Math 122B-008 Worksheet 8

Please write out your solutions on a separate piece of paper. You do not have to print out and staple this page to your assignment.

1. Suppose $g(x)$ is a continuous function with domain all real numbers. Answer the following True/False questions about $g(x)$; provide a brief explanation or counterexample to support your answer.

(a) Every critical point of $g(x)$ is either a local maximum or a local minimum.

(b) Every local maximum or minimum of $g(x)$ is a critical point.

(c) All global maximums (and minimums) of g must also be local maxima or minima.

(d) A local max or min cannot occur at $x = a$ if $g'(a)$ is undefined

(e) Suppose we restrict the domain of $g(x)$ to the closed interval $[a, b]$. Then, $g(x)$ has both a global maximum and minimum on its new domain.

(f) Inflection points of $g(x)$ can only occur when $g'' = 0$.

(g) An inflection point of $g(x)$ can never be a local max or min.

2. Let $f(x) = -4ax^3 - ax^2 + 4ax + 9a$.

(a) Find and classify the local maxes and mins of f. You must use calculus (i.e. the first or second derivative test) to classify each point.

(b) Find the inflection points of f. Again, you must use calculus to show *why* your points are inflection points.

3. Let $f(x) = 3x^3 - x^2 + x - 1$ for $x \in [-1, 1]$.

(a) Find all local maxima and minima on $[-1, 1]$. Be sure to explain using calculus!

(b) Find all global maxes and mins on $[-1, 1]$. Be sure to explain!

4. Do the following optimization problems from the attached worksheet labeled '#25 Optimization Problems 4.4'.

(a) # 1

(b) # 2

(c) # 4

(d) # 7

5. Let $f(x;a) = x^3 - ax$.

(a) If $a > 0$, how many critical points does $f(x;a)$ have? Find the (x, y) coordinates of all local maximums and minimums. Your answer will depend on a.

(b) How does increasing the value of a affect the position of the critical points? Choose 3 values of a and sketch graphs $y = f(x;a)$ to illustrate.

(c) Repeat part (a) if $a < 0$.

(d) How does decreasing the value of a affect the position of the critical points? Sketch 3 graphs.

(e) For what value of a does $f(x;a)$ have a local minimum at $x = 5$?

6. Do the following related rates problems from the attached worksheet labeled '#28 Related Rates 4.6'.

(a) #1

(b) #3

(c) #4

(d) #6

Math 124 Name _______________

#25 Optimization Problems 4.4

1. A square sheet of tin 2 meters on a side is to be used to make an open-top box by cutting a small square of tin from each corner and bending up the sides. How large a square should be cut from each corner in order that the box have as large a volume as possible?

(a) What is to be optimized?

(b) Make sketches. What varies? How are they related? Label your sketches clearly by assigning variables to quantities which vary.

(c) Obtain a formula for the function to the optimized in terms of the variables that you identified in part (b). Choose one variable and write the others in terms of this one. What is the domain of this variable?
NOTE: MAX/MIN PROBLEMS CAN BE SOLVED BY USING IMPLICIT DIFFERENTIATION WITH MORE THAN ONE VARIABLE.

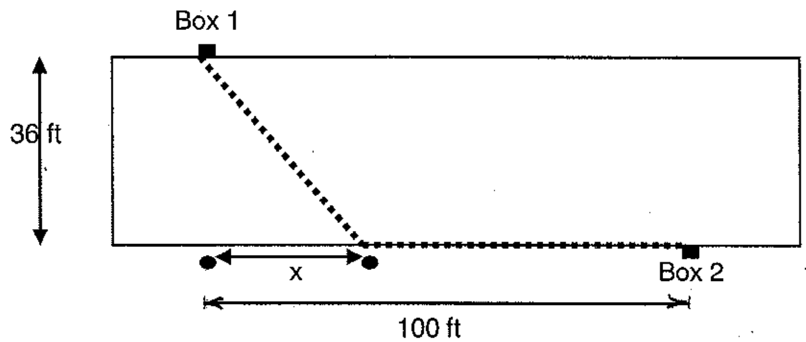
(d) Find the critical points.

(e) Use either the 1st derivative test or the 2nd derivative test to establish whether each critical point is a local max, a local min or neither.

(f) Evaluate the function at these points and at the endpoints of the domain to find the global maxima or minima.

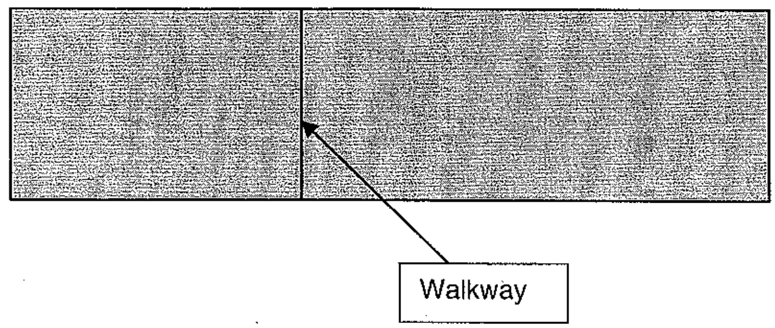
(g) Check to be sure that you have answered the question asked for in the problem

2. A telephone installation crew must run a line underground between two junction boxes. Unfortunately there is a 36 ft wide paved road between the two boxes, and one box is 100 feet down that lane from the other. It costs \$30 per foot to cut and repair the paved road, but only \$24 per foot to dig and refill along the side of the road. The crew will cut and repair the road to a point x feet from the point directly across from the first junction box, and then dig along the road the rest of the way. They are told that they should aim for a point $x = 48$ ft from the point directly across from the junction box. Show that $x = 48$ is the point where the cost of the line will be the least possible.

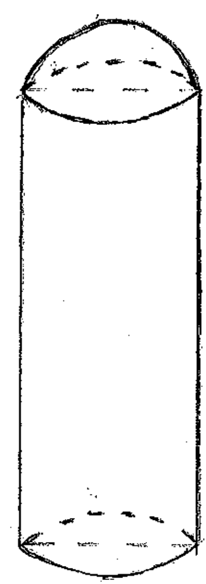


THE 100 FT LINE SHOULD GO FROM THE BEGINNING OF THE "X" SEGMENT TO BOX 2.

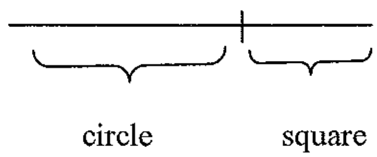
3. An architect designs a rectangular garden. The garden is to be surrounded on all sides by a hedge; a trellised walkway runs between one pair of opposite sides. The hedge costs \$98 per linear foot of length, the walkway costs \$127 per linear foot. The garden must have a total area of 1000 ft^2. What dimensions should the garden have to minimize the cost?



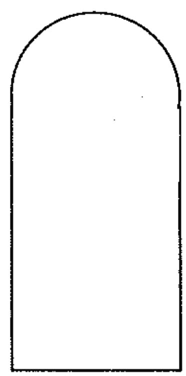
4. A silo is to be made in the form of a cylinder surmounted by a hemisphere. The cost of construction per square foot of surface area is twice as great for the hemisphere as for the cylinder. Determine the dimensions to be used if the volume is fixed and the cost of construction is to be a minimum. Neglect the thickness of the silo and waste in construction. The floor of the silo is not included in this construction.



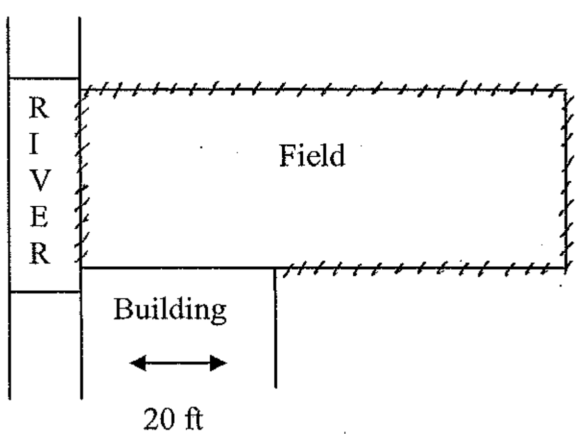
5. A wire of length 12 inches can be bent into a circle, a square, or cut to make both a circle and a square. How much wire should be used for the circle if the total area enclosed by the figure(s) is to be a minimum? A maximum?



6. A window consisting of a rectangle topped by a semicircle is to have a perimeter P. Find the radius of the semicircle if the area of the window is to be a maximum.



7. A rectangular field as shown is to be bounded by a fence. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fencing.



Bridges
Math 122B

Name ___________________

#28 Related Rates 4.6

SHOW ALL WORK FOR EACH OF THESE PROBLEMS. GIVE UNITS IN SIMPLEST FORM.

1. Consider (meditate on or reflect upon) a nice red spherical balloon that is in the process of being blown up (as in *expanded*). Of-course, as the volume increases, then so does the radius; each of these are functions of time.

(a) When the radius of a balloon is 10 cm, how is the volume of the balloon changing with respect to the change in its radius?

(b) If the radius of the balloon is increasing at 0.5 cm/sec, then at what rate is air being blown into the balloon at the instance that the radius is 6 cm?

(c) When the volume of the balloon is 50 cu. cm., how is the radius of the balloon changing with respect to volume?

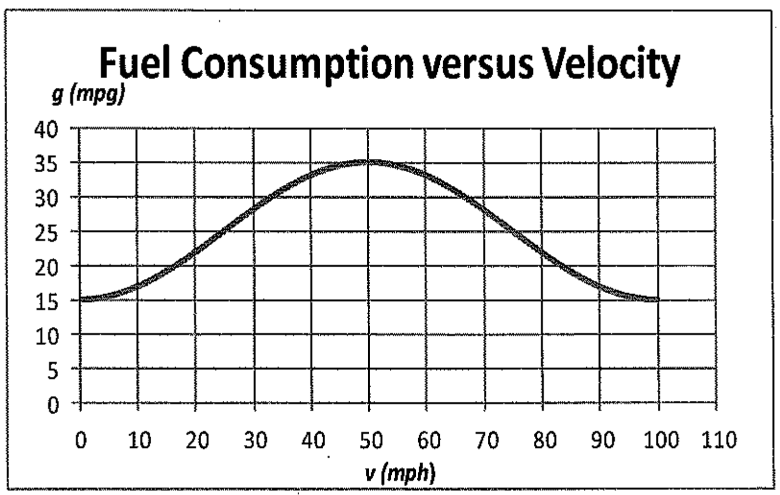
2. The volume of a tree is given by $V = \frac{1}{12\pi}C^2h$. C is the circumference of the tree in meters at ground level and h is the height of the tree in meters. Both C and h are functions of time t in years.

(a) Find a formula for $\frac{dV}{dt}$. What does it represent?

(b) Suppose the circumference grows at a rate of 0.2 meters/year and the height grows at a rate of 4 meters/year. How fast is the volume of the tree growing when the circumference is 5 meters and the height is 22 meters.

3. An airplane, flying at 450 km/hr at a constant altitude of 5 km, is approaching a camera mounted on the ground. Let A be the angle of elevation above the ground at which the camera is pointed. When $A = \pi/3$, how fast does the camera have to rotate in order to keep the plane in view?
ROUND ANSWER TO 1 DECIMAL PLACE.

4. The figure below shows the fuel consumption, g, in miles per gallon, of a car traveling at v mph. At one moment, the car was going 70 mph and its deceleration was 8000 mph per hour. How fast was the fuel consumption changing at that moment? Give answer to the nearest hundred.



5. A spherical snowball melts in such a way that the instant at which its radius is 20 cm, its radius is decreasing at 3 cm/sec. At what rate is the volume of the ball of snow changing at that instant?
GIVE EXACT ANSWER.

6. A 3 meter ladder stands against a high wall. The foot of the ladder moves outward at a speed of 0.1 m/sec when the foot is 1 meter from the wall.

(a)(i) At that moment, how fast is the top of the ladder falling?

(ii) If the foot of the ladder had been 2 meters from the wall and the speed at which it moves outward remains at 0.1 m/sec, how fast is the top of the ladder falling?
ROUND ANSWER TO 3 DECIMAL PLACES.

(b) If the foot of the ladder moves out at a constant speed, how does the speed at which the top falls change as the foot gets father out?