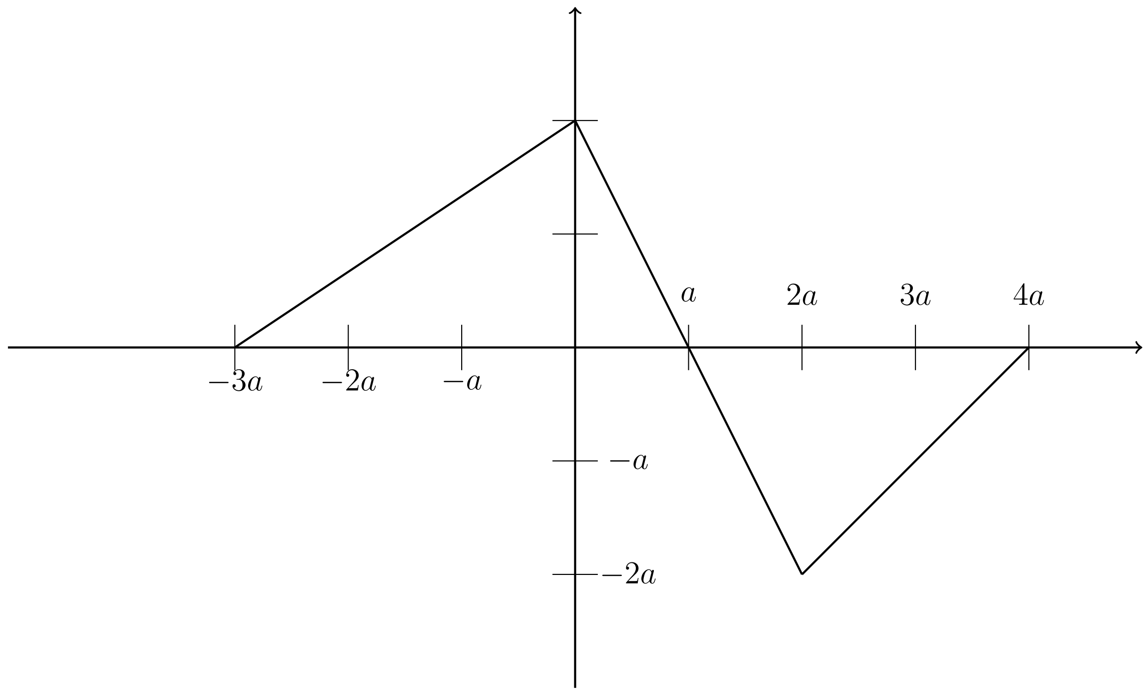
Math 122B

Exam IV PRACTICE

The University of Arizona

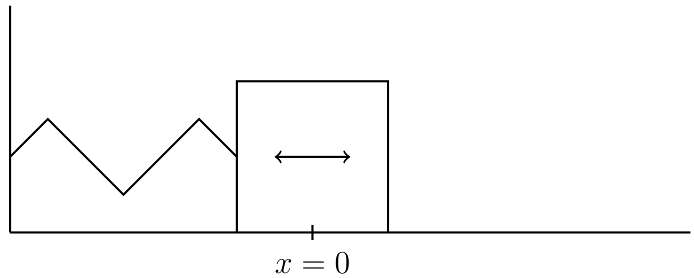
Name: ______________________________

1. (a) For the function f graphed below, compute $\int_{-3a}^{4a} f(t)dt$



(b) Let $g(t)$ be a function such that $g'(t) = f(t)$, and suppose that $g(a) = -1$. Compute $g(4a)$.

2. A spring-mass system is oscillating back and forth on a table (see picture). The velocity of the pendulum is given by the function $v(t) = \cos(t)$, where t is in seconds and positive velocity means the spring is traveling to the *right*.



(a) Let $x(t)$ be the position of the mass at time t, and suppose $x(0) = 1$. Find $x(\pi/4)$. Hint: F.T.O.C.

(b) Write down *but do not evaluate* an expression for the total **distance** D traveled by the mass on the interval $0 \leq t \leq 10$. Hint: total distance traveled is always a *positive* quantity (as opposed to displacement, which can be positive or negative).

3. Let $g(t) = 1 - t^2$ for $t \in [0, 2]$.

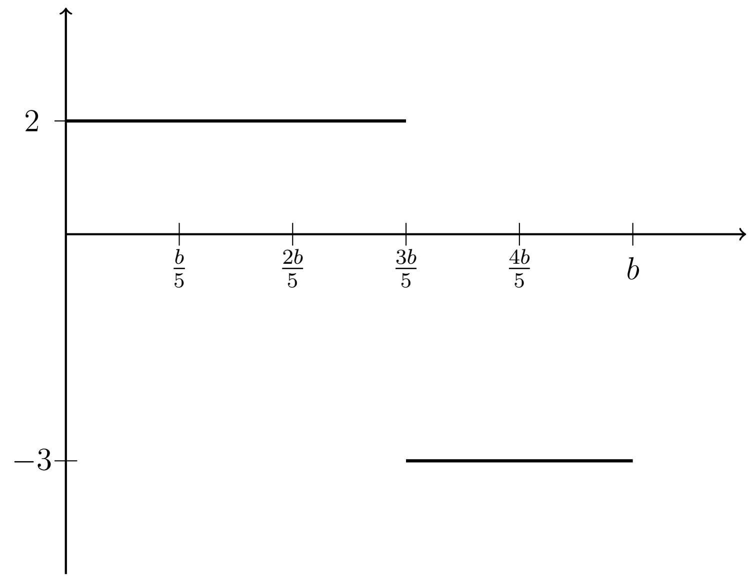
   (a) Suppose we wish to estimate $\int_0^2 g(t)dt$ using a 4-term Riemann sum. What are the t-coordinates we will use to evaluate $g(t)$ if we use a *left* sum? List them as t_0, t_1, t_2 and t_3.

   (b) What are the t-coordinates we will use to evaluate $g(t)$ if we use a *right* Riemann sum? List them as t_1, t_2, t_3 and t_4.

   (c) *Without computing the sums*, which one (left or right) will be an underestimate of $\int_0^2 g(t)dt$ and which will be an over-estimate? Why?

   (d) Suppose we want the difference between the upper and lower sum to be less than 1/10. How many terms in our sums will we need? (Hint: find Δt first).

4. The graph shown below is the velocity of an object moving along the x-axis between $t = 0$ and $t = b$. Positive velocity indicates the object is moving to the right.



(a) At $t = b$, is the object to the left of, to the right of, or at its starting position? Explain.

(b) At $t = 4b/5$, how far is the object from its starting position?

5. Define the *error* function $\text{Erf}(x)$ as follows:

$$\text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

Compute $\frac{d}{dx}\left[\text{Erf}(x^2)\right]$

6. Find the general solution of the differential equation

$$\frac{dy}{dx} = \sqrt{x+1} - \frac{2}{x}$$

7. Suppose $\int_0^2 g(x)dx = -A$, $\int_0^2 f(x)dx = 2$, $\int_2^4 f(x) = -1$, and $\int_2^6 f(x)dx = 3$. Find the following:

   (a) $\displaystyle\int_4^0 f(x)dx$

   (b) $\displaystyle\int_4^6 f(x)dx$

   (c) $\displaystyle\int_0^2 3f(x) - 2g(x)dx$

8. Compute a 4-term left hand Riemann sum for the integral

$$\int_{-\pi}^{\pi} \sin(x - \pi/4)dx$$

Note: This last question is just for fun - I won't put something like this on the exam because it's a bit too advanced!

9. In class, we discussed differential equations of the sort

$$\frac{dF}{dx} = f(x), \quad f(0) = C$$

We discovered that the general solution to this equation was

$$F(x) = \int_0^x f(x)dx + C$$

(a) What do you think is the general solution to differential equation below?

$$\frac{dI}{dx} = -\mu I, \quad I(0) = I_0$$

The numbers μ and I_0 are *constants*, and $I = I(x)$ is a function of x. Hint: think of a simple function whose derivative is a constant multiple of itself. What should we do with the constant I_0? Hint: we don't *add* it - try several things and check the two conditions $I' = -\mu I$ and $I(0) = I_0$.

(b) When an X-ray travels through an object, its energy is *attenuated*. Denser objects attenuate X-rays more, which is why we can use them to image the inside of the human body (bones are very dense). The *Beer-Lambert law* describes the relationship between an X-ray's intensity I and the object's *attenuation* μ. This law is exactly the differential equation given above:

$$\frac{dI}{dx} = -\mu I, \quad I(0) = I_0$$

The units of x are centimeters. Suppose the attenuation coefficient of bone is $\mu = 5$, and the initial X-ray intensity at the source is $I_0 = 100$ (don't worry about the units of μ and I_0). What is the X-ray intensity after passing through 2cm of bone? Hint: find I as a function of x using your answer from part (a), and evaluate at $x = 2$.