Fibonacci Patterns in Plants

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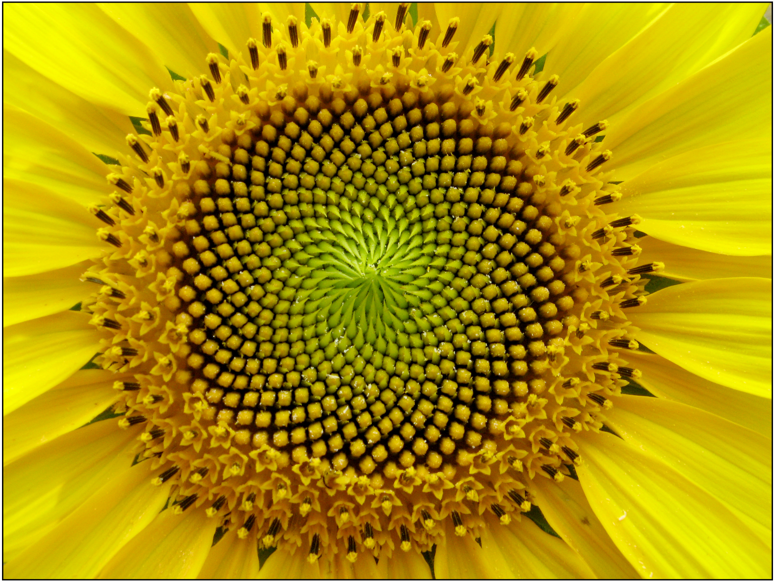
An Example



History

Placement of primordia is determined by a set of rules, the most important being:

> *New primordia are formed periodically in the largest available space left by the previous ones.*

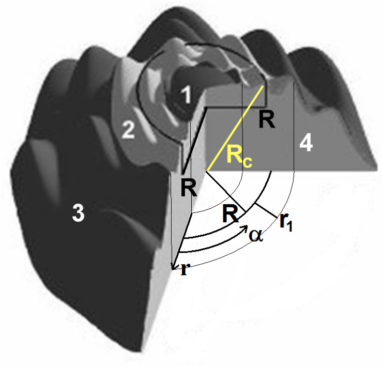
Pursuing this approach leads to the conclusion:

> *The ordering is explained due to the system's trend to avoid rational (periodic) organization, thus leading to a convergence towards the golden mean.*

Hofmeister (1868); Douady and Couder (1996); Atela, Golé, and Hotton (2002).

The Shoot Apical Meristem

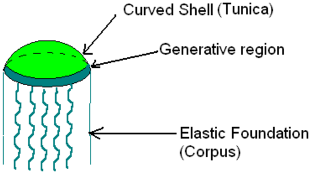
1. Rapid mitosis produces undifferentiated cells.
2. Primordia form on the generative annulus.
3. Existing primordia move outward and continue their development.

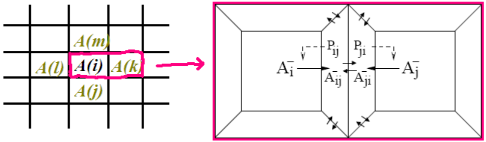


Model Statement

Mechanics and biochemistry are the primary contributors to primordium formation.

Growth Mechanisms





Mechanics:

> *Primordia appear as buckling of the surface of the SAM, which can be modeled as a hard shell on an elastic foundation.*

Biochemistry:

> *Primordium locations are mediated by local concentrations of the plant hormone auxin, which is in turn regulated by the PIN1 protein.*

Greene, Steele, and Rennich (1996); Jönsson, et al (2005).

The Model Equations

Let w be the surface deformation field, f be the fluctuation of the Airy stress potential, and g be the growth strain field.

Model Equations

$$w_t = -\Delta^2 w - \kappa w - \gamma w^3 - P\Delta_\chi w + \nu^{-1}[f, w] - C\Delta_\rho f$$
$$\Delta^2 f - C\Delta_\rho w + (2\nu)^{-1}[w, w] + \Delta g = 0$$
$$g_t = -Lg - H\Delta g - \Delta^2 g - \bar{\kappa}_1 \nabla(g\nabla g) - \bar{\kappa}_2 \nabla(\nabla g \Delta g) - \delta g^3 + \beta \Delta f$$

Near the onset of pattern formation, the behavior is governed more by the overall symmetries. We simplify this model to include only the main ingedients.

Swift-Hohenberg Equation

$$u_t = -(\nabla^2 + k_0^2)^2 u + \varepsilon u + \vartheta(\nabla u)^2 - 2\vartheta\nabla(u\nabla u) - \gamma u^3$$

Aside: Why Hexagons?

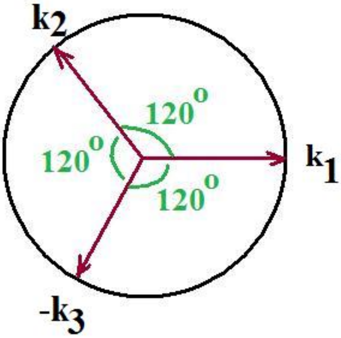
Another Swift-Hohenberg Equation

$$u_t = -(\nabla^2 + 1)^2 u + \varepsilon u + \vartheta u^2 - \gamma u^3$$

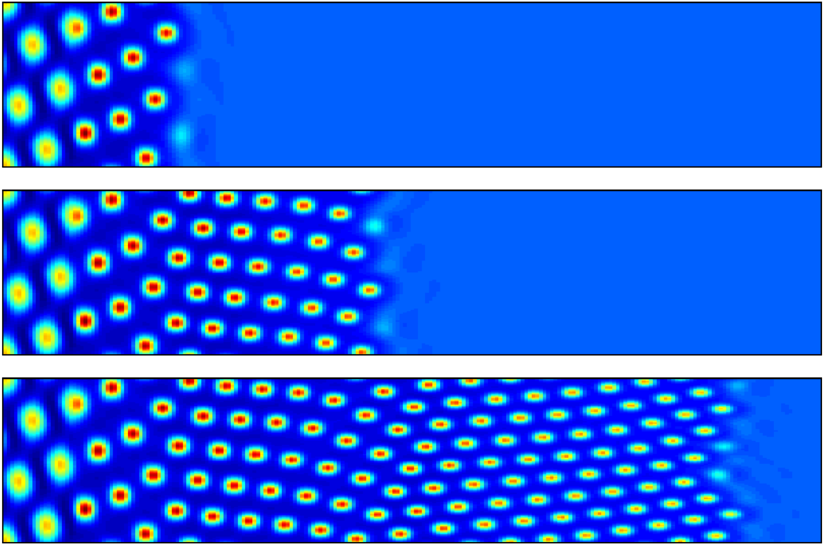
Examine the the uniform solution to find that modes with

$$\varepsilon - (k^2 - 1)^2 > 0$$

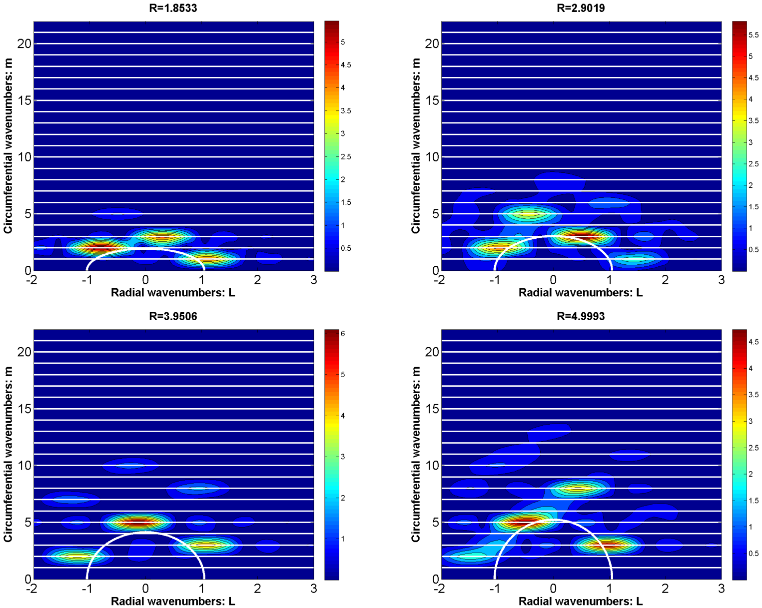
are unstable. When do k_1, k_2, and $k_3 = k_1 + k_2$ all lie on the unit circle?



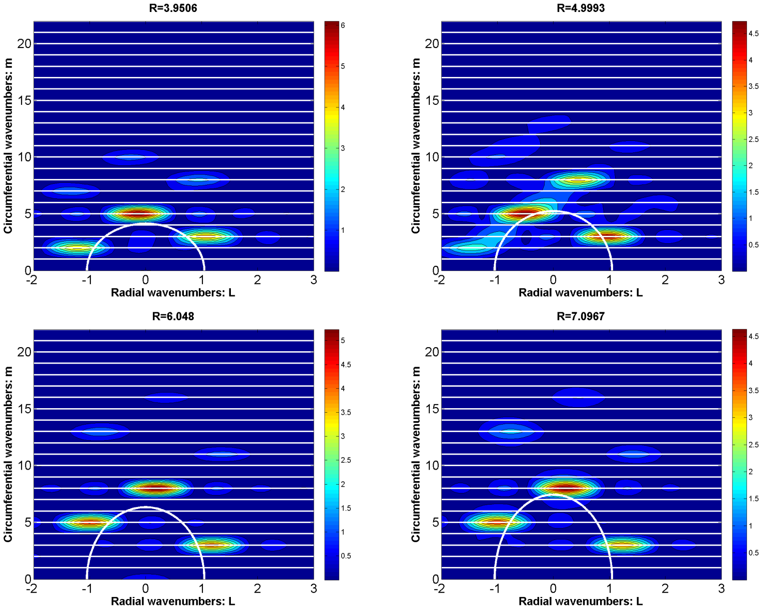
Simulation Results



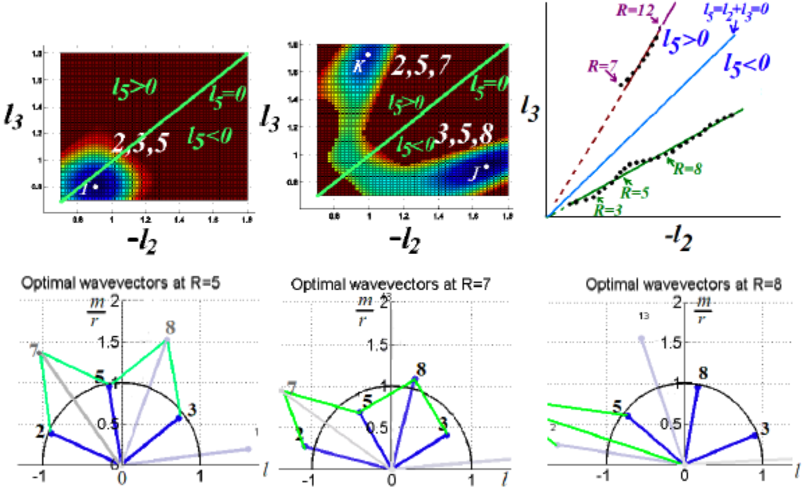
Local Spectra of Results



Local Spectra of Results



Why Fibonacci?



Thank You