2/12/15 Math 263 Lecture 7

Today: 4.2, 4.5

Webassign: 4.2 due tomorrow

4.3, 4.4, 4.5 next Fri

Last time: * Sample space & events
* Probability as the limit of sample distributions as the # of samples gets large.

Sample space: set of possible outcomes.

Ex Vending machine w/ 10 items

$S = \{$ Snickers, M&Ms, Doritos, ... $\}$

Ex Temperature in Tucson (Assuming infinite precision measurement) in °F

$S = \{$ All #'s between 0 & 120 $\} = [0, 120]$

(Record max was 115°, Record min 6°)

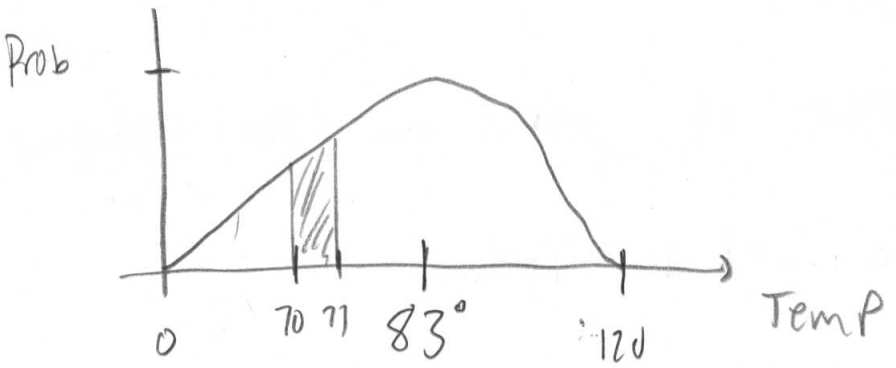
Eg. Temp can be 39.7521°, 50°, etc.

Event: A subset of S.

Why do we want events to be subsets of outcomes & not just outcomes?

* We will want to ask what the probability of an outcome being in some range of possible outcomes

Ex what is the probability that the temp in Tucson is between 70° & 77°?




What is a probability?

A probability is number between 0 & 1 assigned to every event

* $0 \leq P(A) \leq 1$ For every event A
* $P(S) = 1$ "something" must happen

* If A & B are <u>disjoint</u> (share no outcomes) Then

$$P(A \text{ or } B) = P(A) + P(B)$$

* The Probability That an event A does <u>not</u> happen is $1 - P(A)$

IE. $P(A^c) = 1 - P(A)$.

<u>Examples</u>

1) Suppose I Select A Card @ random. What is Wrong With The following Statement?

"The Probability That I Choose a Spade is 25%"

A: Probabilities Are given In decimal form Always.

2) Suppose I Select UA Students @ random & Ask If They are L or R handed (Assume no one is ambidextrous)

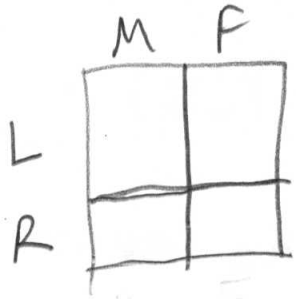
Pr(Female Left) = 0.13
Pr(Female Right) = 0.87

Pr(male Left) = 0.13
Pr(Male R) = 0.87

What Is Wrong?

$S = \{$ All UA Students $\}$

$S =$




Pr(S) should be 1, Not 2!

3) Suppose we Draw from a deck of Cards. What Is The Prob That I draw <u>either</u> a Red King or a Spade?

A = {Red Kings} & B = {Spades} are <u>disjoint</u> so Pr(A or B) = Pr(A) + Pr(B)

$$= \frac{2}{52} + \frac{13}{52}$$

$$= \frac{15}{52} \approx 0.2885$$

4) When making a medical diagnosis, we have the following Possibilities:

S = {TRUE Positive, TRUE negative, FAlse Positive, False Negative}.

Suppose the probabilities of TP, TN, & FP are:

| TP | TN | FP | FN |
|---|---|---|---|
| 0.48 | 0.49 | 0.01 | ? |

What is the probability of giving a false negative diagnosis?

*Note: In a finite sample space, we can assign a probability to each outcome. These must sum to 1!

Ex If all outcomes are equally likely, & the sample space is of size N, what is the probability of any given outcome? Any event?

$$Pr(A) = \frac{\#\text{ outcomes in } A}{\#\text{ outcomes in } S}$$

Eg. In a (fair) deck of cards,

$$Pr(\text{Dealing a face card}) = \frac{12 \leftarrow \#\text{ face cards}}{52 \leftarrow \#\text{ total cards}}$$

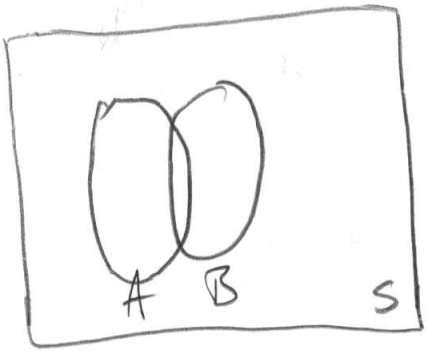
# Independence

Suppose I roll 2 dice, one red & one blue.

$S = \{11, 12, \ldots, 66\}$ (36 possibilities)

Consider $A = \{$ first die is a 3 $\}$

$B = \{$ second die is a 2 $\}$




A & B are not disjoint - why?

"32" is in <u>both</u> A & B.

What is $Pr(A \text{ and } B)$?

- $Pr(A) = \frac{1}{6}$
- $Pr(B) = 1/6$

So "1/6 of 1/6" or $\frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6}$.

OF the time.

This can only happen if the dice rolls are <u>independent</u> IE they do not influence each other.

If A & B are independent,

$Pr(A \text{ and } B) = Pr(A)\,Pr(B)$. OR, with Conditional Probability:

$Pr(B|A) = Pr(B)$

*Note*

Conditional Probability "Reduces your sample space"

<u>Ex</u> Draw 2 cards from a deck (<u>W/o Replacement</u>) What is the probability of the <u>second</u> card being Red given that the first is Red?

* After the first draw, there are only 51 cards left, 25 of which are red! So

$Pr(2^{nd} \text{ red} \mid 1^{st} \text{ red}) = \frac{25}{51}$.

These are NOT independent since the first draw affects the second.

EX Stock prices are not independent (from day-to-day)

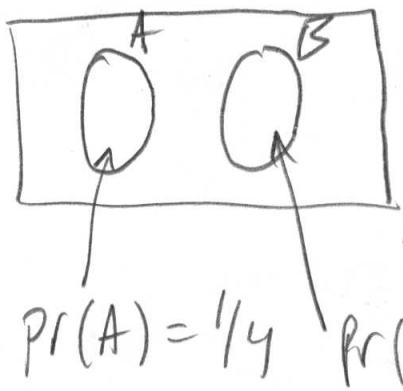
Review

* $Pr(A \text{ or } B) = Pr(A) + Pr(B)$ if A & B cannot BOTH happen (disjoint)

* $Pr(A \text{ And } B) = Pr(A)\,Pr(B)$ if A & B are independent

Disjoint events cannot be independent!

EX




$Pr(A) = 1/4$ $Pr(B) = 1/4$

$Pr(A|B) = 0$ since "If we're in B, we cannot be in A"!!

EX Independent events with nonzero probabilities MUST overlap.

EX (HIV testing) Suppose the false positive Rate for a certain HIV test is 2% (who do NOT have HIV). Suppose a clinic tests 50 people. What is the probability that at least 1 false positive occurs?

$$\begin{aligned}
\Pr(\text{at least 1}) &= 1 - \Pr(\text{No pos}) \\
&= 1 - \Pr(\text{50 neg}) \\
&= 1 - 0.98^{50} \quad (*) \\
&\approx 1 - 0.3642 \\
&= 0.6358
\end{aligned}$$

$(*)$ $\Pr(\text{Negative}) = 1 - \Pr(\text{Positive})$

$$\begin{aligned}
&= 1 - 0.02 \\
&= 0.98
\end{aligned}$$