Synchrony in Stochastic Pulse-coupled Neuronal Network Models

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March 27, 2010

Frontiers in Nonlinear Waves
The University of Arizona

Motivation and Goals

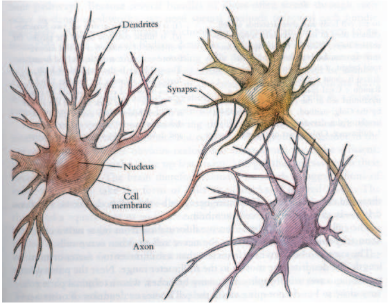
- Motivation: Neuronal Model Behavior
  - Study the most basic point neuron models, network models, and coarse-grained models for use in computations
  - Develop new analytical techniques
  - Look for simple, universal network mechanisms
- Goals: Understand Oscillatory Dynamics of IF Model
  - Generalize classical problem of perfectly synchronous Integrate-and-Fire (IF) network oscillations to stochastic setting
  - Quantitative analysis of mechanism sustaining synchrony
  - Lessons for understanding more complicated models and types of oscillations?

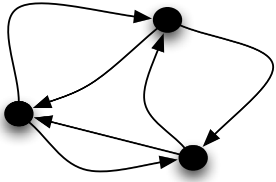
Outline

- Network dynamics: model and simulations
- Characterization of mechanism for sustaining stochastically driven synchronized dynamics
- Computation of firing rates based on first passage time
- Computation of probability of repeated total firing events

Neuronal Network Model

- Point-Neuron Model
  - Integrate-and-fire
  - Poisson process external driving current
- Simple Network Model
  - Excitatory neurons
  - All-to-all connected
  - Instantaneous spike transmission



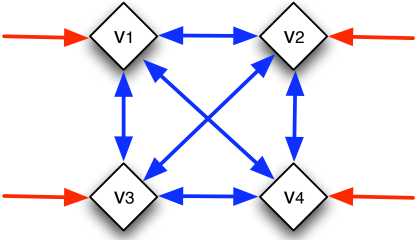


Excitatory Neuronal Network

All-to-all connected, current based, integrate-and-fire (IF) network

$$\frac{dv_i(t)}{dt} = -g_L(v_i(t) - V_R) + I_i(t), \quad i = 1, \ldots, N$$

$$I_i(t) = f \sum_k \delta(t - t_{ik}) + \frac{S}{N} \sum_{j \neq i} \sum_k \delta(t - \tilde{t}_{jk})$$



$V_R \leq v_i(t) < V_T$

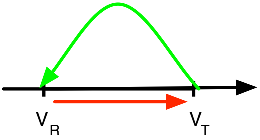
- V_R: reset potential (=0)
- V_T: firing threshold (=1)

g_L: leakage rate (=1)
f and S: coupling strengths (> 0)

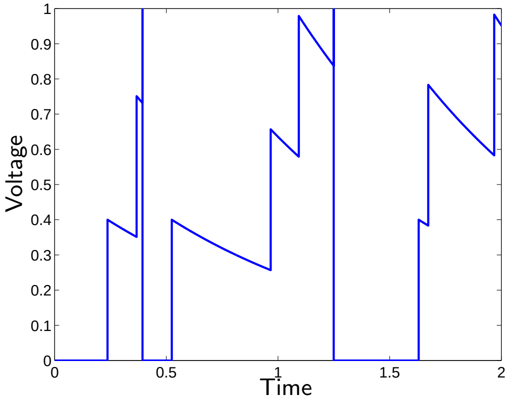
Integrate & Fire Dynamics

- $v_i(t)$ evolves according to the voltage ODE until $\tilde{t}_{ik}$, when $v_i(\tilde{t}_{ik}) \geq V_T$
- At $\tilde{t}_{ik}$, $v_i(t)$ is reset to V_R: $v_i(\tilde{t}_{ik}^+) = V_R$
- S/N is added to the voltage, v_j, for $j = 1, \ldots, N, j \neq i$

Reset:

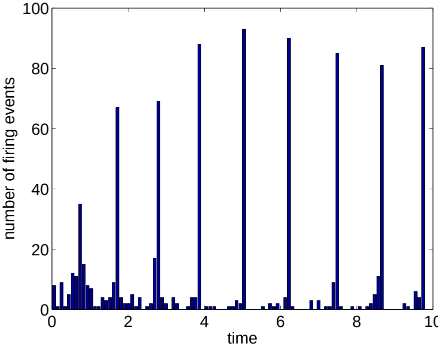


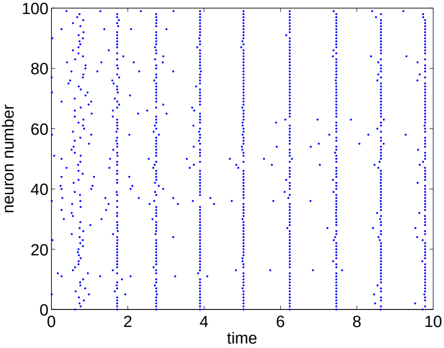
Membrane Potential:



Types of Synchronous Firing: Partial Synchrony

All-to-all connected network of $N = 100$ neurons

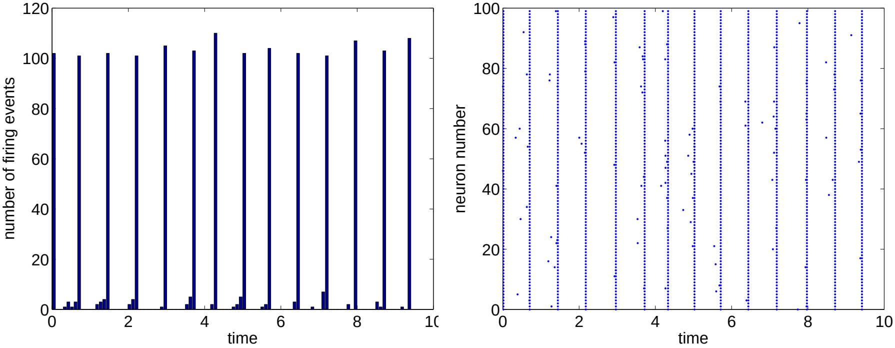




$$f = 0.01, f\nu = 1.2, S = 0.4$$

Types of Synchronous Firing: Imperfect Synchrony

All-to-all connected network of $N = 100$ neurons

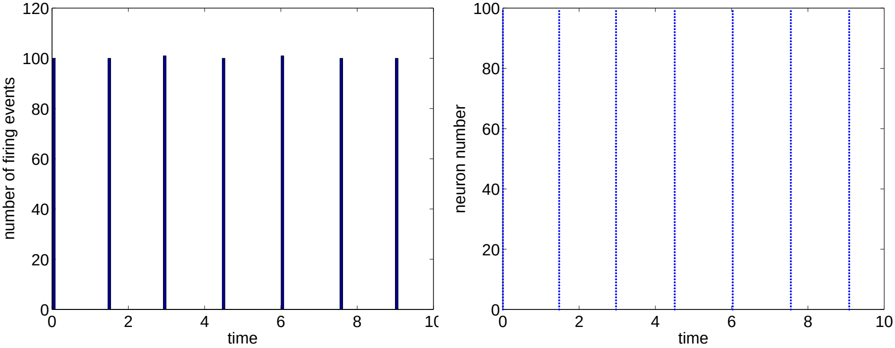


$$f = 0.1, f\nu = 1.2, S = 2.0$$

Types of Synchronous Firing: Perfect Synchrony

All-to-all connected network of $N = 100$ neurons

Consistent total firing events; "synchronizable network"

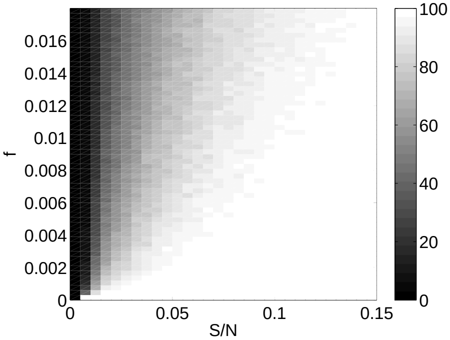


$$f = 0.001, f\nu = 1.2, S = 2.0$$

Average Size of Cascading Firing Event

Superthreshold

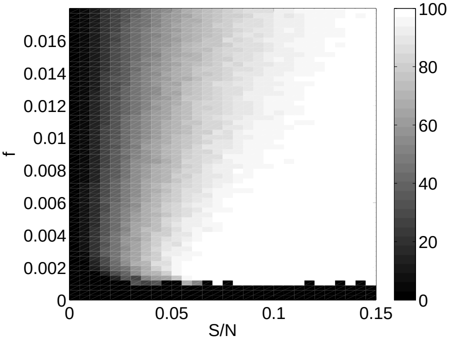
$V_R + f\nu/g_L > V_T$



$N = 100,\ f\nu = 1.2 > 1$

Subthreshold

$V_R + f\nu/g_L < V_T$



$N = 100,\ f\nu = 0.9 < 1$

Model for Self-Sustaining Synchrony

Fluctuations in input desynchronize the network

- Between total firing events N neurons behave independently

Pulse coupling synchronizes the network

- First neuron firing causes cascading event, pulling all other neurons over threshold due to synaptic coupling (S)

After each total firing event, all neurons reset, process repeats

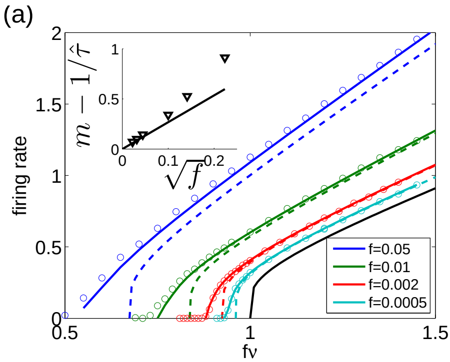
Which systems are synchronous?
What is the mean firing rate of the network?

Deterministic Firing Rate

- Replace stochastic driving with mean, $f\nu$

$$\frac{dv}{dt} = -g_L(v - V_R) + f\nu$$

- Solve $v(\hat{\tau}) = V_T$ for $\hat{\tau}$ with $v(0) = V_R$



$$\hat{\tau} = -\frac{1}{g_L} \log\left(\frac{f\nu}{f\nu - g_L(V_T - V_R)}\right)$$

$N = 100, S = 5.0$

black line: $1/\tau$

- ▷ Synchronous stochastic network fires at a faster rate than synchronous deterministic network
- ▷ Synchronous stochastic network is not periodic

[Mirollo and Strogatz 1990]

Synchronous Gain Curves

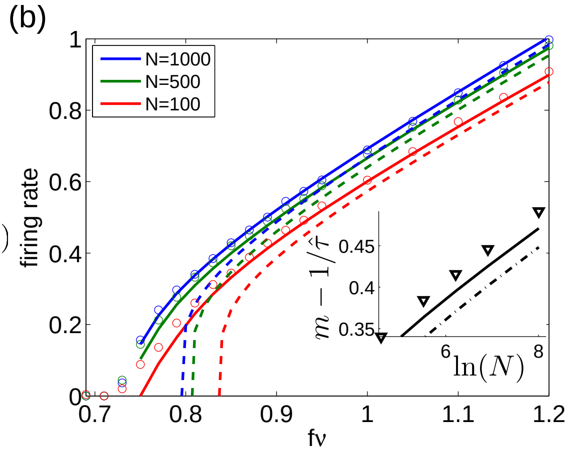
solid - Mean first passage time of N neurons

$$\langle t\rangle = \int_0^\infty t p_T^{(1)}(t)dt$$

$$p_T^{(1)}(t) = N p_T(t)(1-F_T(t))$$

dashed - Gaussian approx.
circles - simulations

- For fixed (superthreshold) $f\nu$, dependence $\sim \ln N$

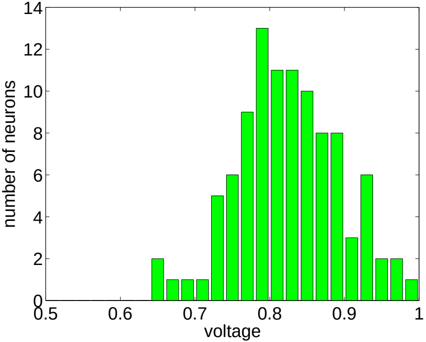


$f = 0.01, S = 20.0$

Model for Synchronous Firing

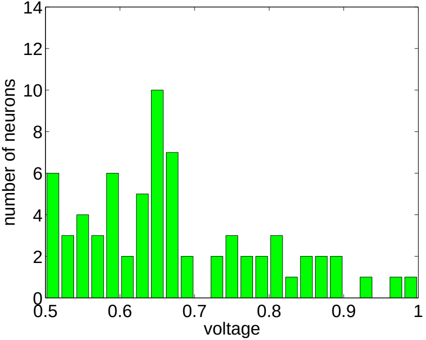
Voltages of the other $N-1$ neurons when the first neuron fires:

Cascade-Susceptible



$f = 0.008, f\nu = 1.0$

Not Cascade-Susceptible

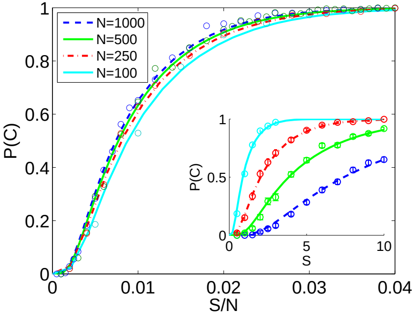


$f = 0.08, f\nu = 1.0$

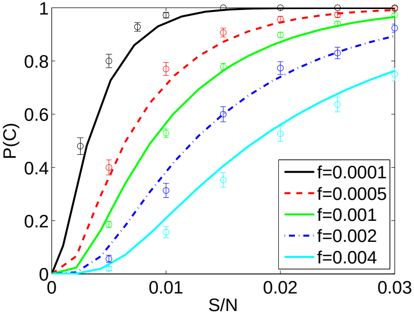
$S = 2, N = 100$ (Bin size $= S/N$)

Probability that Network is Cascade-Susceptible

Larger fluctuations need larger coupling to maintain synchrony



$$f\nu = 1.2, f = 0.001$$

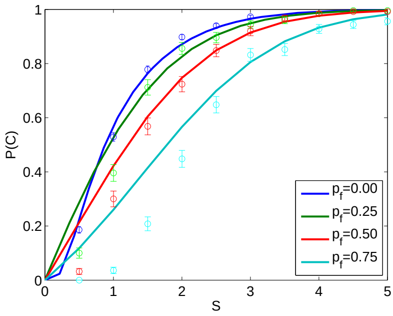


$$f\nu = 1.2, N = 100$$

Probability that Network is Cascade-Susceptible

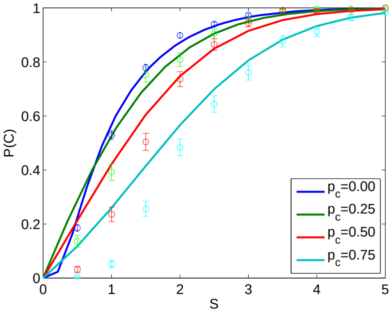
What about for networks where connections are not all-to-all?

Synaptic failure: p_f



$N = 100, f\nu = 1.2, f = 0.001$

Sparsity: p_c

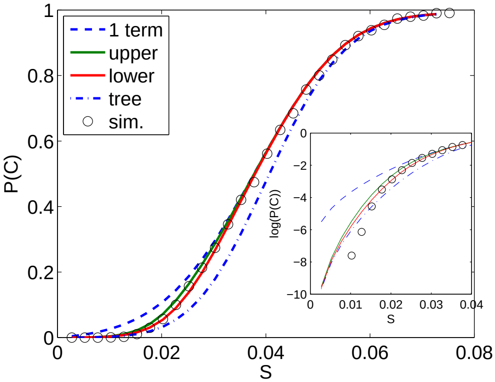


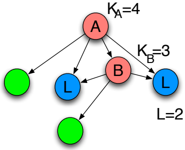
$N = 100, f\nu = 1.2, N = 100$

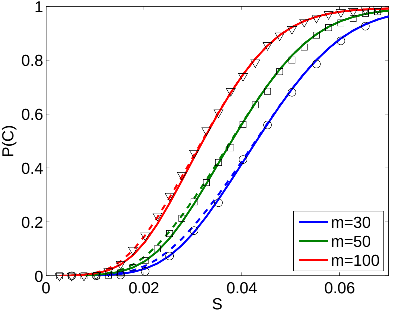
Adjust $P(C)$ by taking $p_{v|V^{(N)}}(x,t) \rightarrow \left\{ \begin{array}{l} (1-p_f) \\ (1-p_c) \end{array} \right. \; p_{v|V^{(N)}}(x,t)$

Probability that Network is Cascade-Susceptible

What about complex directed networks? Effect of local topology





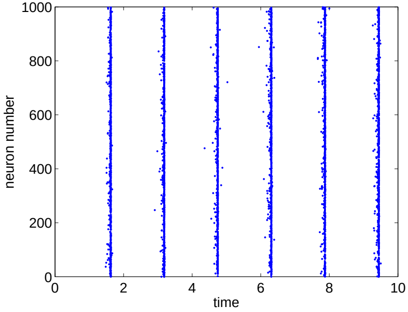


$N = 4000, f\nu = 1.2, f = 0.001$

Probability that Network is Cascade-Susceptible

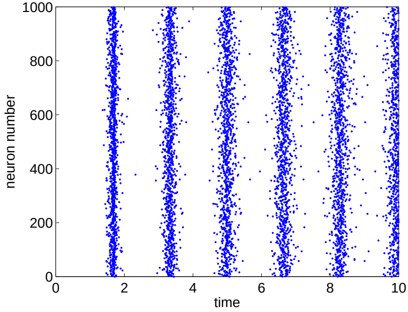
What about non instantaneous synaptic coupling? $P(C) = 0$

Synaptic Delay: T_D



$$N = 1000, S = 0.1, \langle T_D \rangle = 0.002$$
$$f\nu = 1.2, f = 0.001$$

Synaptic Time Course: $H(t)\frac{t}{\tau_E^2}e^{-t/\tau_E}$



$$N = 1000, S = 0.1, \tau_E = 0.002$$
$$f\nu = 1.2, f = 0.001,$$

Future Directions

- Understand interaction of nonlinear dynamics with the network topology
  - Find attracting, statistically stable states
  - Determine dominant network property interacting with the dynamics of interest

- Investigate stochastic synchrony for non-instantaneously coupled models
  - Perturbation away from perfectly synchronous state
  - Cascading total firing events become "waves" that propagate through the network

This work was supported by a NSF graduate fellowship
(Newhall & al. (2010) *Comm in Math Sci*, Vol. 8, No. 2, pp. 541-600)
(Newhall & al. (2010) *Phys. Rev. E*, submitted)
Thanks to Peter Kramer, Gregor Kovacic, Aaditya Rangan, Max Shkarayev, and David Cai