

$$a = \frac{16}{3} \left(1 + \frac{23}{32} \frac{\tilde{\eta}}{\eta} + \frac{\zeta}{2\eta r_0} \right)$$