The role of coherence in near-field optics

Eugenio R. Méndez

División de Física Aplicada, CICESE,
Ensenada, Mexico

Outline

- Introduction
- A principle of equivalence
- Partial coherence in near field optics
- Conclusions.

In collaboration with:

- J-J. Greffet and R. Carminati, Ecole Centrale (Paris)
- A. A. Maradudin and T. A. Leskova, UC Irvine

Introduction

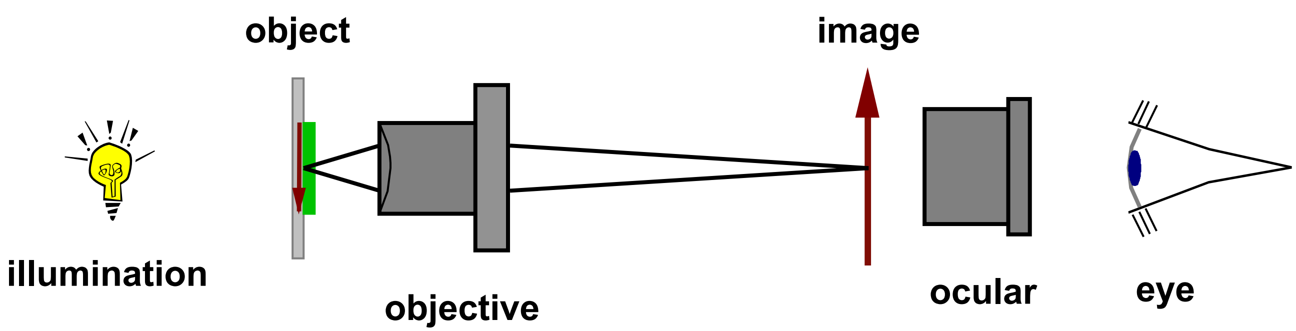
The Conventional Microscope

Mode of image formation determined by the coherence of the illumination

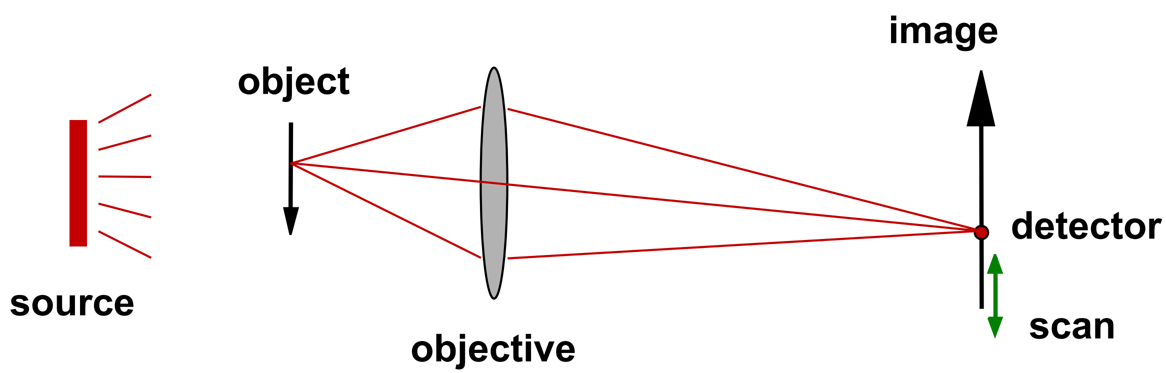
Resolution determined by size of PSF

In the image plane:

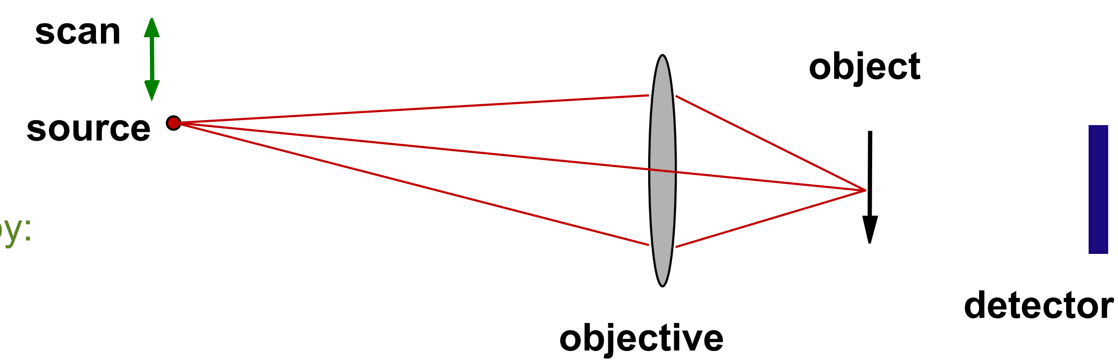
- Photographic film
- CCD array, etc.



Conventional Optical Scanning Microscopy



Resolution determined by:
- Size of PSF



Mode of image formation determined by:
- Size of source
- Size of detector

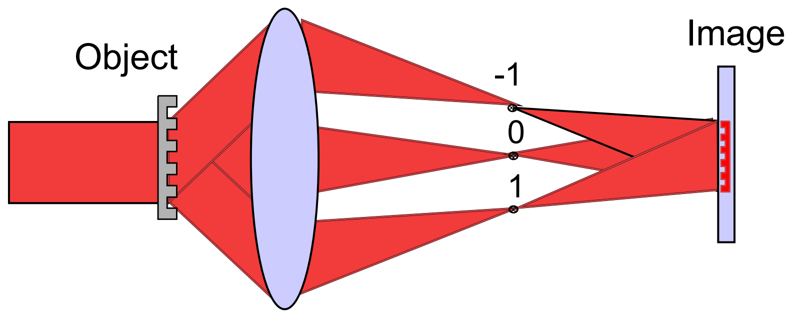
Principle of equivalence (based on reciprocity) **Linear optics!**

Resolution

Periodic object: Period T

Grating equation

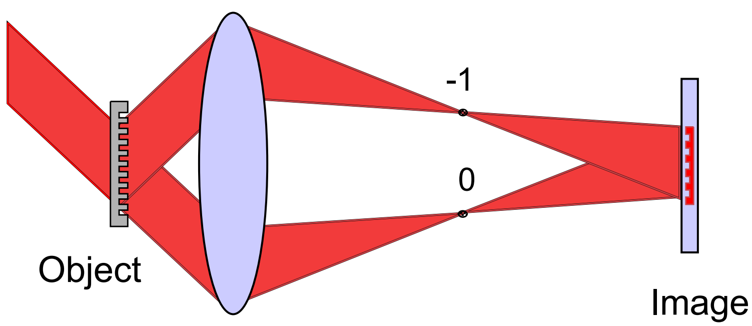
$$\sin\theta_s = \sin\theta_0 + \frac{m\lambda}{T}$$



If $\theta_0 = 0$

$$\sin\theta_s = \frac{m\lambda}{T}$$

for $T \le \lambda$ no orders go through

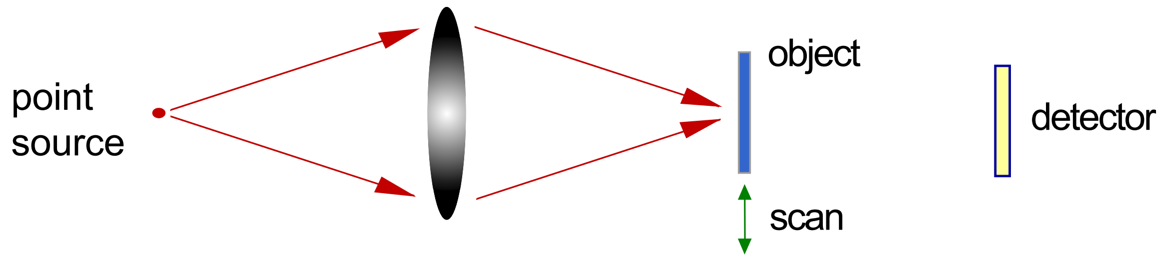


if $\theta_0 \neq 0$

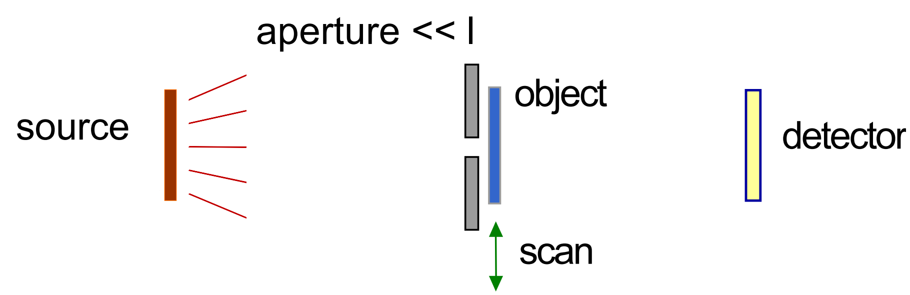
$$\sin\theta_s - \sin\theta_0 = \frac{m\lambda}{T}$$

for $T \le \lambda/2$ no orders go through

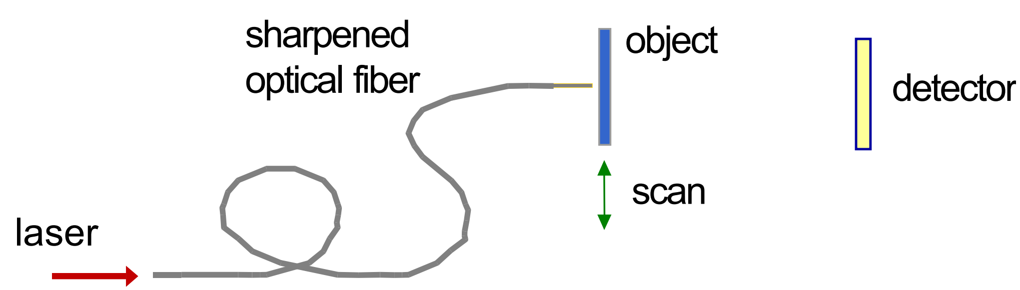
Near-field optics



Synge (1929)
Ash (1982)

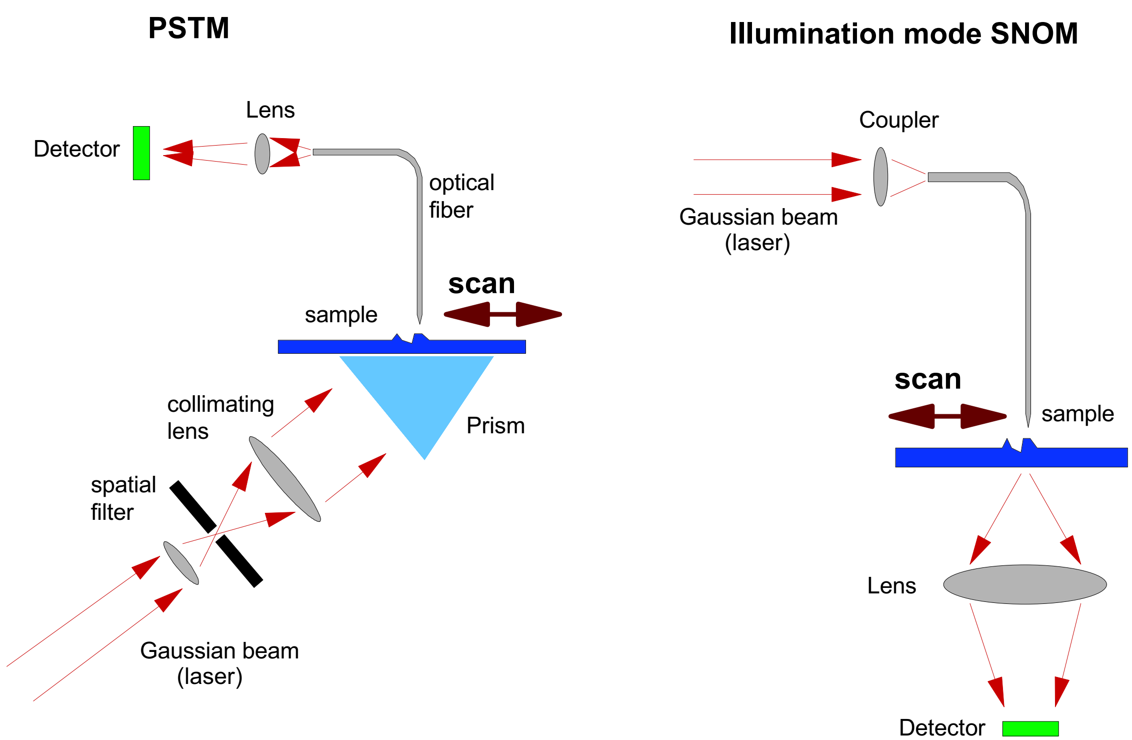


Scanning Near-field Optical Microscopy

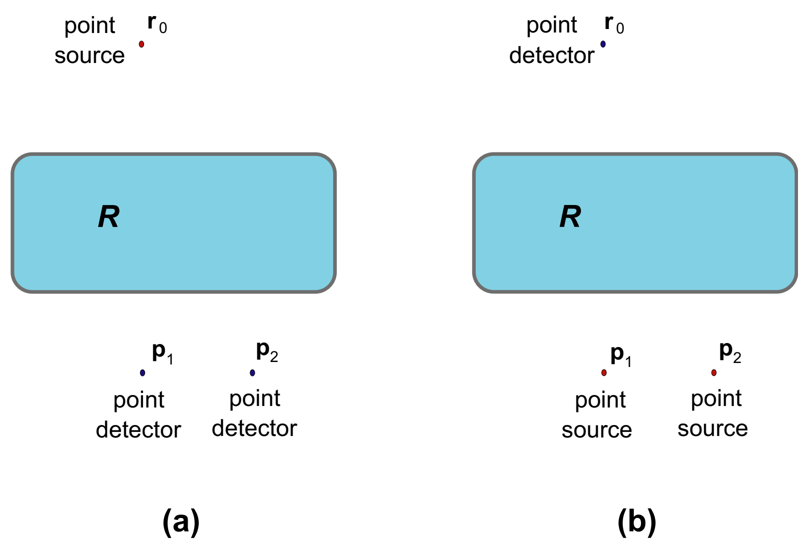


It is commonly believed that:

- The PSTM and the illumination mode SNOM are fundamentally different instruments
- The coherence of the illumination in the collection mode SNOM influences the resolution [OL **18**, 2090 (1993)]



Reciprocity and equivalence

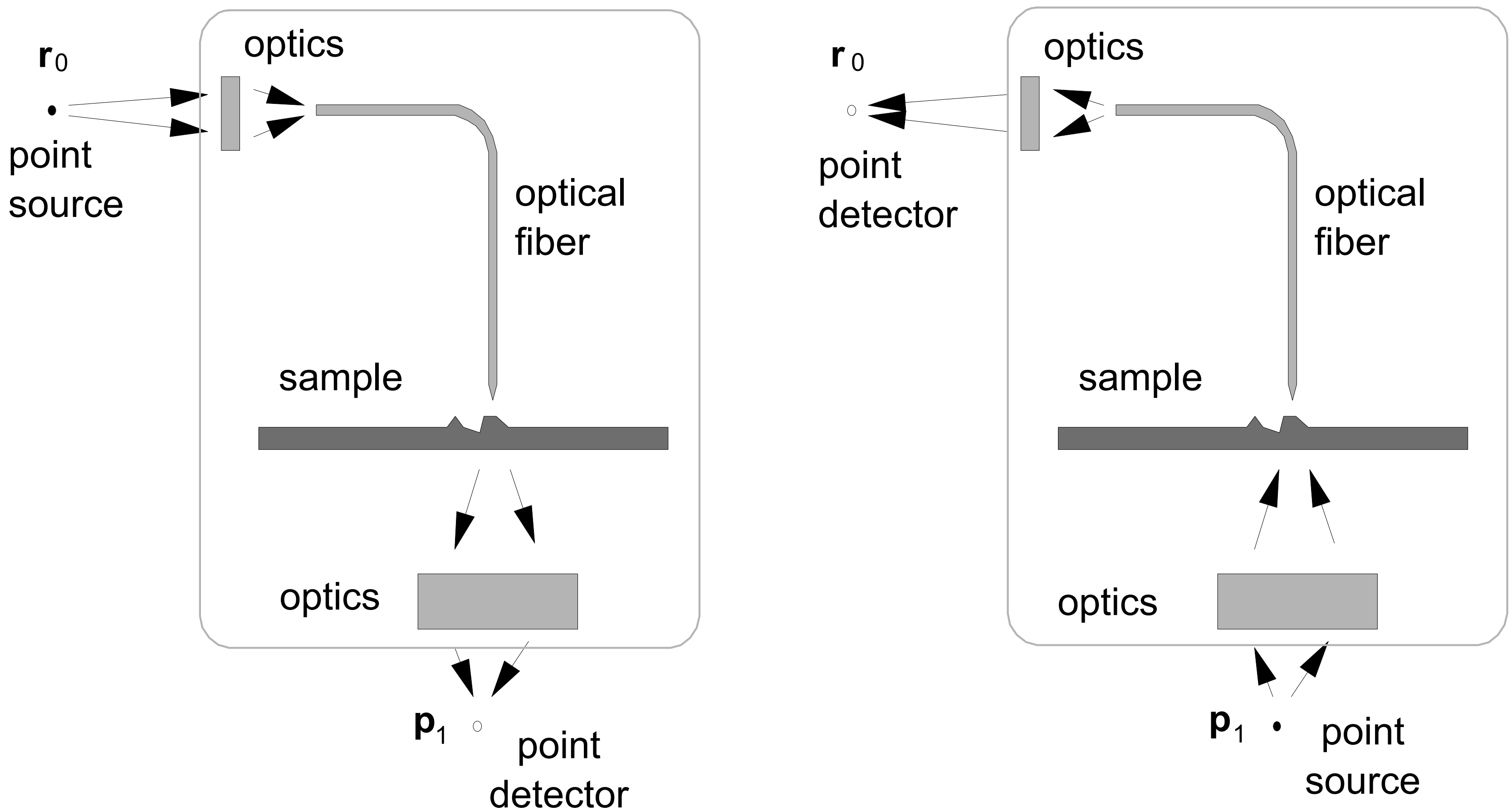


$$\mathbf{A}_1 \cdot \mathbf{E}(\mathbf{p}_1; \mathbf{r}_0) = \mathbf{A}_0 \cdot \mathbf{E}(\mathbf{r}_0; \mathbf{p}_1)$$

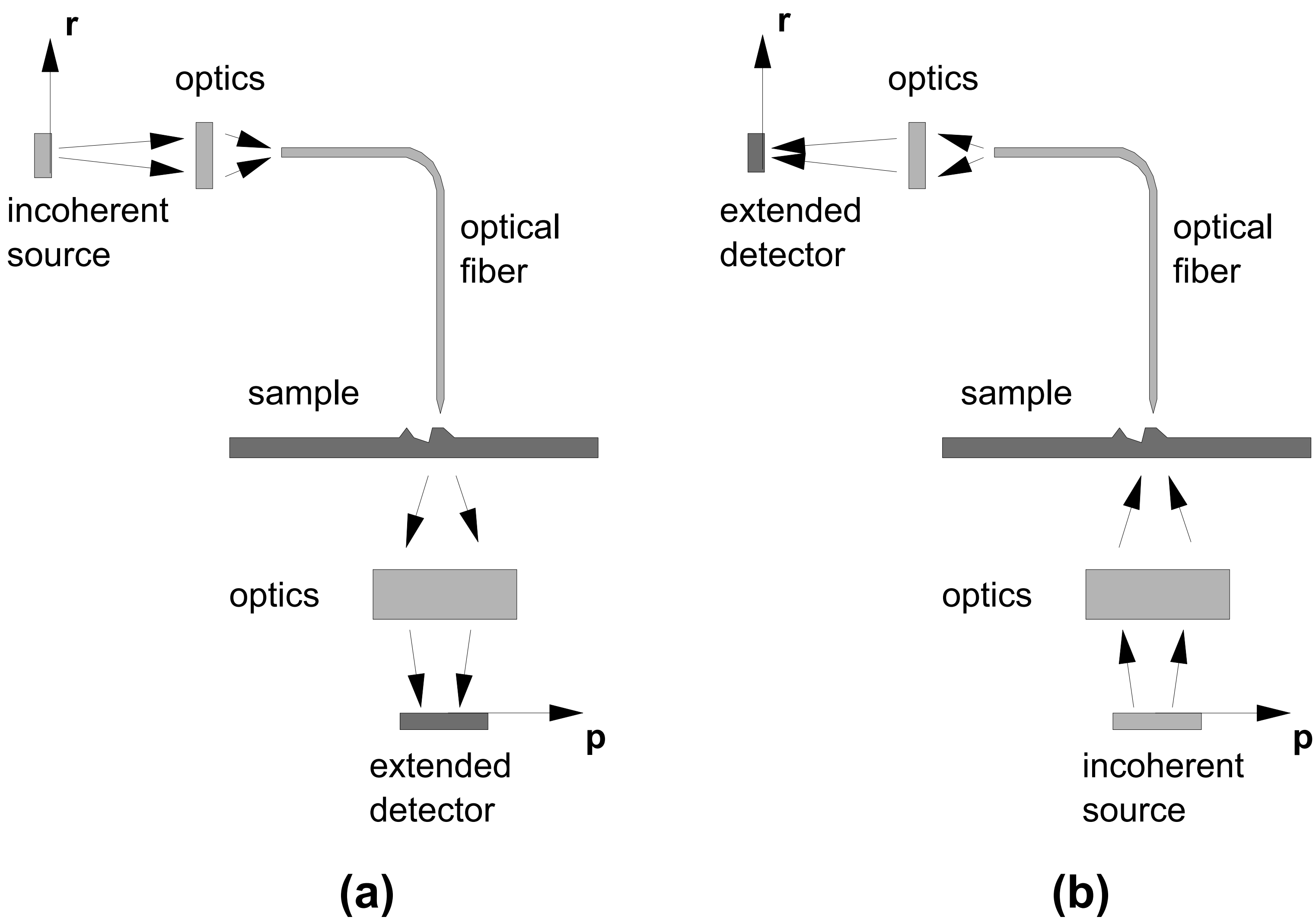
(a) $\mathbf{E}(\mathbf{p}_1; \mathbf{r}_0)$ electric field at $\mathbf{p}_1$ due to a dipole of amplitude $\mathbf{A}_0$ at $\mathbf{r}_0$.
(b) $\mathbf{E}(\mathbf{r}_0; \mathbf{p}_1)$ electric field at $\mathbf{r}_0$ due to a dipole of amplitude $\mathbf{A}_1$ at $\mathbf{p}_1$.

For randomly orientated dipoles of equal strengths,

$$I(\mathbf{p}_1; \mathbf{r}_0) = I(\mathbf{r}_0; \mathbf{p}_1)$$

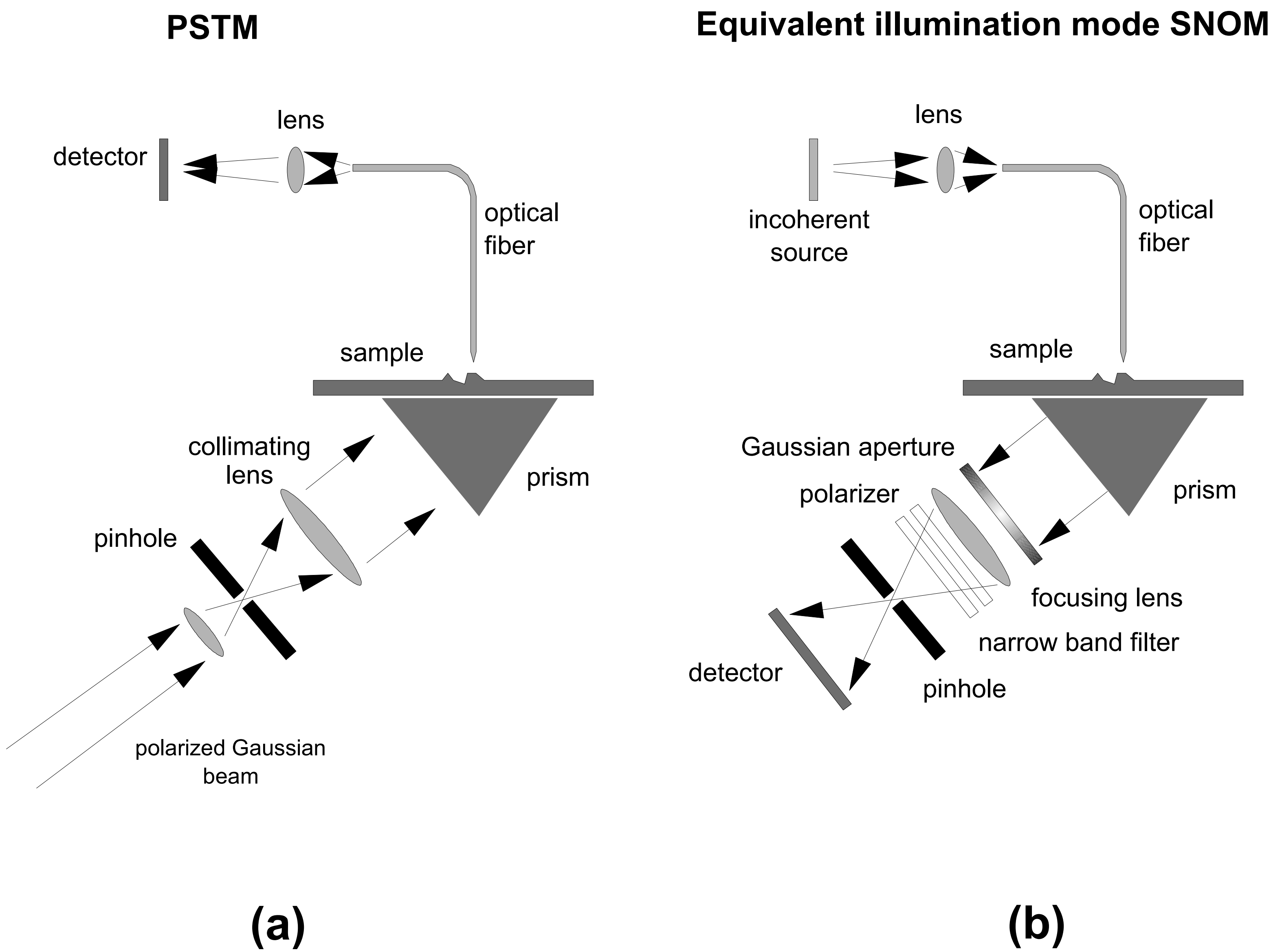

r0
point
source
optics
optical
fiber
sample
optics
p1
point
detector
r0
point
detector
optics
optical
fiber
sample
optics
p1
point
source

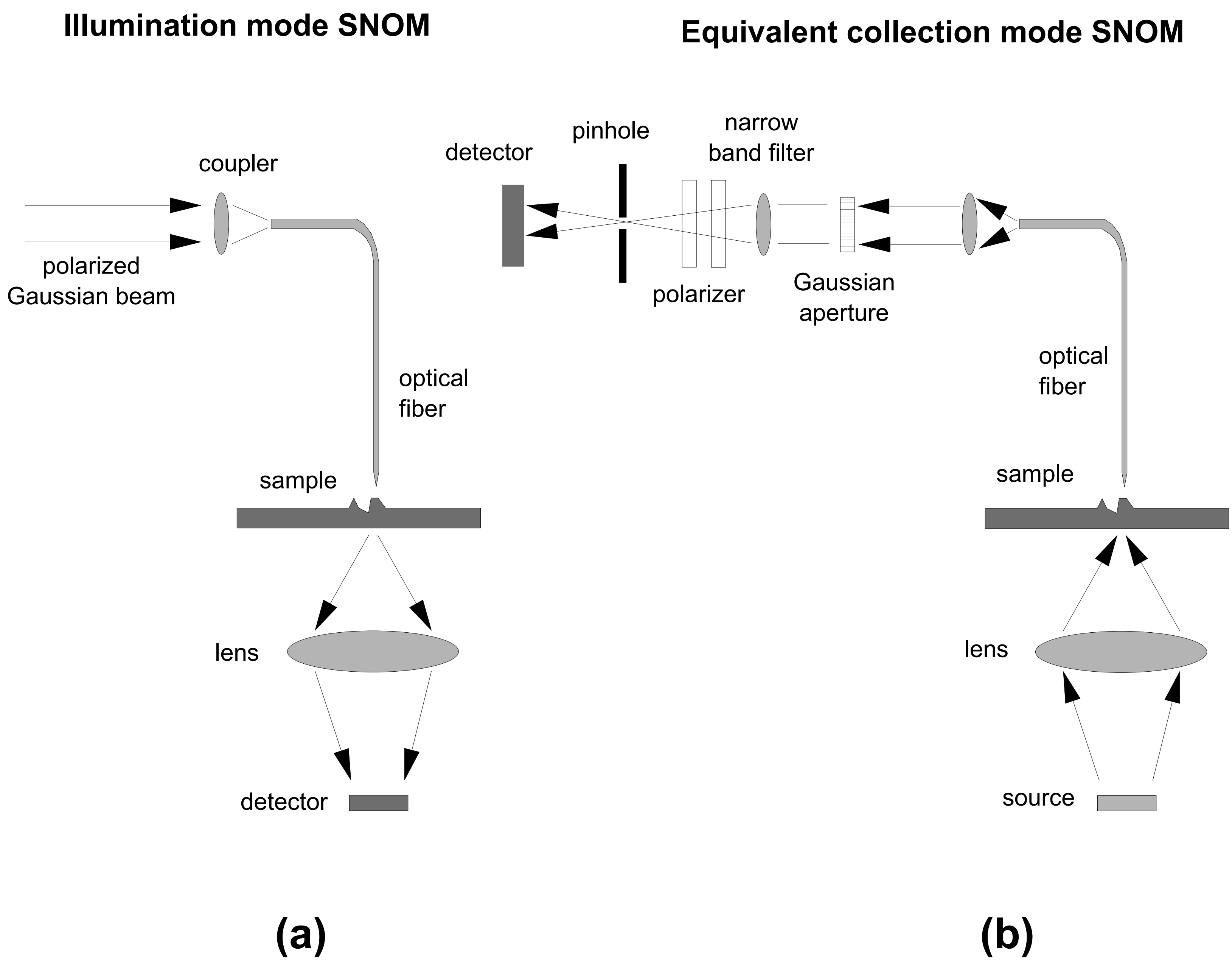
(a) (b)



Incoherent sources and “incoherent detectors”

Principle of equivalence (aas in conventional scanning microscopy).


PSTM
detector
lens
optical
fiber
sample
collimating
lens
prism
pinhole
polarized Gaussian
beam
(a)
Equivalent illumination mode SNOM
lens
incoherent
source
optical
fiber
sample
Gaussian aperture
polarizer
prism
focusing lens
narrow band filter
detector
pinhole
(b)

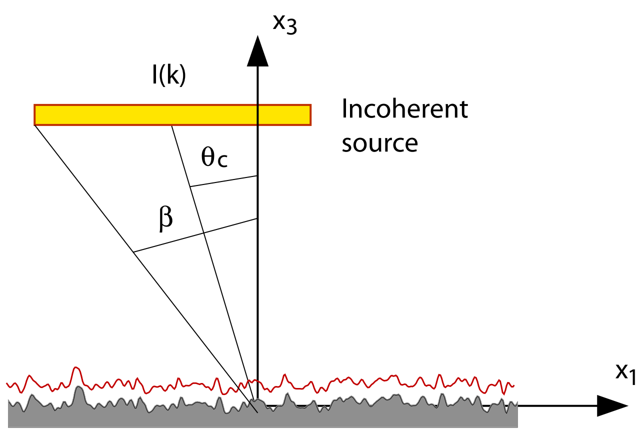


Illumination mode SNOM
coupler
polarized
Gaussian beam
optical
fiber
sample
lens
detector
(a)
Equivalent collection mode SNOM
detector
pinhole
narrow
band filter
polarizer
Gaussian
aperture
optical
fiber
sample
lens
source
(b)

The near-field intensity under partially coherent illumination

Assumptions:

- Perfectly conducting one-dimensional surface.
- S-polarized incoherent quasimonochromatic ($\Delta\nu << \nu$) source
- Passive probe.



Objective:

Find a relation between $\zeta(x_1)$ and the near-field intensity.

Partially coherent illumination

- $\Gamma_{12}(\tau) = \langle \psi(P_1, t+\tau)\psi(P_2, t)\rangle$ - mutual coherence function.
- $J_{12} = \Gamma_{12}(0)$ - mutual intensity.

These quantities involve averages over an ensemble of realizations.

Employing Hopkin's formula, we can write the incident mutual intensity

$$J_{p,s}(x_1, x_3; x_1', x_3'|\omega)_{inc} = \int_\sigma dk\, \Psi_{p,s}(x_1, x_3|k|\omega)_{inc}\Psi^*_{p,s}(x_1', x_3'|k|\omega)_{inc}\,,$$

where

$$\Psi_{p,s}(x_1, x_3|k|\omega)_{inc} = \sqrt{I(k)}e^{ikx_1 - i\alpha_0(k,\omega)x_3}\,,$$

For $(x_1, x_3) = (x_1', x_3')$, we obtain the incident intensity

$$I_{p,s}(x_1, x_3)_{inc} = \int_\sigma dk\, |\Psi_{p,s}(x_1, x_3|k|\omega)_{inc}|^2 = \int_\sigma dk\, I(k)\,.$$

The total and scattered intensities

The total field in the region $x_3 > \zeta(x_1)_{max}$, is

$$\Psi_{p,s}(x_1, x_3|k|\omega)_{tot} = \sqrt{I(k)} e^{ikx_1 - i\alpha_0(k,\omega)x_3} + \Psi_{p,s}(x_1, x_3|k|\omega)_{sc} \,,$$

The total intensity can be written in the form

$$\begin{aligned} I_{p,s}(x_1, x_3)_{tot} &= I_{p,s}(x_1, x_3)_{inc} + I_{p,s}(x_1, x_3)_{sc} + \\ &+ \int_\sigma dk \, 2\Re e\{\Psi_{p,s}(x_1, x_3|k|\omega)_{inc} \Psi^*_{p,s}(x_1, x_3|k|\omega)_{sc}\} \,, \end{aligned}$$

where we have defined

$$I_{p,s}(x_1, x_3)_{sc} = \int_\sigma dk \, |\Psi_{p,s}(x_1, x_3|k|\omega)_{sc}|^2 \,,$$

and

$$I_{p,s}(x_1, x_3)_{tot} = \int_\sigma dk \, |\Psi_{p,s}(x_1, x_3|k|\omega)_{tot}|^2 \,.$$

First order perturbation

The scattered field due to a plane wave component of the incident field is given by

$$\Psi_{p,s}(x_1, x_3|k|\omega)_{sc} = \sqrt{I(k)} \int_{-\infty}^{\infty} \frac{dq}{2\pi} R_{p,s}(q|k) e^{iqx_1 + i\alpha_0(q,\omega)x_3}.$$

To first order in $\zeta(x_1)$, the scattering amplitude has the form

$$R_{p,s}(q|k) = -2\pi\delta(q-k) + 2i\alpha_0(k,\omega)\hat{\zeta}(q-k).$$

where

- $\hat{\zeta}(Q) = \int_{-\infty}^{\infty} dx_1 e^{-iQx_1}\zeta(x_1)$

The Scattered Intensity

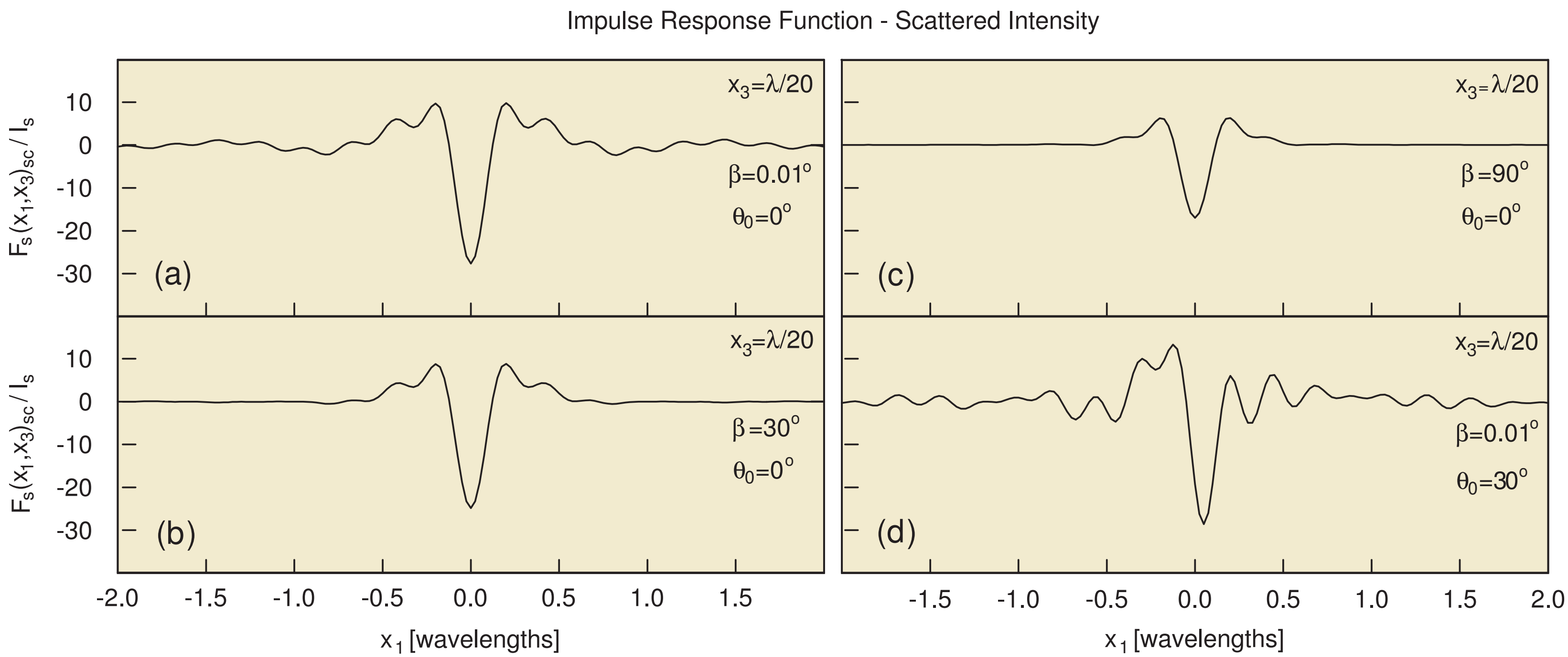
The integrated scattered intensity at the point (x_1, x_3) is

$$I_s(x_1, x_3)_{sc} = I_s + \int_{-\infty}^{\infty} du \zeta(x_1 - u) F_s(u, x_3)_{sc},$$

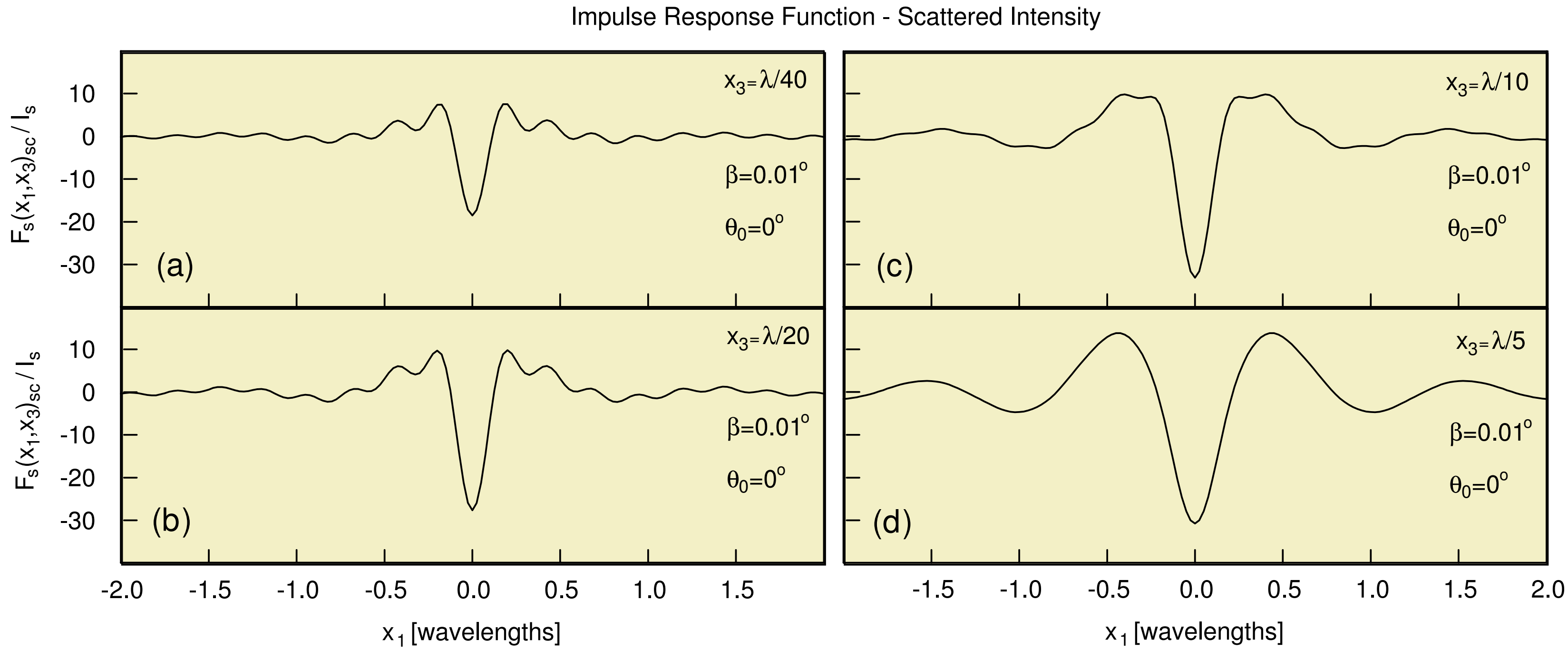
where the functions I_s and $F_s(u, x_3)_{sc}$ are given by

$$I_s = I_0 \Delta k = 2 I_0 \left(\frac{\omega}{c}\right) \cos\theta_c \sin\beta \,,$$

$$F_s(u, x_3)_{sc} \;\; = \;\; 4 I_0 Im \int_{k_c - \Delta k/2}^{k_c + \Delta k/2} dk \alpha_0(k, \omega) e^{-iku - i\alpha_0(k,\omega)x_3} \int_{-\infty}^{\infty} \frac{dq}{2\pi} e^{iqu + i\alpha_0(q,\omega)x_3}.$$



Dependence of the impulse response function of the scattered field on the kind of illumination.



Dependence of the impulse response function of the scattered field on the distance from the surface.

$F_s(u, x_3)_{sc}$ is strongly peaked at $u = 0$ for small values of x_3.

Thus,

$$I_s(x_1, x_3)_{sc} \simeq I_s + \zeta(x_1) \int_{-\infty}^{\infty} du F_s(u, x_3)_{sc},$$

It can be shown, however, that the integral vanishes.

Returning to the convolution integral, we then expand $\zeta(x_1 - u)$ in powers of u, and integrate term-by-term, the first nonzero term yields

$$I_s(x_1, x_3)_{sc} \simeq I_s + \frac{1}{2}\zeta''(x_1) \int_{-\infty}^{\infty} du u^2 F(u, x_3)_{sc}.$$

So, the scattered intensity at constant height will resemble more closely the second derivative of the surface profile, rather than the profile itself.

The claim that $I_s(x_1, x_3)_{sc}$ as a function of x_1 for a fixed value of x_3 follows the surface profile function $\zeta(x_1)$ is not generally valid.

The total intensity

To first order in $\zeta(x_1)$, the total intensity at the point (x_1, x_3) is

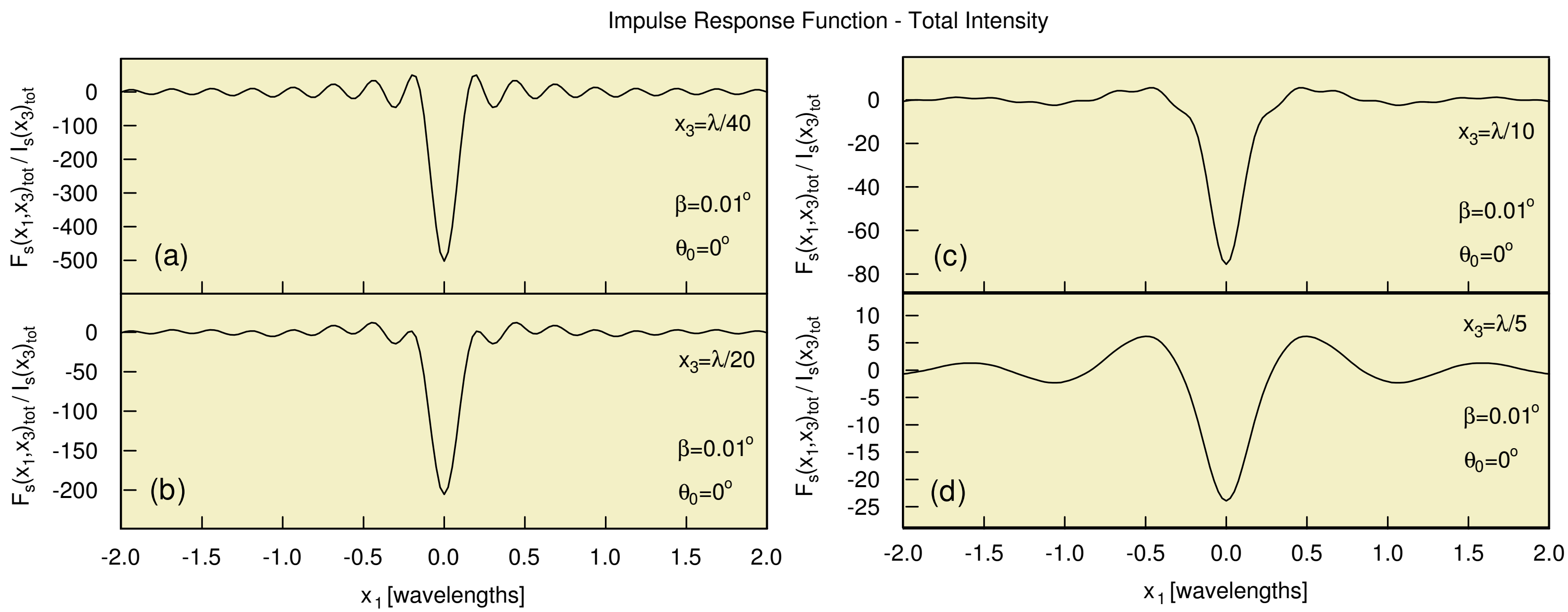
$$I_s(x_1, x_3)_{tot} = I_s(x_3)_{tot} + \int_{-\infty}^{\infty} du \zeta(x_1 - u) F_s(u, x_3)_{tot},$$

where

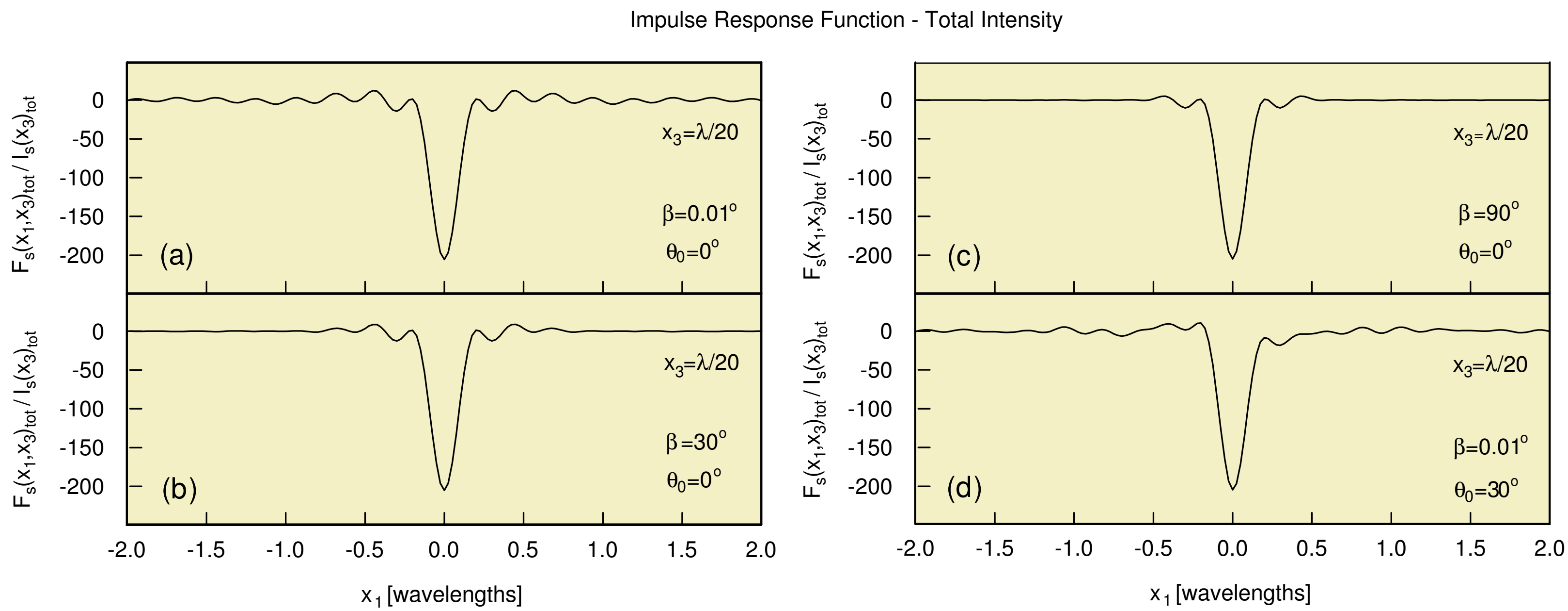
$$I_s(x_3)_{tot} = 4I_0 \int_{k_c-\Delta k/2}^{k_c+\Delta k/2} dk \sin^2 \alpha_0(k,\omega) x_3,$$

and

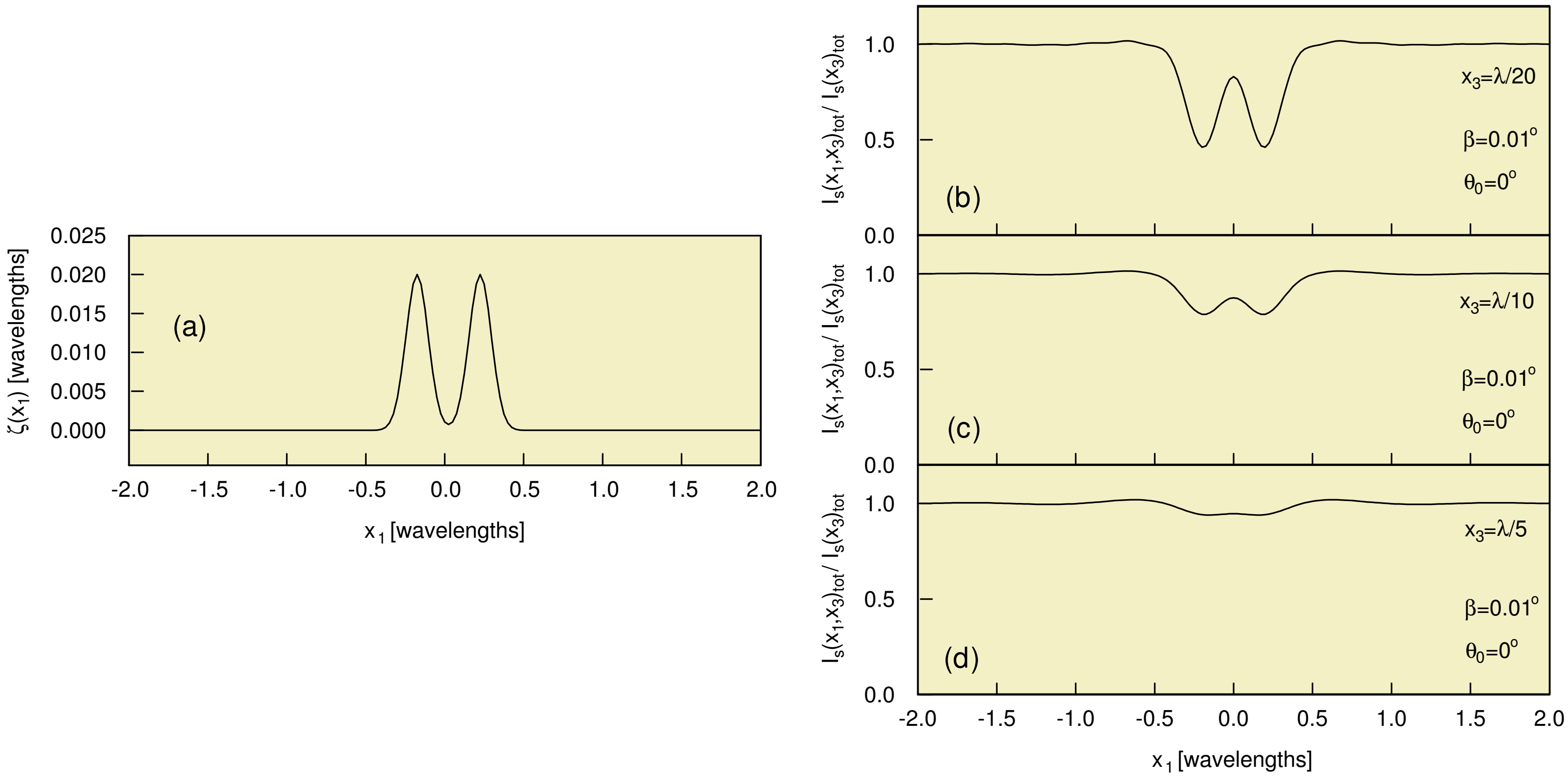
$$\begin{aligned} F_s(u, x_3)_{tot} &= -8I_0 Re \int_{k_c-\Delta k/2}^{k_c+\Delta k/2} dk e^{-iku} \alpha_0(k,\omega) \sin \alpha_0(k,\omega) x_3 \times \\ &\times \int_{-\infty}^{\infty} \frac{dq}{2\pi} e^{iqu + i\alpha_0(q,\omega) x_3}. \end{aligned}$$



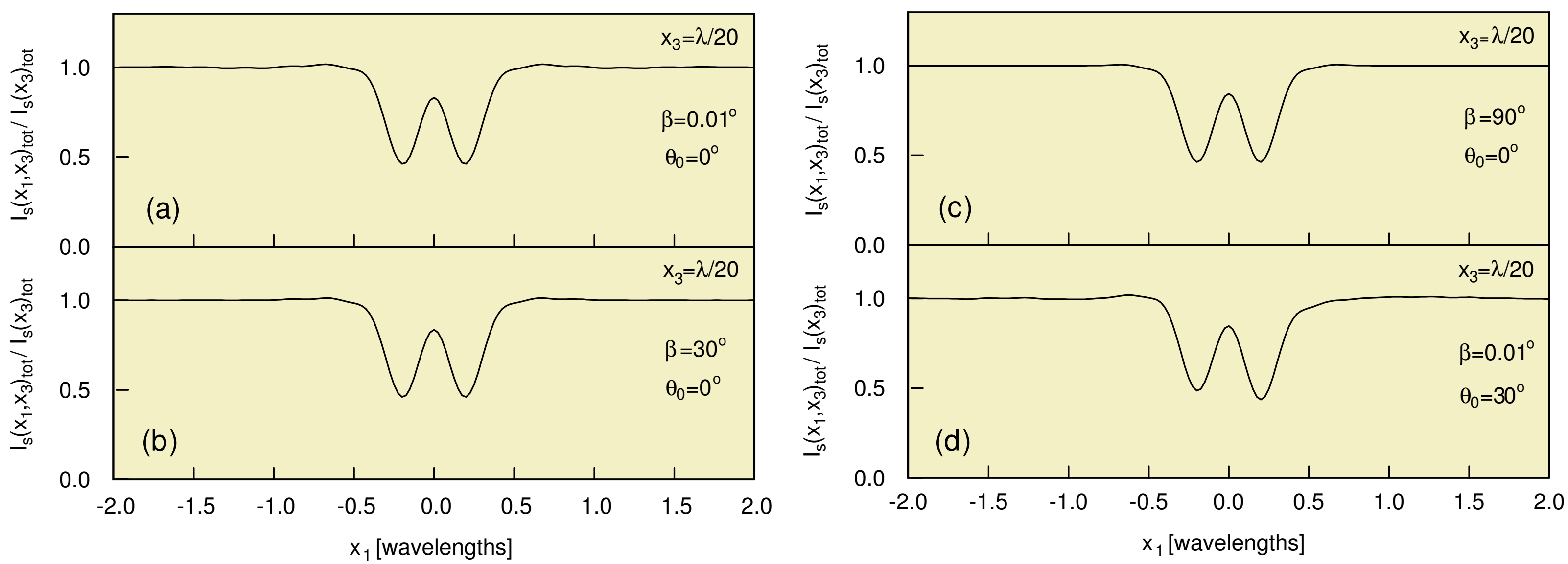
Dependence of the impulse response function of the total field on the distance from the surface.



Dependence of the impulse response function of the total field on the kind of illumination.



The surface profile and the total intensity for a surface profile function of the form $\zeta(x_1) = A\exp(-(x_1 - B)^2/R^2) + A\exp(-(x_1 + B)^2/R^2)$ with, $A = 0.02\lambda$, $R = 0.1\lambda$, and $B = 0.2\lambda$.



The total near field intensity for the double Gaussian groove as a function of the kind of illumination.

For small values of x_3, the impulse response function is a sharply peaked function of x_1 and we can write

$$\begin{aligned} I(x_1, x_3)_{tot} &\simeq I(x_3)_{tot} + \zeta(x_1)\int_{-\infty}^{\infty} du F(u, x_3)_{tot} - \zeta'(x_1)\int_{-\infty}^{\infty} du u F(u, x_3)_{tot} + \\ &+\frac{1}{2}\zeta''(x_1)\int_{-\infty}^{\infty} du u^2 F(u, x_3)_{tot}. \end{aligned}$$

It can be easily verified that:

- the term proportional to $\zeta'(x_1)$ vanishes for symmetrical illumination modes.

- the term proportional to $\zeta''(x_1)$ provides a small correction.

It follows that we can estimate the surface profile from the equation

$$\zeta(x_1) \simeq \frac{I(x_1, x_3)_{tot} - I(x_3)_{tot}}{\int_{-\infty}^{\infty} du F(u, x_3)_{tot}}.$$

Conclusions

- Equivalence between PSTM and SNOM.
- The resolution is not affected by the coherence of the illumination.
- The image (and resolution) deteriorates as one moves away from the surface.
- The scattered intensity does not resembles the profile $[\zeta''(x_1)]$.
- The total intensity does resemble the profile.