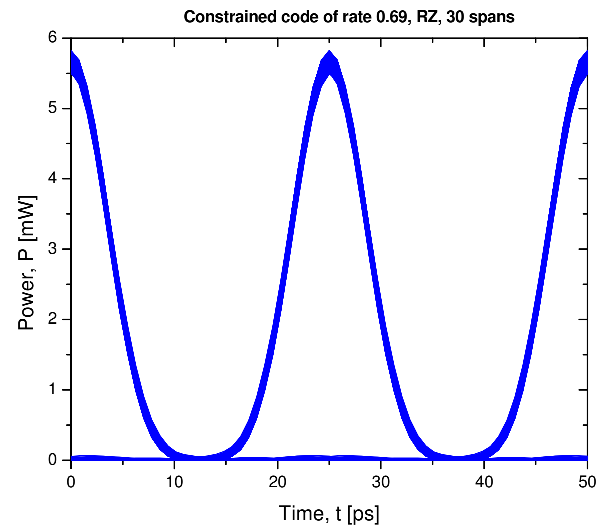
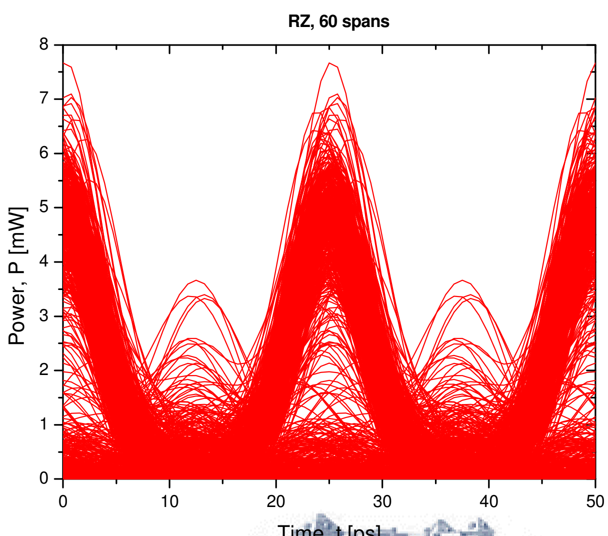
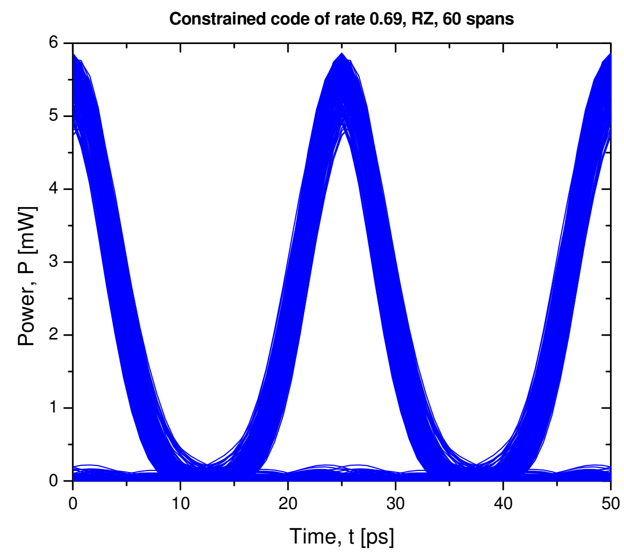
Constrained Codes of Rate 0.69









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Conclusion

- The use of constrained codes to counter the effects of IFWM and IXPM is proposed
- Significant Q-factor improvement up to 9.75 dB, and significant eye-opening penalty improvement of more than 10.24 dB, depending on code rate and number of spans, are demonstrated.
- At 40 Gb/s and above, constrained codes can significantly improve the transmission distance and system capacity.
- The constrained codes are capable of improving the FEC threshold in systems with severly degraded performance due to intrachannel nolinearities.
- Notice that the complexity of the constrained encoder/decoder is significantly simpler than any state of the art FEC scheme employed in long-haul optical transmission.