

10/15

MATH 263 LECTURE 6

Today: 4.1-4.2 basic Probability

THURS: Probability, cont'd

NEXT week: Probability, cont'd

WebAssign for 4.1-4.2 & 4.5 due FRI 2/13

What is a "random" event?

- An event that we don't know the outcome of.

EXAMPLES

1) What will the weather be tomorrow?

$$\Pr(\text{rain}) = \frac{?}{?}$$

$$\Pr(\text{sun}) = \frac{?}{?}$$

$$\Pr(\text{snow}) = \frac{?}{?}$$

- When tomorrow comes, the weather will do something. We're not sure what, but the possible events follow a distribution!

2) What number will the die show on its next roll?

$$\Pr(1) = 1/6$$

$$\Pr(2) = 1/6$$

''''

} If the die is fair!



How do we determine Probabilities without Prior knowledge of the distribution?

- We do "experiments" & Record outcomes

- The "frequentist" defⁿ:

"The probability that an event occurs is the Proportion of times the event occurs in *many* repeated trials" (*many \equiv "infinitely" many)

IE. Roll a die 10,000 times:

Roll	1	2	3	4	5	6
# Times	1661	1672	1660	1659	1683	1665

$$Pr(\text{Die shows 1}) \approx \frac{1661}{10000} = 0.1661$$

↑
Need a "larger" sample for =

Alternate defⁿs exist, in particular the Bayesian approach, which doesn't rely on a "law of large #'s" in other words, it works can work "better" on small sample sizes

Idea behind frequentist defⁿ: If the process follows some "TRUE" distribution, "random" fluctuations

will average out over a large # of samples
e.g. lots of 6's will eventually be balanced by lots of other rolls.

2

Independence

Suppose we ask two people what their favorite color is. Does the first person's answer affect the second's?

"Conditional" Probability: Probability that an event occurs given that another has occurred.

IE. $\Pr(\text{2nd person says blue} \mid \text{first person says red})$

"A" & "B" are independent if

$$\Pr(A|B) = \Pr(A)$$

Sample Spaces & Events

A sample space is a set

eg. 1 coin flip

$$S = \{H, T\}$$

2 coin flip

$$S = \{HT, TH, HH, TT\}$$

An event is some subset of the Sample Space

Ex. "The event that coin 1 is Heads"

$$A = \{HT, HH\}$$

* Sample Spaces can contain either Quantitative or Categorical Variables!

Ex Throw a dart at a dart board.

$$S = \{\text{Red, Black}\}$$

$$S = \{\text{distance from center}\} \quad (\text{in cm})$$

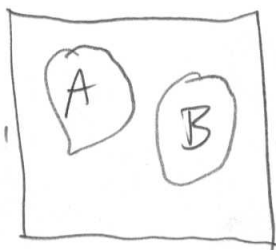
$$= \{\text{all real \#s between 0 \& 20}\}$$

Eg. "2.75239" or "π cm from center".

$$S = \{\# \text{ Hits}\} = \{1, 2, 3, \dots, 20\}$$

Venn diagrams and disjoint/mutually exclusive events:

"MUTUALLY EXCLUSIVE"



Ex.

$$A = \{HT, HH\}$$

$$B = \{TT\}$$

Probabilities

A Probability is a number between 0 & 1
Assigned to each event.

$$\text{E. } \Pr(\underbrace{\{HTHH, HHTT\}}_{\text{Event}})$$

Probability "Rules" / Axioms:

- 1) $0 \leq \Pr(A) \leq 1$ For all events A .
- 2) $\Pr(S) = 1$ IE The Probability of
Something Happening is 1
- 3) If A & B are mutually exclusive,
 $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$
- 4) $\Pr(A \text{ does NOT occur})$
 $= 1 - \Pr(A \text{ does occur})$

"A does not occur" = " A^c "

EX Flip a coin 3 times. Find the \longrightarrow

Probability That The first flip is NOT Heads,

EX What is wrong w/ the following?

A person is selected @ random. We collect their gender & whether they are left or right handed. Sample Space?

$$\Pr(\text{female left}) = 0.13$$

$$\Pr(\text{female right}) = 0.87$$

$$\Pr(\text{male left}) = 0.13$$

$$\Pr(\text{male right}) = 0.87$$

↓
{ male left,
male right,
female left,
female right }

Doesn't make sense because $\Pr(S) = 2 !!$

EX Suppose we draw from females only.

The Prob