Modeling U(ltra)V(iolet) filamentation

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In honor of Volodja Zakharov,
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Motivation

The motivation for the proposed research is to gain a complete understanding of the filamentation of intense UV **picosecond** pulses in air. On the theoretical side, of interest is first to find stationary solutions of the modeling equations and determine their stability. Thus we expect to give insight to the experimental conditions for propagation in long distances.

Of benefit is that in the UV regime, less intense, longer pulses are needed, thus simplifying the model.

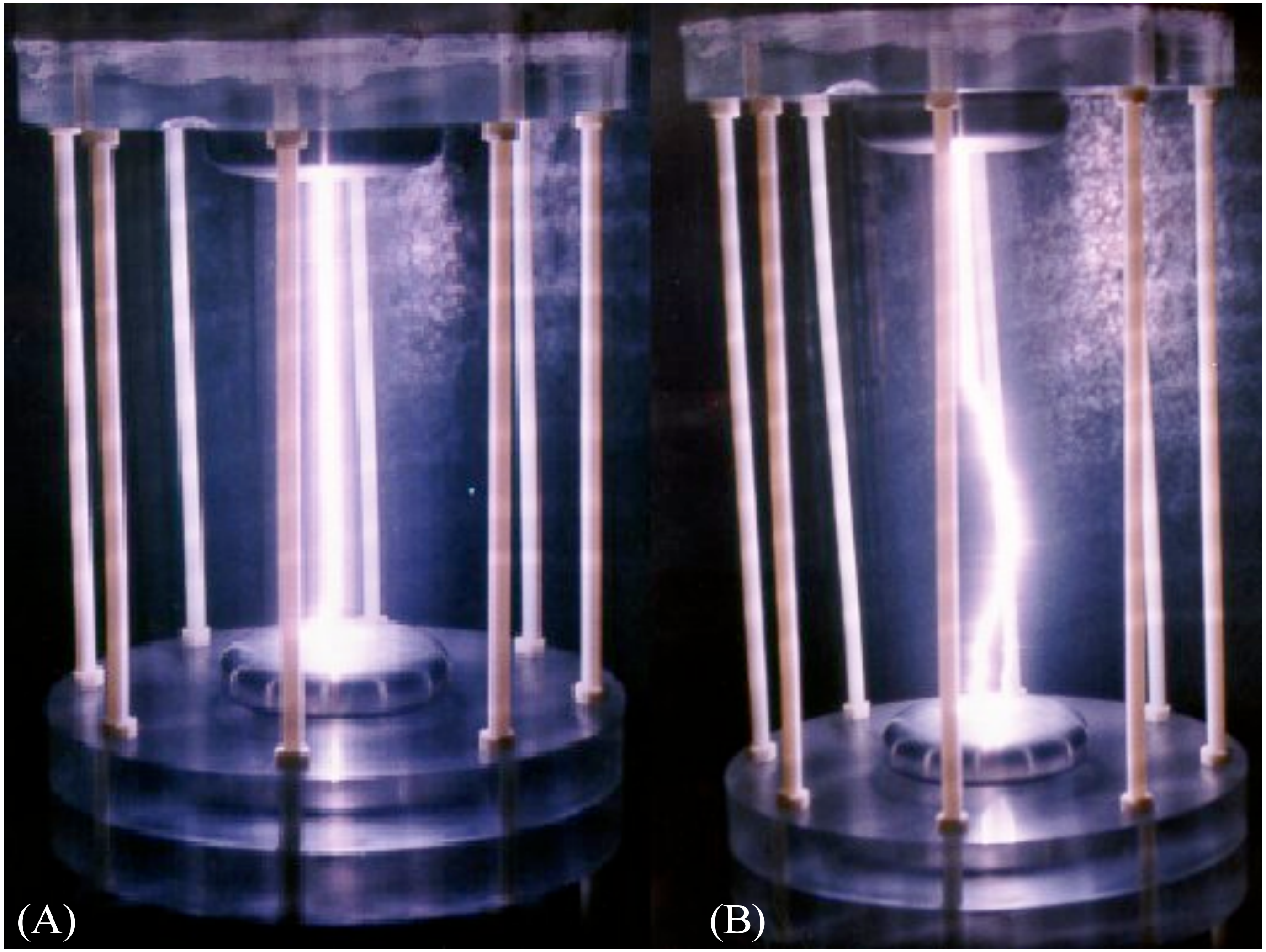
Applications

Light Detection and Ranging(LIDAR)

Directed Energy

Remote Diagnostics

Laser guide stars



(A) The discharge is triggered by the laser filament

(B) There is no filament which bring a random pass between the electrodes.

Formation of a filament

High power pulses self-focus during their propagation through air due to the nonlinear index of refraction.

At some critical power the self-focusing can overcome diffraction and possibly lead to a collapse of the beam.

Short pulses of high peak intensity create their own plasma due to multi-photon ionization of air, which defocuses beam.

When the laser intensity exceeds the threshold of multiphoton ionization, the produced plasma will defocus the beam. If the self-focusing is balanced by multiphoton ionization defocusing, a stable filament can form.

Equation for the plasma

The number of electrons N_e in the medium is the function of time and the intensity of the beam

$$\frac{dN_e}{dt} = \sigma^{(3)}\varepsilon^6 N_0 - \beta_{ep} N_e^2 - \gamma N_e$$

where $\sigma^{(3)}$ is the third order multiphoton ionization coefficient,

β_{ep} the electron-positive-ion recombination coefficient

γ the electron oxygen attachment coefficient

$\sigma^{(3)}$ third order multi-photon ionization coefficient

N_0 atom density at sea level

Equation for the model

The model to be considered is an unidirected beam described by an envelope approximation that leads to the following equation:

$$\frac{\partial \varepsilon}{\partial z} = -\frac{i}{2k}\left(\frac{1}{r}\frac{\partial \varepsilon}{\partial r} + \frac{\partial^2 \varepsilon}{\partial r^2}\right) - i \in_0 \frac{\pi \bar{n}_2 n_0}{377\lambda}|\varepsilon|^2 \varepsilon + i\frac{n_0 e^2 \mu_0}{c^2 m} N_e \varepsilon$$

where the second and third terms on the right-hand side describe the second and third order nonlinearities of the propagation which respectively introduce the focusing and defocusing phenomena

$$\frac{dN_e}{dt} = \sigma^{(3)} \varepsilon^6 N_0 - \beta_{ep} N_e^2 - \gamma N_e$$

2D NLS Equation
(no plasma)

$$\frac{\partial \varepsilon}{\partial z} = -\frac{i}{2k}\Delta\varepsilon - i\in_0 \frac{\pi \bar{n}_2 n_0}{377\lambda}\left|\varepsilon\right|^2 \varepsilon + \cancel{i\frac{n_0 e^2 \mu_0}{c^2 m} N_e \varepsilon}$$

$$\frac{dN_e}{dt} = \sigma^{(3)}\varepsilon^6 N_0 - \beta_{ep} N_e^2 - \gamma N_e, \quad \text{where } \sigma^{(3)} = 0$$

Back to the plasma formation case.
Search for steady state solutions

$$\frac{1}{r}\frac{\partial \varepsilon}{\partial r} + \frac{\partial^2 \varepsilon}{\partial r^2} + C_1 \varepsilon |\varepsilon|^2 - C_2 N_e \varepsilon = \beta \varepsilon \qquad (1)$$

$$\frac{\partial N_e}{\partial t} = C_3 \varepsilon^6 - C_4 N_e^2 - C_5 N_e \qquad (2)$$

where

$$C_1 = \frac{2k\pi n_0^2 \overline{n}_2}{377\lambda} r_0^2 |\varepsilon_0|^2, \qquad C_2 = \frac{k\mu_0 e^2}{c^2 m} \varepsilon_0 N e_0$$

$$C_3 = \frac{\sigma^{(3)} N_0 t_0 \varepsilon_0^6}{N e_0}, \quad C_4 = \beta_{ep} N e_0 t_0, \quad C_5 = \gamma t_0$$

C1 = 1.155, C2 = 3.5405, C3 = 1.62 × 10−4, C4 = 1.3 × 10−4, C5 = 1.5 × 10−4

Search of the stationary solution

$$\Downarrow$$

$$\frac{\partial N_e}{\partial t} = C_3 \varepsilon^6 - C_4 N_e^2 - C_5 N_e = 0$$

$$N_e(\varepsilon) = \frac{-C_5 + \sqrt{C_5^2 + 4C_3C_4\varepsilon^6}}{2C_4}$$

Equation for ε becomes a nonlinear eigenvalue problem

$$\frac{1}{r}\frac{\partial \varepsilon}{\partial r}+\frac{\partial^2 \varepsilon}{\partial r^2}+C_1\varepsilon\left|\varepsilon\right|^2-C_2N_e\varepsilon=\beta\varepsilon$$

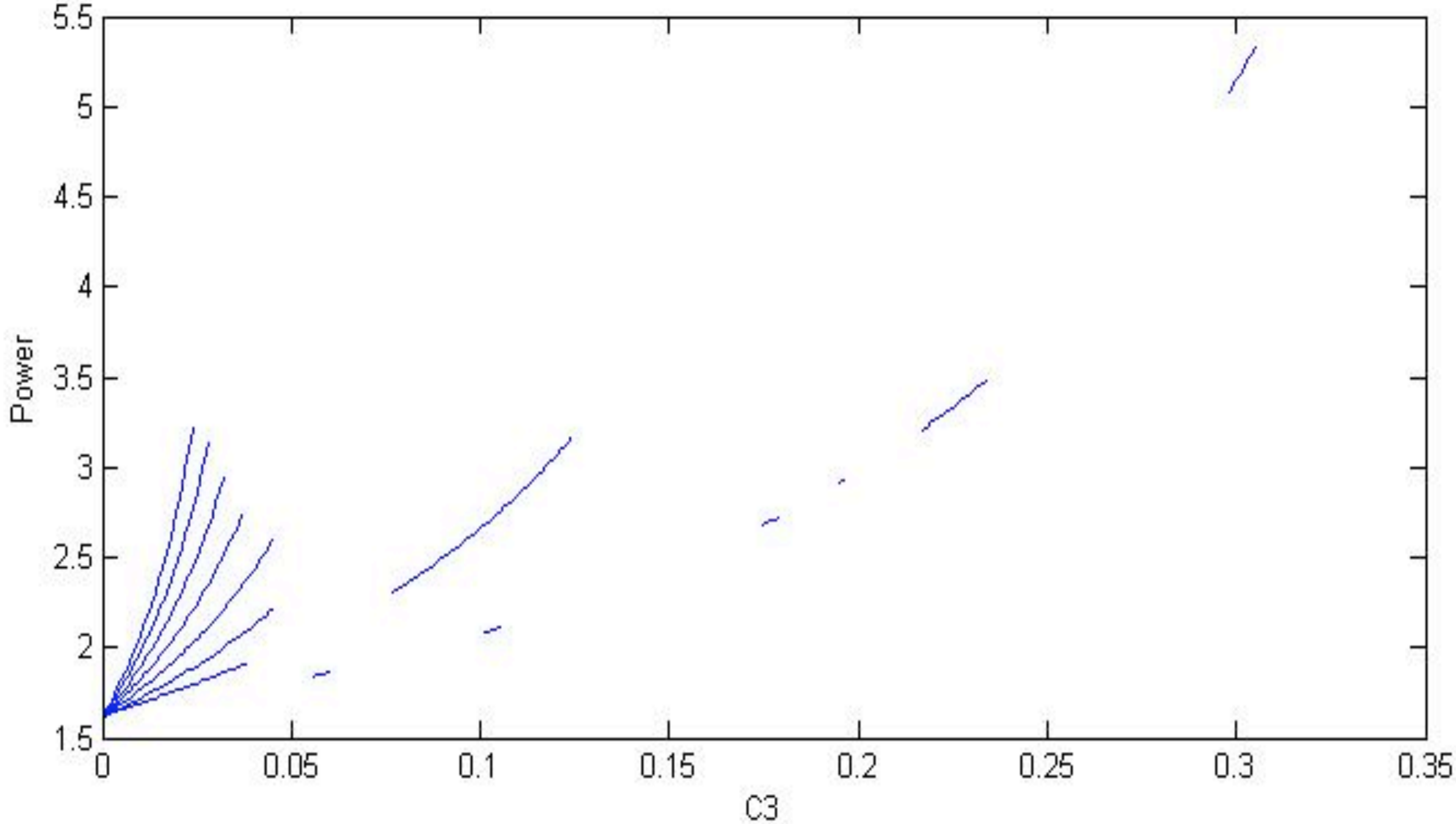
where β is an eigenvalue and $N_e(\varepsilon)=\dfrac{-C_5+\sqrt{C_5^2+4C_3C_4\varepsilon^6}}{2C_4}$

Our approach is a continuation method beginning from the Townes soliton of 2D NLSE which is also the solution of our model if $C_3=0$

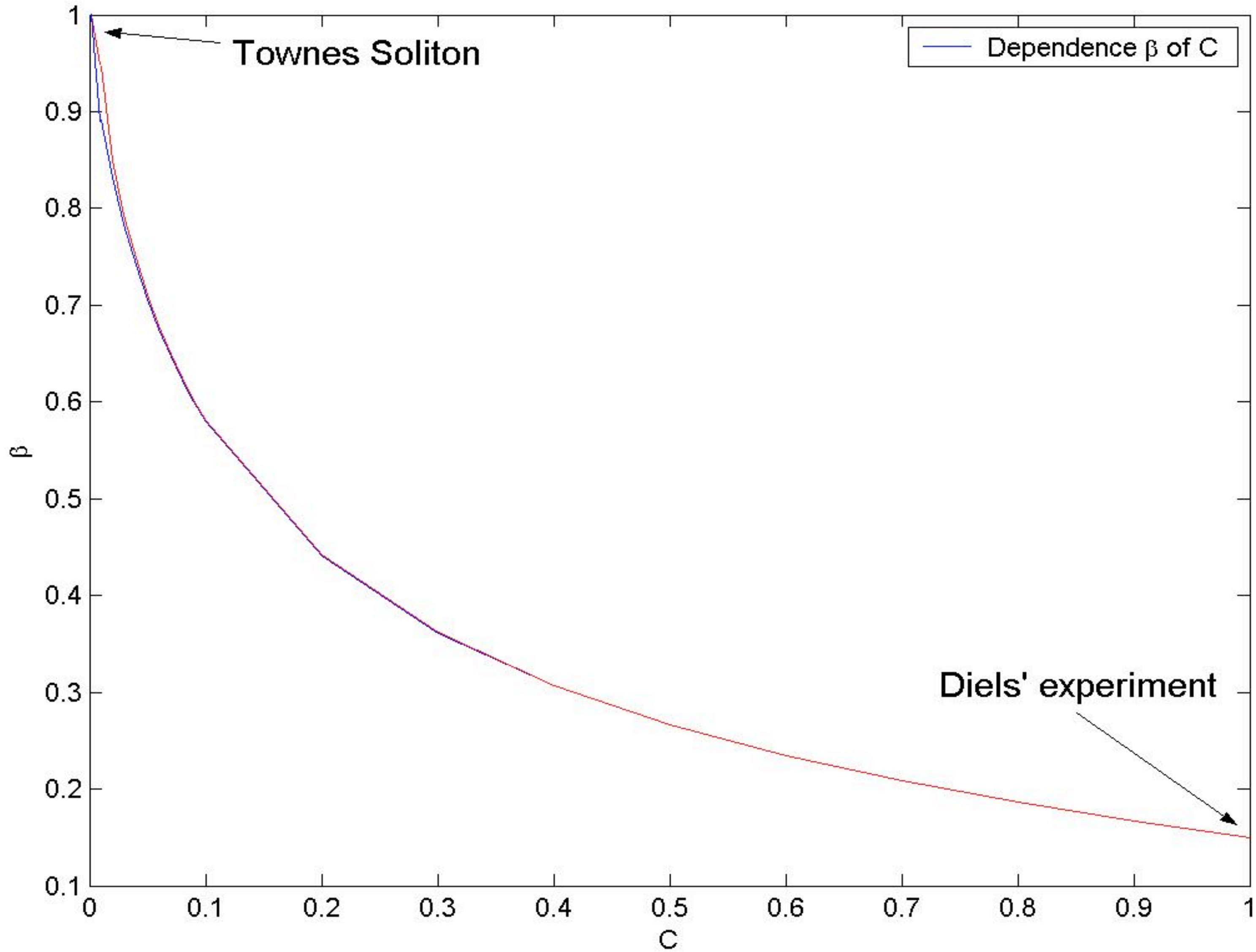
boundary conditions: $\varepsilon(r,t)\approx A_R r^{-\frac{1}{2}}e^{-r}$ $\quad A_R\cong 3.52 \quad r>>1$

$$\varepsilon_r(0,t)=0$$

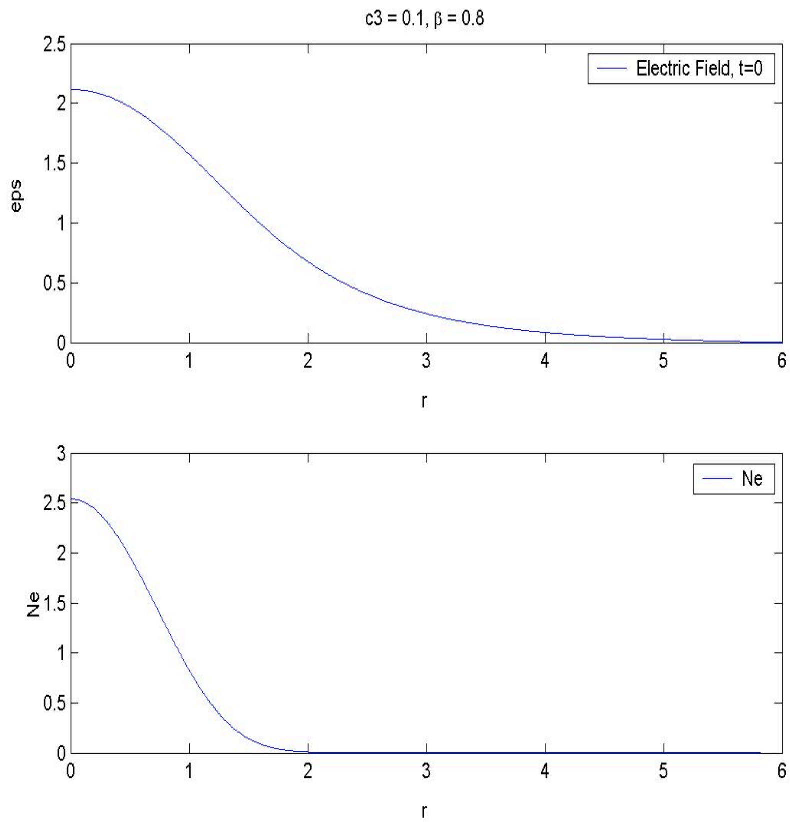
Multiple branches



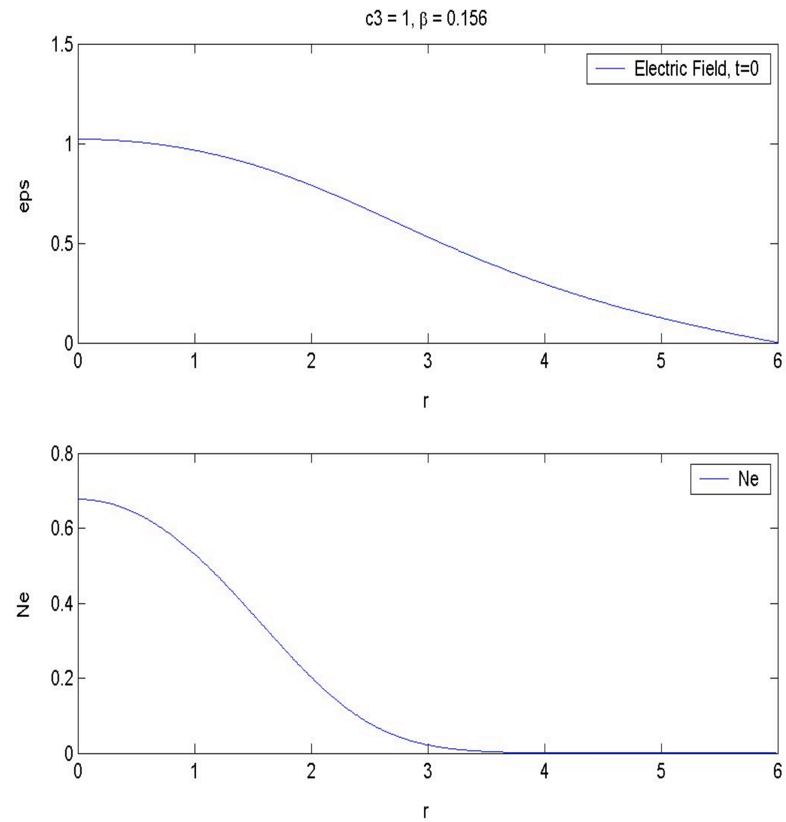
C_3 vs β


Townes Soliton
Dependence β of C
Diels' experiment
β
C
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

$C_3 = 10^{-1} \quad \beta = 0.8$

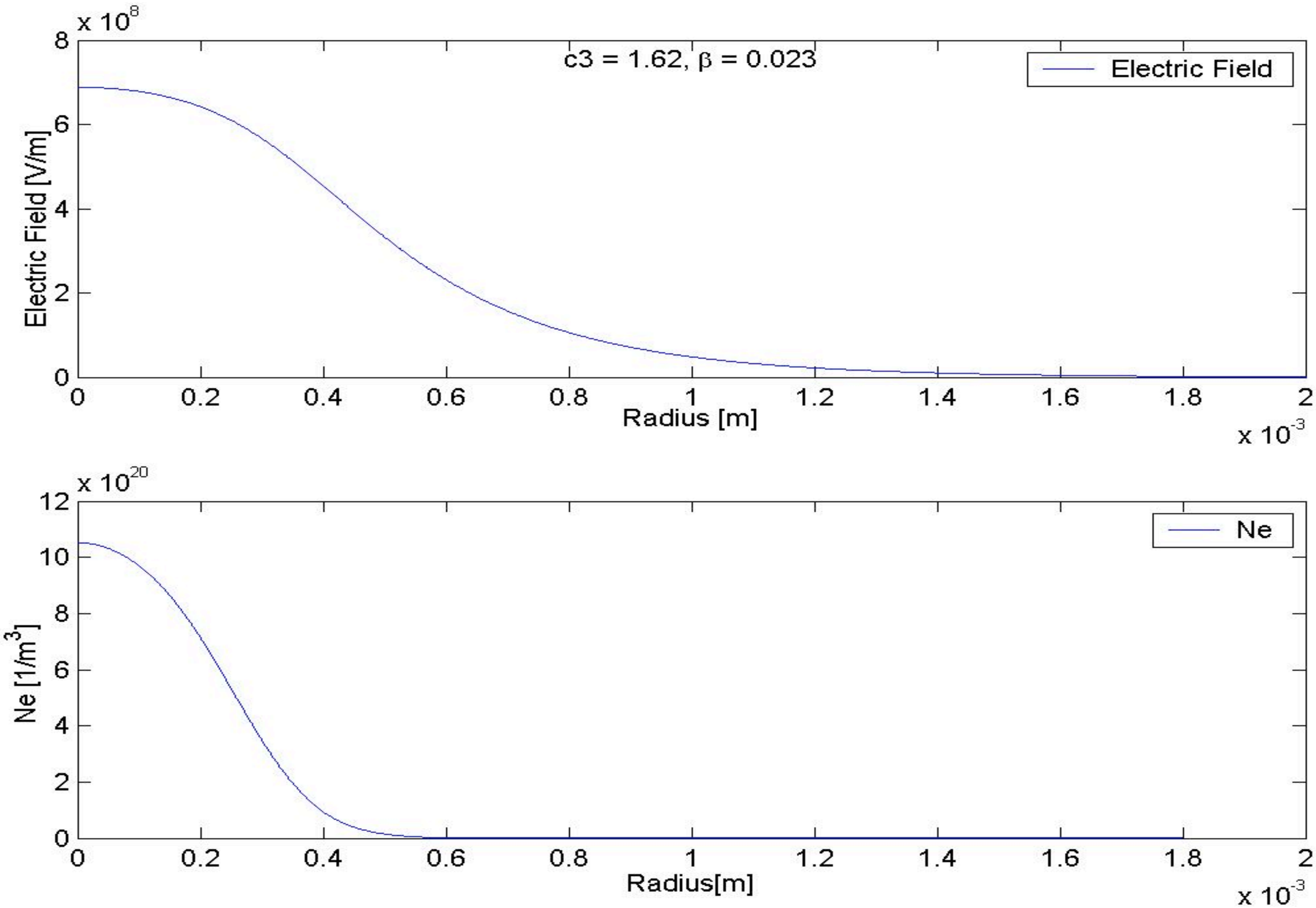


$C_3 = 1 \qquad \beta = 0.156$



Results
(relevant to the experimental realization)

$$C_3 = 1.62 \quad \beta = 0.023$$



SUMMARY

MAXWELL

$$\left[\Delta_{tr} + \partial^2_{zz} - \frac{n^2}{c^2}\partial^2_{tt}\right]\mathcal{E}e^{(i\omega t - kz)} = 0$$

$$\partial^2_{tt} \rightarrow -\omega^2 \qquad\qquad n = n_0 + n_2 I - \Delta n$$

Δn due to plasma diffraction has been added

$$2ik\partial_z\tilde{\mathcal{E}} = \partial^2_{rr}\tilde{\mathcal{E}} + \frac{1}{r}\partial_r\tilde{\mathcal{E}} + \frac{\omega^2}{c^2}2n_0\left(n_2 I - \Delta n\right)\tilde{\mathcal{E}}$$

Normalization: $\mathcal{E}_0 = \sqrt{\frac{\eta_0}{n_2}}$ and $\chi = kr$

$$\mathcal{E} = \mathcal{E}_s e^{-i\beta_s z}$$

solution of an eigenvalue equation (dimensionless form):

$$\boxed{2\beta_s\mathcal{E}_s = \frac{d^2\mathcal{E}_s}{d\chi^2} + \frac{1}{\chi}\frac{d\mathcal{E}_s}{d\chi} + |\mathcal{E}_s|^2\mathcal{E}_s - f(\mathcal{E}_s)\mathcal{E}_s}$$

Beam propagation

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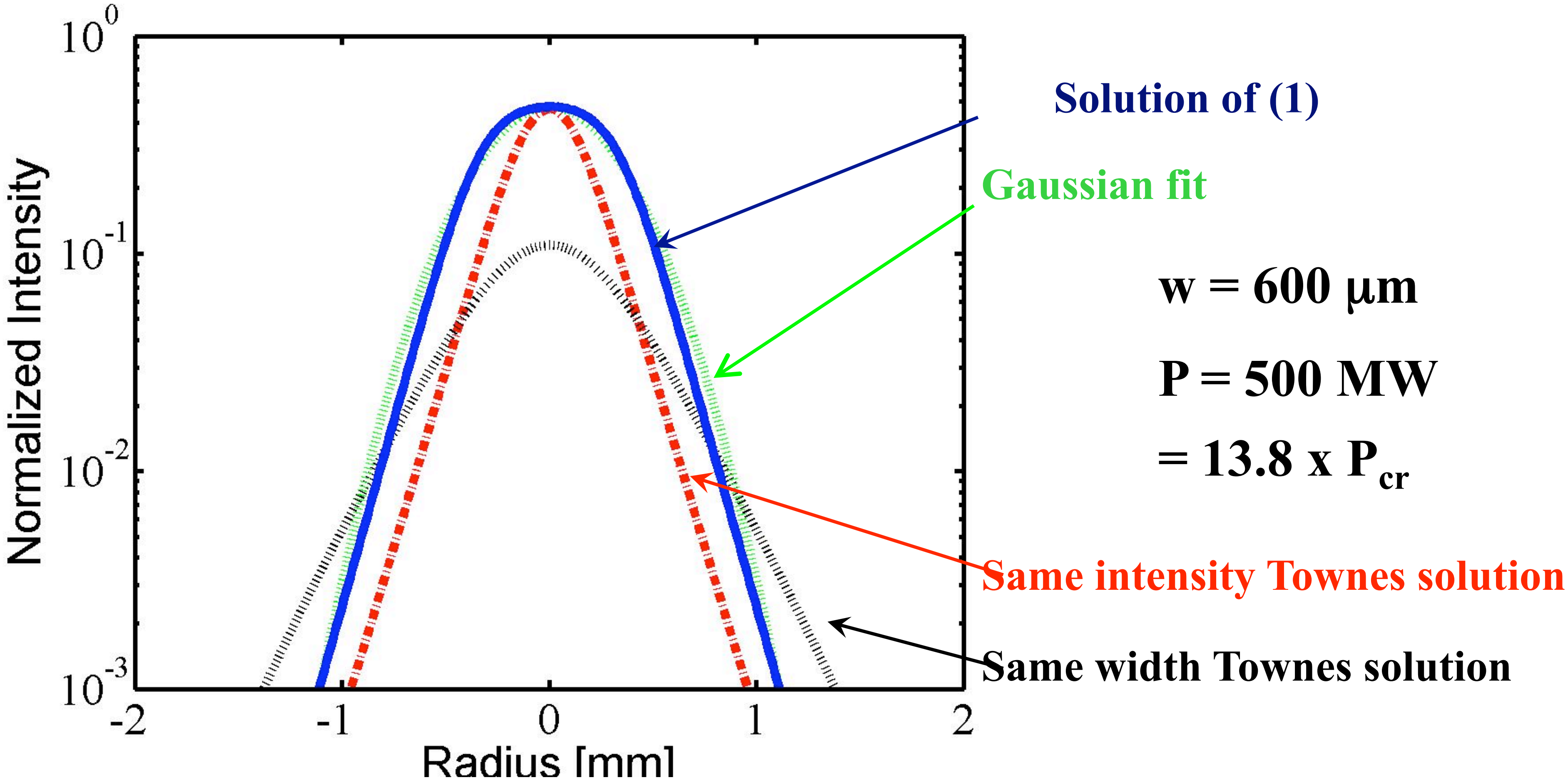
$$\boxed{2\beta_s\mathcal{E}_s = \frac{d^2\mathcal{E}_s}{d\chi^2} + \frac{1}{\chi}\frac{d\mathcal{E}_s}{d\chi} + |\mathcal{E}_s|^2\mathcal{E}_s - f(\mathcal{E}_s)\mathcal{E}_s} \quad (1)$$



w = 600 μm

P = 500 MW

= 13.8 x P_{cr}

Numerical Approach

$$Ne(\varepsilon) = \frac{-C_5 + \sqrt{C_5^2 + 4C_3C_4\varepsilon^6}}{2C_4}$$

$$\frac{1}{j\,h}\left(\frac{\varepsilon_{j+1}^{n} - \varepsilon_{j-1}^{n}}{2h}\right) + \frac{\varepsilon_{j+1}^{n} - 2\varepsilon_{j}^{n} + \varepsilon_{j-1}^{n}}{h^2} + C_1\varepsilon_j^n\left|\varepsilon_j^n\right|^2 - C_2N_e(\varepsilon)_j^n\varepsilon_j^n = \beta\varepsilon_j^n$$

for $j = 2 \ldots m-1$

Near r equal to zero we have $\frac{1}{r}\frac{\partial\varepsilon}{\partial r} \approx \frac{\partial^2\varepsilon}{\partial r^2}$

$$2\frac{2\varepsilon_1^n - 2\varepsilon_0^n}{h^2} + \varepsilon_0^n\left|\varepsilon_0^n\right|^2 - N_e(\varepsilon)_o^n\varepsilon_0^n = \beta\varepsilon_0^n$$

$$F(\varepsilon) = A\varepsilon + f(\varepsilon) = 0$$

where

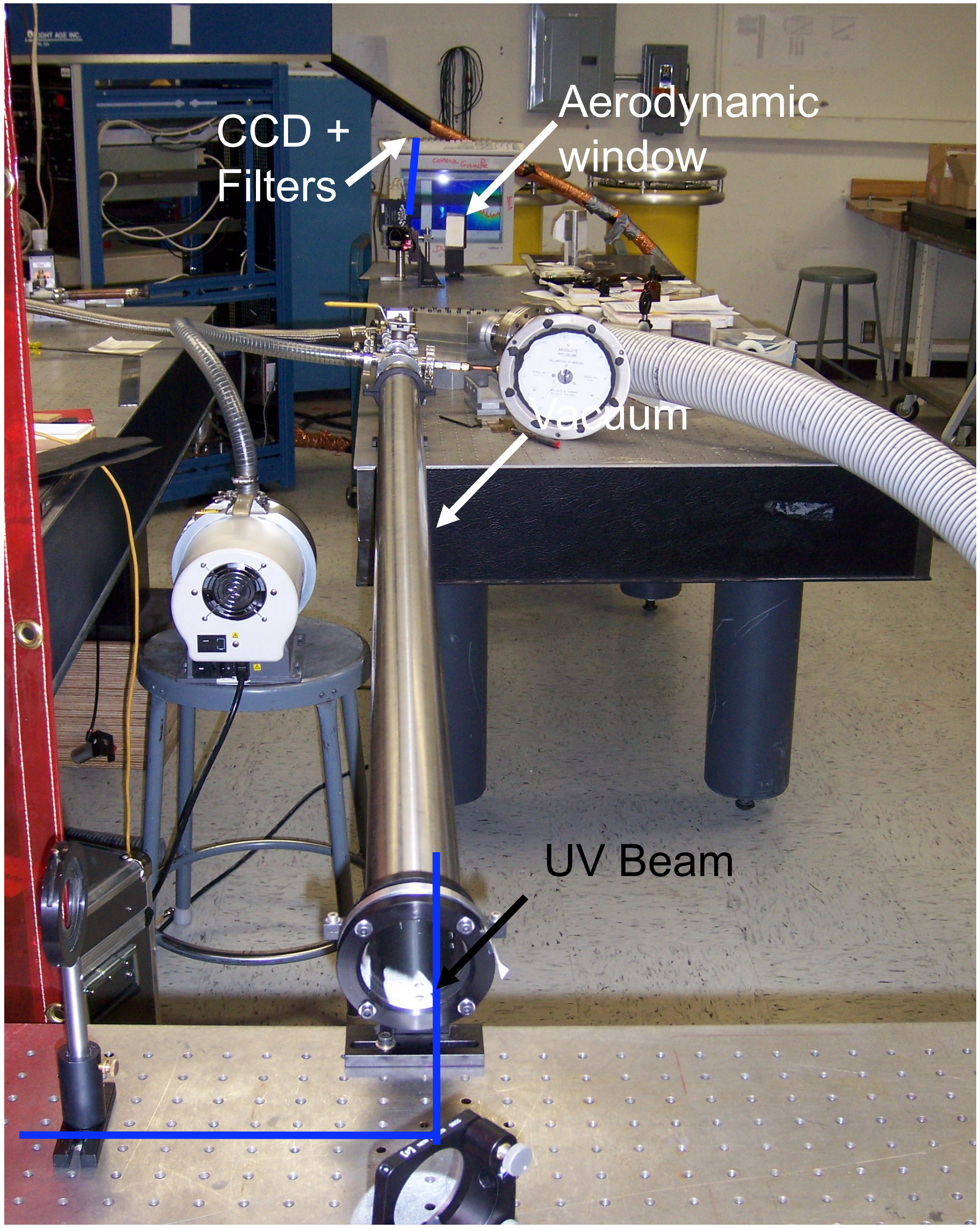
$$A = \frac{1}{h^2}\begin{pmatrix} -4 & 4 & & & \\ 1 & -2 & \frac{3}{2} & & \\ & \frac{3}{4} & -2 & \frac{5}{4} & \\ & & & & \\ & & & \frac{2m-3}{2m-2} & -2 \end{pmatrix} \quad \varepsilon = \begin{pmatrix} \varepsilon_0^n \\ \varepsilon_1^n \\ \\ \\ \varepsilon_{m-1}^n \end{pmatrix} \quad f(\varepsilon) = \begin{pmatrix} \left|\varepsilon_0^n\right|^2 \varepsilon_0^n - N_e(\varepsilon)_0^n \varepsilon_0^n \\ \left|\varepsilon_1^n\right|^2 \varepsilon_1^n - N_e(\varepsilon)_1^n \varepsilon_1^n \\ \\ \\ \left|\varepsilon_{m-1}^n\right|^2 \varepsilon_{m-1}^n - N_e(\varepsilon)_{m-1}^n \varepsilon_{m-1}^n \end{pmatrix}$$

Using the continuation method along with Newton's method we can find the solution

$$\varepsilon^{(n+1)} = \varepsilon^{(n)} - \Delta\varepsilon = \varepsilon^{(n)} - \left(\nabla F(\varepsilon^{(n)})\right)^{-1} F(\varepsilon^{(n)})$$

$$\left\|\Delta\varepsilon\right\| \leq tol$$

Experimental results



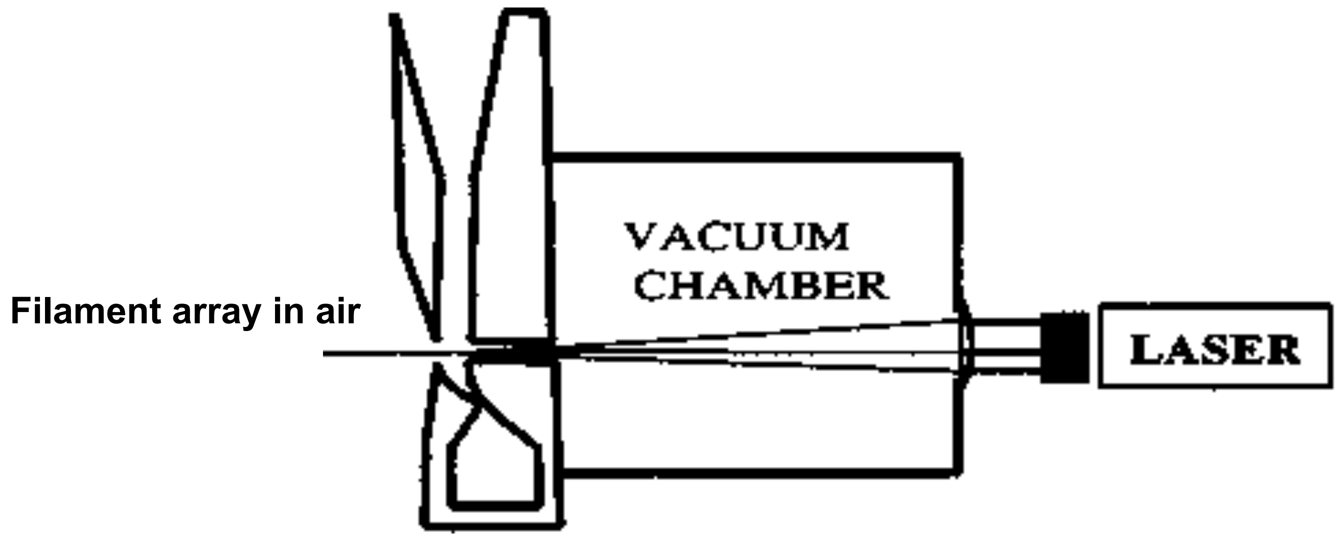
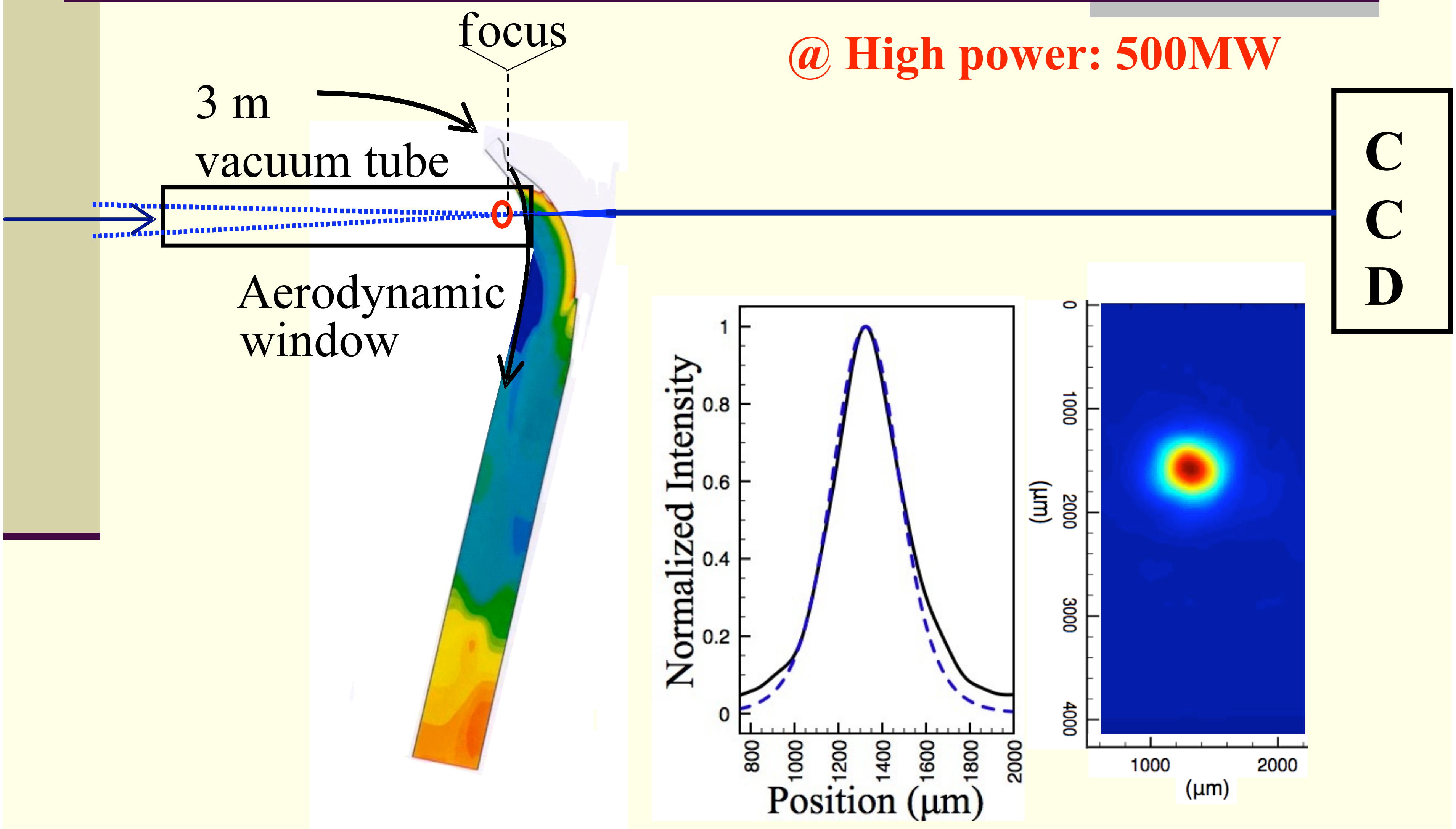


Figure. Setup of the Aerodynamic window, Focus of the beam into the vacuum then propagation of the filament in atmospheric pressure

The possible propagation of filament is dependent on input power. Most of the energy loss occurs in the formation of the filament. The propagation of the filament once formed, is practically lossless. If we match the shape of the intensity at the input we can minimize loss of energy in the filament as it propagates in Aerodynamic window.

"On axis" observation of a filament

@ High power: 500MW



Ground State Stability

$$\varepsilon(z,r) = \varepsilon_s(r)e^{i\beta z}$$, where

$\varepsilon_s(r)$ is the ground state solution

$$\varepsilon(z,r) = (\varepsilon_s(r) + \kappa(z,r)e^{is(z,r)})e^{i\beta z}$$

or

$$\varepsilon(z,r) = (\varepsilon_s(r) + u(z,r) + iv(z,r))e^{i\beta z}$$

After linearization we have

$$\partial_z \begin{pmatrix} u \\ v \end{pmatrix} = N \begin{pmatrix} u \\ v \end{pmatrix} \quad \text{where} \quad N = \begin{pmatrix} 0 & L_0 \\ -L_1 & 0 \end{pmatrix}$$

$$L_0 = -\Delta + \beta - \varepsilon_s^2 + C(F(\varepsilon_s))$$

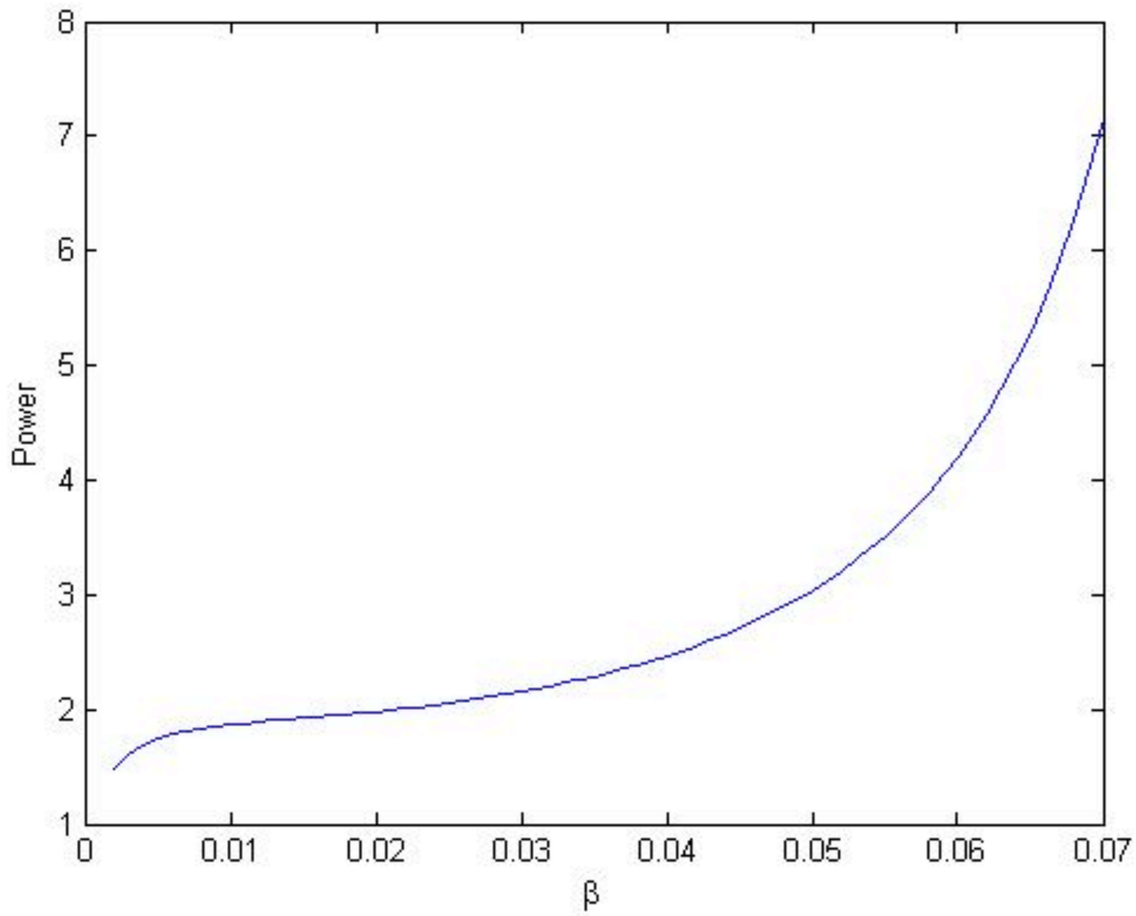
$$L_1 = -\Delta + \beta - 3\varepsilon_s^2 + C(F(\varepsilon_s))$$

$$u, v \sim e^{i\Omega z} \quad \Rightarrow \quad \Omega^2 u = L_0 L_1 u$$

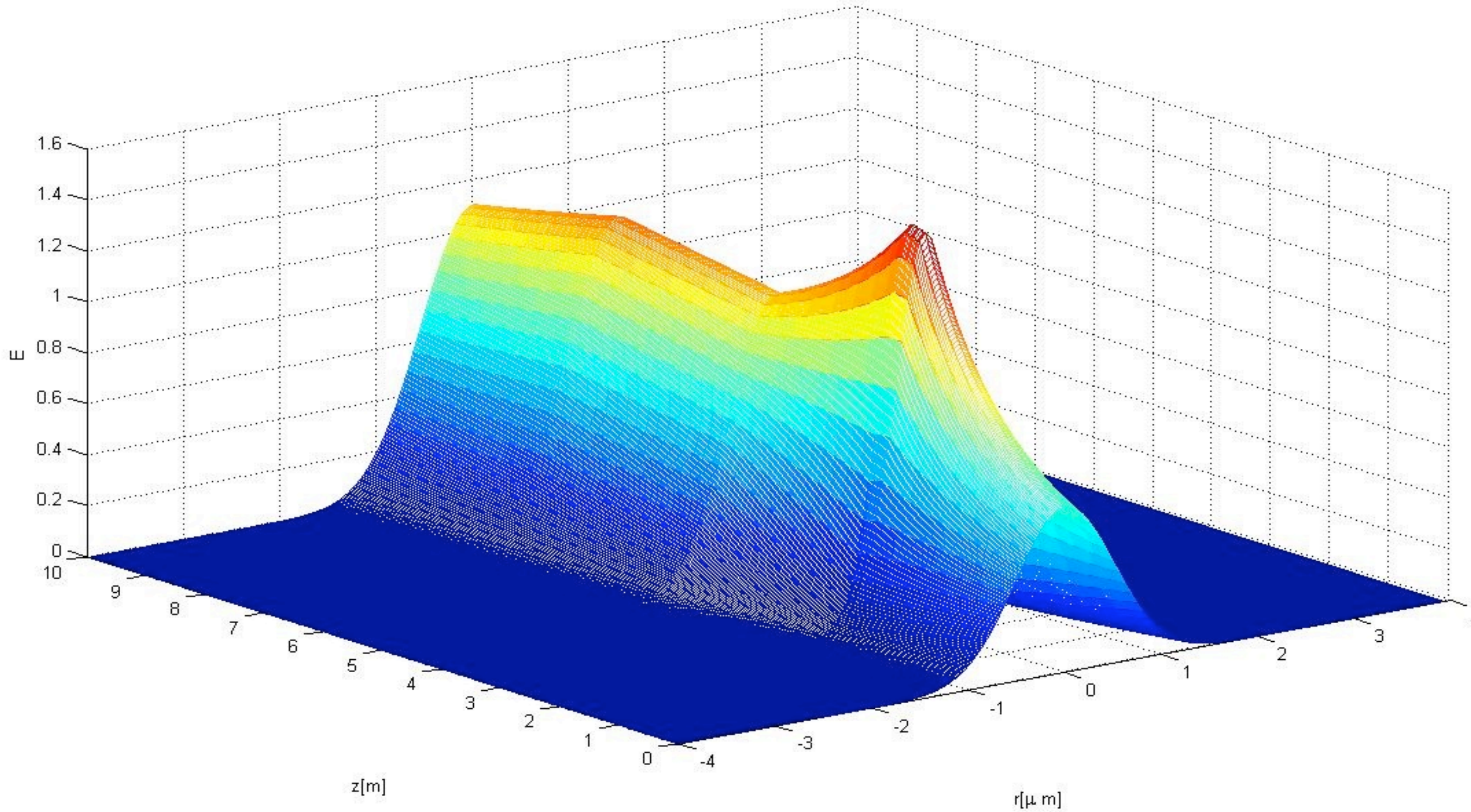
$$\int u L_0 u dx \geq 0$$

$$f(\mu) = \int \varepsilon_s (L_1 - \mu)^{-1} \varepsilon_s dx$$

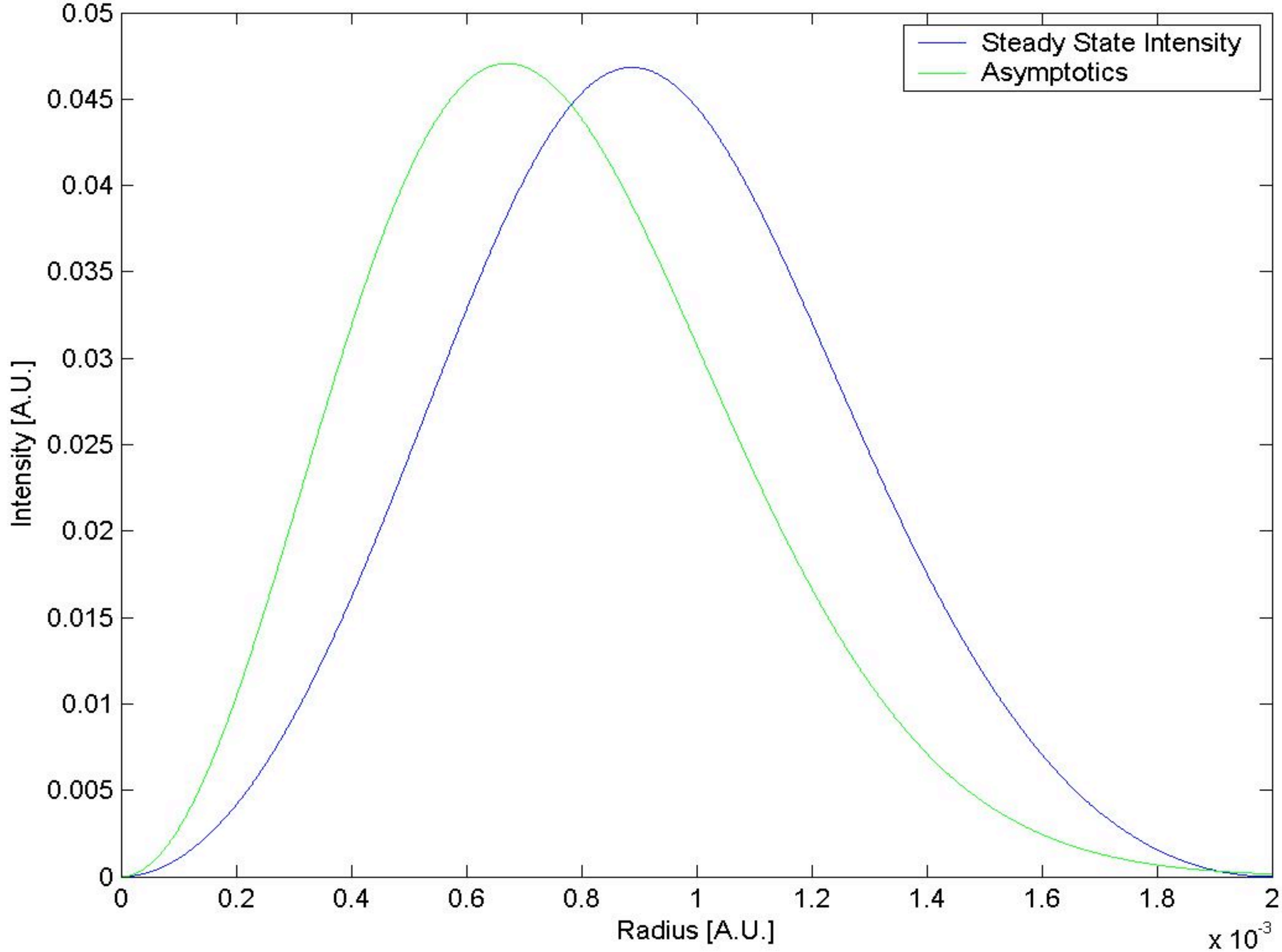
Hence, $$f(0) = -\frac{1}{2}\frac{\partial N}{\partial \beta} \quad \text{where} \quad N = \int \varepsilon_s^2 \, dx$$



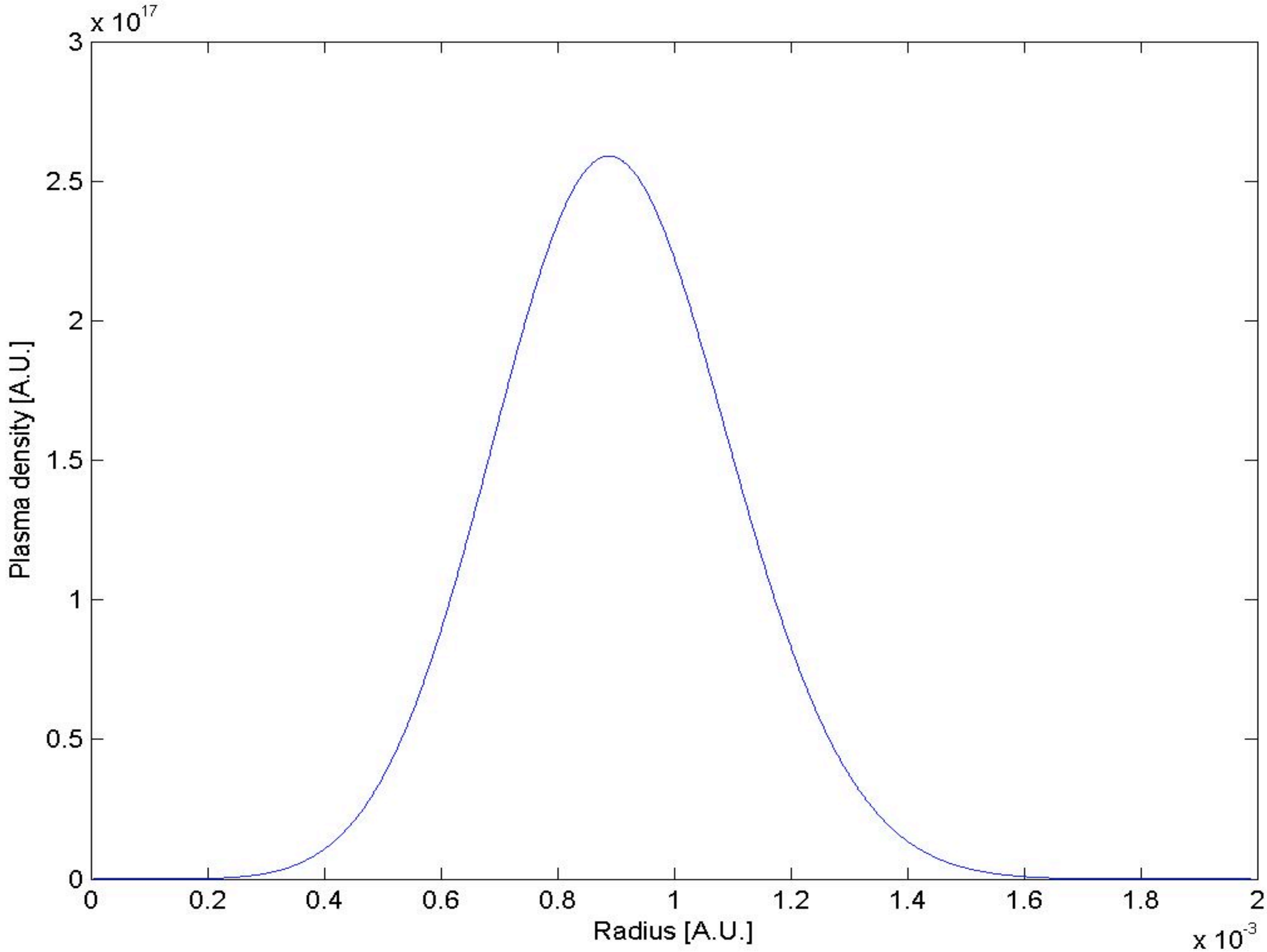
Propagation of an initial gaussian beam



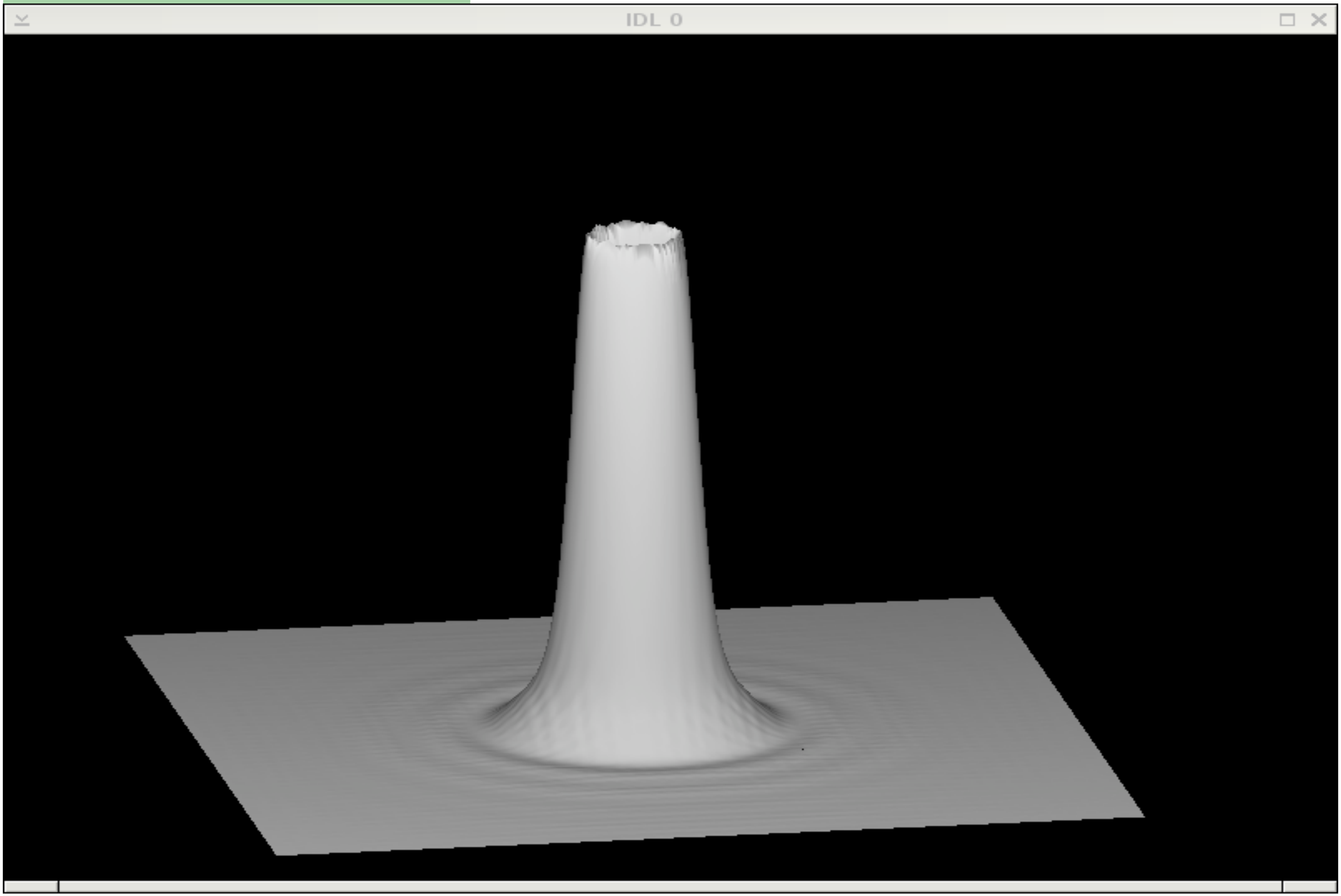
Vortex (m=1) solution



Plasma solution for (m=1) vortex



Vortex (m=1) dynamics



Conclusions/Future work

0. Shown good agreement between theory and experiment for UV filaments. Evidence of stability for ~ 10 m. propagation.

1. Complete stability analysis.

2. Full (z,t,r) simulation. (see the buildup of the plasma leading towards steady state)

3. Vortex dynamics

4. Circular vs. linear polarization

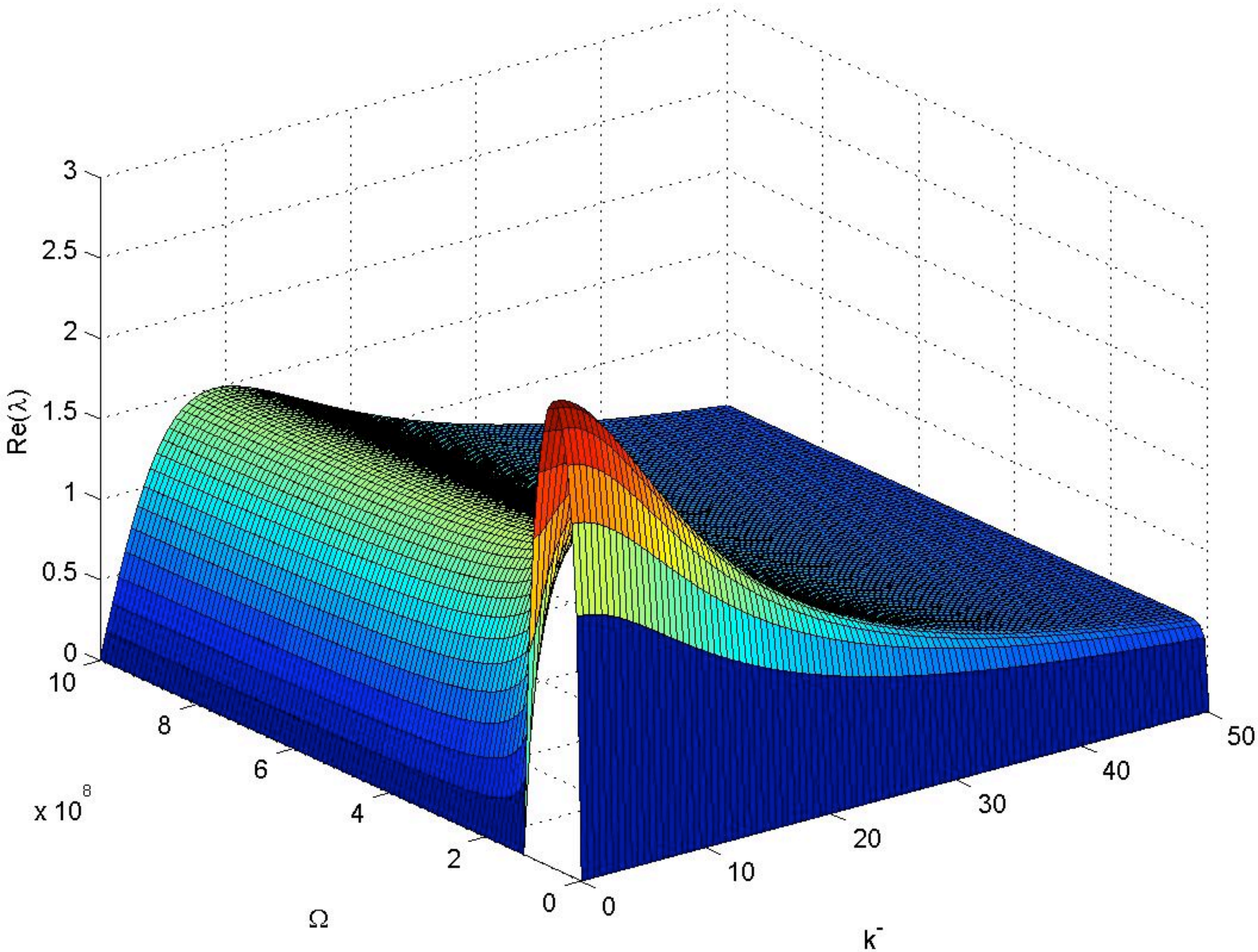
Plane Wave Stability

$$\varepsilon(z,r) = ue^{i\bar{k}z}$$

$$\varepsilon(z,r,t) = \varepsilon_0(z,r) + \varepsilon_+(z,r)e^{-i\Omega t} + \varepsilon_-(z,r)e^{i\Omega t}$$

$$Ne_0(r) = \frac{-\gamma + \sqrt{\gamma^2 + 4\beta_{ep}\sigma^{(3)}N_0 \frac{n_0^3}{8\eta_0^3}|u|^6}}{2\beta_{ep}}$$

$$Ne(z,r,t) = Ne_0(r) + Ne_+(z,r)e^{-i\Omega t} + Ne_+^*(z,r)e^{i\Omega t}$$



3
2.5
2
1.5
1
0.5
0
Re(λ)
10
8
6
4
2
0
x 10^8
Ω
0
10
20
30
40
50
k^-