

Math 122B-008 Worksheet 7

Please write out your solutions on a separate piece of paper. You do not have to print out and staple this page to your assignment.

0. Respond to the anonymous feedback form link in your email or use the url below:

<https://www.surveymonkey.com/s/328VDLK>

Because it's anonymous, this won't be graded, but please do it - spring break is a great time for me to make adjustments if necessary!

1. Compute the following derivatives. Simplify your answer as much as possible.

(a)  $f'(t)$  for  $f(t) = \frac{t}{\sqrt{t^3+1}}$

(b)  $\frac{dz}{dx}$  for  $z = (x+1)^3(5-x)^4$

(c)  $f'(\theta)$  for  $f(\theta) = \frac{1}{2\sec(2\theta)}$

(d)  $R'(\alpha)$  for  $R(\alpha) = \frac{\lambda^\alpha e^\alpha}{1-\alpha}$

(e)  $\frac{d}{dz}[\ln(\ln(2z^3))]$

(f)  $h'(y)$  for  $h(y) = \frac{\ln y}{1-\ln y}$

(g)  $f'(x)$  for  $f(x) = \sinh(x^2 + 1)$

(h)  $f'(t)$  for  $f(t) = \sin^{-1}\left(\frac{2}{t}\right)$

(i)  $a'(t)$  for  $a(t) = \ln\left(\left(\frac{1-\cos t}{1+\cos t}\right)^4\right)$

(j)  $\frac{d}{dx}[\operatorname{arcsec}(x)]$ . Your answer will be an *algebraic* function, i.e. it will not involve trigonometric or inverse trigonometric functions.

(k)  $y'$  for  $y = x^x$ . Big hint: take the natural logarithm of both sides of  $y = x^x$  and use implicit differentiation.

2. The following table gives samples of a differentiable function and its derivative.

$x$	$f(x)$	$f'(x)$
-1	2	0
0	5	-1.5
5	-1	-3

(a) Use the table to evaluate  $(f^{-1})'(5)$ .

(b) Does  $(f^{-1})'(2)$  exist?

3. Consider the implicit function given by

$$x^{2/3} + y^{2/3} = 8$$

- (a) Draw a sketch of the plane curve defined by this implicit function. Hint: Solve for  $y$ , but remember that you need a  $\pm$  when you take the 3/2-root.
- (b) Find all points  $(x, y)$  where the slope of the tangent line to the curve is  $-1$ .

4. A cable or chain strung between two points hangs in a hyperbolic shape called a *catenary* which has equation

$$y = \frac{T}{w} \cosh\left(\frac{wx}{T}\right)$$

where  $T$  is the tension and  $w$  is the weight. Show that the following equation is satisfied:

$$\frac{d^2y}{dx^2} = \frac{w}{T} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

the expression on the right side is related to the (arc) length of the chain. Hint: compute both sides of the equation and simplify until they look identical.