Bubble break-up as a two dimensional hydrodynamics problem

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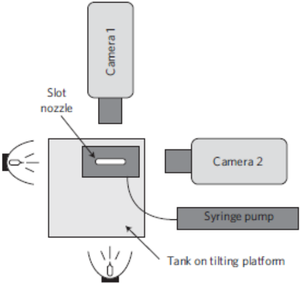
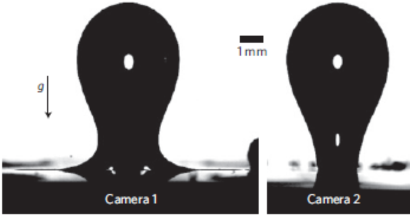
K.Turitsyn, L.Lai, W.Zhang, Phys.Rev.Lett. 123, 124501(2009)

Experiment





L. Schmidt, N. Keim, W. Zhang, S. Nagel, Nature Physics 2009

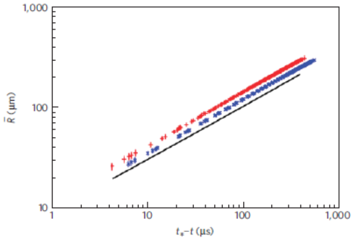
Air bubble is blown from the bottom of water tank. The bubble disconnection process is observed with high-speed video cameras.

Motivation

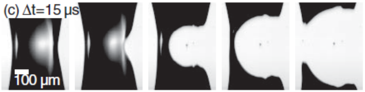
Disconnection

- Natural process: formation and breaking of bubbles
- Singularity formation: Fluid velocity grows like $V \propto (t_c - t)^{-1/2}$
- Non-universal behavior: axial symmetry is inevitably broken

Neck radius



Final Shape



N. Keim et.al. Phys. Rev. Lett. 2006

Theoretical considerations

- In axisymmetric situation the neck radius goes like $R \propto (t_c - t)^\alpha$, where $\alpha \approx 0.56 \pm 0.01$ (experiment: Keim et.al. PRL 2006) or $\alpha = 1/2 + 1/4\sqrt{t_c - t}$ (theory: Eggers et.al. PRL 2007)
- Neck becomes more and more slender: slope $R/z \propto (t_c - t)^{\alpha - 1/2} \to 0$
- Reynolds number: $\mathrm{Re} \propto (t_c - t)^{2\alpha - 1}$, initially $\mathrm{Re} \approx 200$
- Weber number ($\mathrm{We} = \rho V^2 R/\sigma$): $\mathrm{We} \propto (t_c - t)^{3\alpha - 2} \to \infty$

Theory: Hydrodynamic equation

In the sleder neck approximation the dynamics of neck cross-section as a two-dimensional cavity surrounded by ideal fluid: $\vec{v} = \nabla\Phi$ with $\Phi \to -Q \log r$ when $r \to \infty$ and

$$\nabla^2 \Phi = 0$$

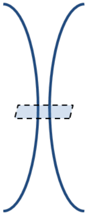
Surface dynamics is governed by two boundary conditions. Kinematic:

$$\frac{d\vec{x}}{dt} = (\partial_t + \nabla\Phi \cdot \nabla)\vec{x} = \nabla\Phi|_S$$

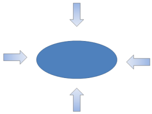
Dynamic:

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 \Big|_S = 0$$

Crosssection



2D bubble

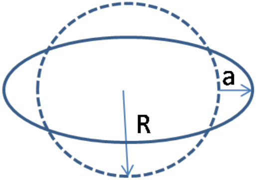


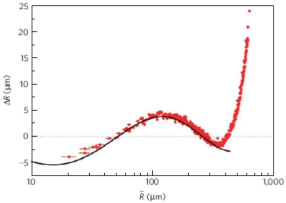
Linear Stability

There exists a perfect axisymmetric solution: $R(t) = C(t_c - t)^{1/2}$. This solution is unstable. If the initial surface is deformed by $a = \sum a_m(0)\cos(m\varphi)$, then within linear approximation

$$a_m(t) = a_m(0)\cos\left(\sqrt{m-1}\log\frac{R(0)}{R(t)}\right)$$
$$\propto \mathrm{Re}(t_c - t)^{i\sqrt{m-1}/2}$$

Relative distortion grows in logarithmic time: $a_m/R \propto (t_c - t)^{-1/2}$ Full memory about initial perturbation is preserved!



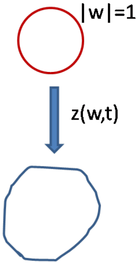


Schmidt et.al., Nature Phys. 2009

Conformal Mapping

Two-dimensional free surface problem can be effectively solved by mapping the physical surface to a circle in mathematical plane (Dyachenko et.al. 1996).

Mapping



$$z \equiv x + iy = z(w,t), \quad \Phi = \mathrm{Re}\Psi(w,t)$$

$$w \to \infty : \quad z \to Rw, \ \Psi \to -QR \log w$$

where $z(w,t)$ and $\Psi(w,t)$ are analytic in the disk $1 < |w| < \infty$. It is more convenient to use "Dyachenko" variables:

$$\mathcal{R}(w,t) = \frac{1}{w\partial_w z(w,t)}, \quad \mathcal{V}(w,t) = \frac{\partial_w \Psi(w,t)}{\partial_w z(w,t)}$$

Dyachenko Equations

The dynamical equations can be written as

$$\partial_t \mathcal{R} = w(\partial_w \mathcal{R})\mathcal{A}\left\{\mathrm{Re}[\mathcal{R}\mathcal{V}^*]\right\} - w\mathcal{R}\partial_w \mathcal{A}\left\{\mathrm{Re}[\mathcal{R}\mathcal{V}^*]\right\}$$

$$\partial_t \mathcal{V} = w(\partial_w \mathcal{V})\mathcal{A}\left\{\mathrm{Re}[\mathcal{R}\mathcal{V}^*]\right\} - w\mathcal{R}\partial_w \mathcal{A}\left\{|\mathcal{V}|^2\right\}/2$$

where $\mathcal{A}$ is analytic continuation operator:

$$\mathcal{A}\left\{F\right\}(w) = \oint_{|w'|=1} \frac{dw'}{2\pi i w'} \frac{w + w'}{w - w'} F(w')$$

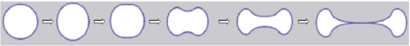
General form of the solution:

$$\mathcal{R}(w,t) = \sum_{n=0}^{\infty} \frac{r_n(t)}{w^{1+n}}, \quad \mathcal{V}(w,t) = \sum_{n=0}^{\infty} \frac{v_n(t)}{w^{1+n}}$$

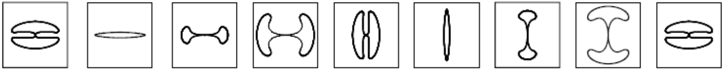
Convenient for numerics: low-order nonlinearity; FFT to perform non-local operation $\mathcal{A}$.

Numerical results

Typical dynamics (rescaled)



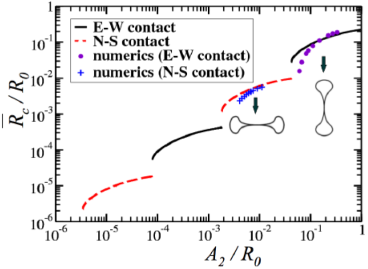
Final shapes



We started from a small elliptical ($m = 2$) perturbation on top of circular bubble. The shape of the bubble remained smooth throughout the dynamics and typically evolved in one or two contacts.

Outcomes

- The outcome depends periodically on initial amplitude of the perturbation.
- Final shape has one or two contact points
- Strongly elongated shape observed for special initial conditions

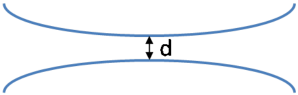


Three lobe shape

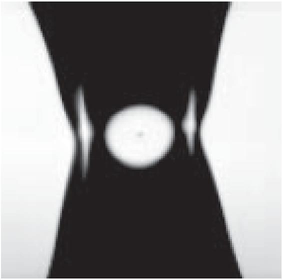
Aspect ratio $L/d \approx 90$

Contact event

- Perfectly regular dynamics withing free surface approximation: layer width behaves like $d \propto t_c - t$, surface curvature $K \to const$
- Air pressure grows like $p \propto (t_c - t)^{-1}$ and excites capillary waves resulting in satellite bubble with $R \approx 5 - 10\mu m$
- Similar to processes happening during droplet splashing on solid surfaces (Mandre et.al., PRL 2009)

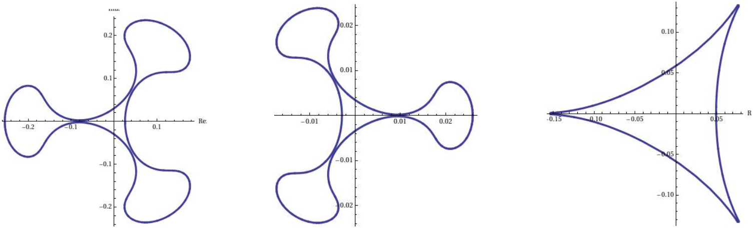


Satellite bubble



Three fold symmetry

$m = 3$ perturbation



Smooth surface contact is the generic outcome for $m = 3$ symmetry. However, singular cusp shapes are observed for special initial conditions.

Conclusions

- Bubble formation is accompanied by a singular neck reconnection process
- Cross section evolution can be effectively described by a 2D free surface hydrodynamics model
- Dynamics is very sensitive to initial perturbations, but generic outcome is smooth contact

Open questions

- Dynamics after the contact
- 3D effects
- Analytical structure of the solutions ?