Math 122B **Quiz** **April 18, 2014** **Name:** KEY

1. Consider the following limit

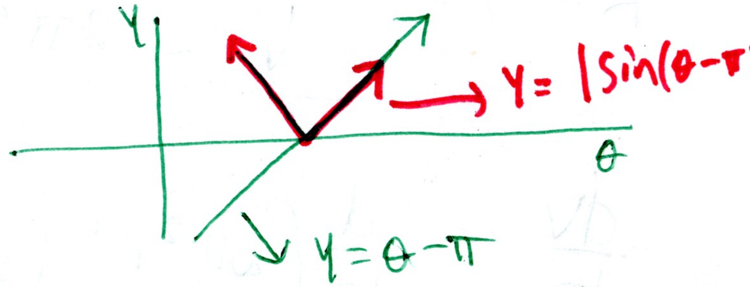
$$\lim_{\theta \to \pi} \frac{\theta - \pi}{|\sin(\theta - \pi)|}$$

Does l'Hôpital's rule apply? If so, compute the limit using l'Hôpital's rule. If not, why does it not apply?

No, it does not apply. The bottom function is not differentiable @ $\theta = \pi$!

Geometry: "near" $\theta = \pi$,

$$\lim_{\theta \to \pi^-} \neq \lim_{\theta \to \pi^+}$$



2. Consider the following limit

$$\lim_{x \to 0^+} x \ln(x)$$

Does l'Hôpital's rule apply? If so, compute the limit. If not, try to modify the limit so l'Hôpital's rule applies. Hint: $x = (1/x)^{-1}$.

l'Hôpital's Rule doesn't apply @ first since $\lim_{x \to 0^+} x\ln(x)$ "=" $0 \cdot \infty$. Want $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$

However, $x\ln(x) = \frac{\ln(x)}{1/x}$. Let $f(x) = \ln(x)$, $g(x) = 1/x$

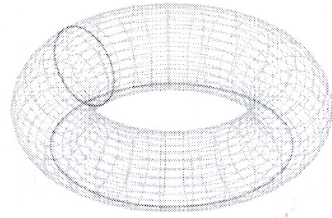
$f(x)$ & $g(x)$ are differentiable, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} g(x) =$

~~So l'Hôpital applies~~ And

$$\lim_{x \to 0^+} \frac{f'(x)}{g'(x)} = \lim_{x \to 0^+} \frac{1/x}{-1/x^2} = \lim_{x \to 0^+} \frac{1}{x} \cdot \left(-\frac{x^2}{1}\right) = \lim_{x \to 0^+} -x = 0$$

So l'Hôpital applies to give $\lim x\ln(x) = 0$

3. A *torus* is the mathematical name for a doughnut/bagel shape:



The volume of a torus with major radius R and minor radius r is $V = 2\pi^2 r^2 R$. Suppose I have a torus with major radius 10cm. If the volume increases at 1cm^3 per minute, how fast is the minor radius r changing when $V = 20\pi^2\text{cm}^3$? Leave your answer *exact*.

$R = 10$, so

$$V = 2\pi^2 r^2 \cdot 10 = 20\pi^2 r^2 \quad \bigstar$$

$$\frac{dV}{dt} = \frac{d}{dt}\left[20\pi^2 r^2\right] \Rightarrow \boxed{\frac{dV}{dt} = 20\pi^2 \cdot 2r\,\frac{dr}{dt}} \quad \diamondsuit$$

$\frac{dV}{dt} = 1\,\text{cm}^3/\text{min}$, & when $V = 20\pi^2$ we have

$$20\pi^2 = 20\pi^2 r^2 \qquad (\text{from } \bigstar)$$

$$\Rightarrow \quad \underline{r = 1}$$

Now, $\diamondsuit$ gives $1 = 20\pi^2\, 2 \cdot 1 \cdot \frac{dr}{dt}$

$$\Rightarrow \boxed{\frac{dr}{dt} = \frac{1}{40\pi^2}}$$