Math 122B

Exam II PRACTICE

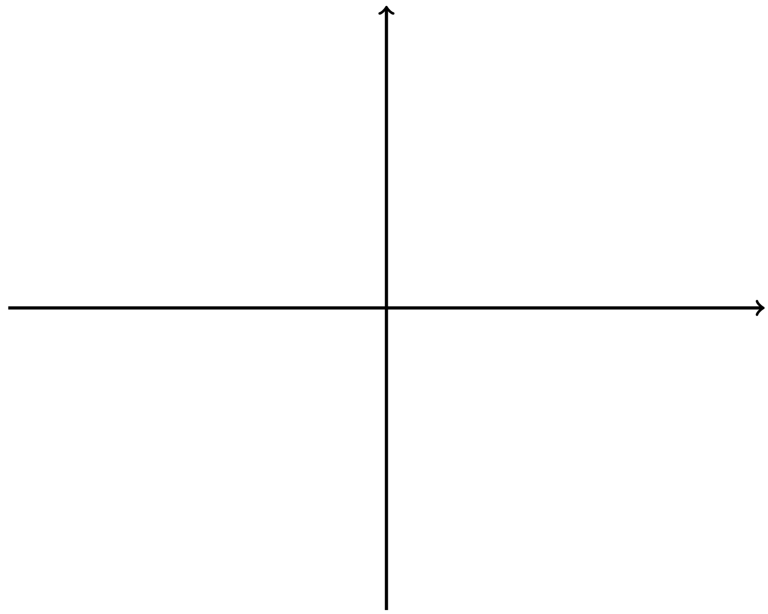
March 28th 2014

The University of Arizona

Name: ____

1. Consider the implicit curve defined by $(ax)^3 + y^2 = 1$, where $a > 0$.

   (a) Plot the curve when $a = 2$. Hint: solve for y to obtain *two* explicit functions describing the top and bottom half. You may use your calculator, but be sure to label your graph.



   (b) Find $\frac{dy}{dx}$.

(c) Where is $\frac{dy}{dx}$ undefined? Use the graph from part (a) to explain *why* this derivative is not defined.

(d) Find the equation of the tangent line to the curve when $y = 2$.

2. Compute the following derivatives and simplify your answer.

(a) $\dfrac{d}{dz}\left[z \log_2(b + 1/z)\right]$

(b) $\frac{d}{da}\left[\frac{R}{Ya + \sinh(1+a^2)}\right]$

(c) $A'(\theta)$ if $A(\theta) = \arcsin(\theta^{2/3})$.

(d) $(y^{-1})'(t)$ if $y(t) = t^{1/3}$

(e) y' if $y = x^{x^2}$. Hint: take a logarithm of both sides of $y = x^{x^2}$.

3. Use the table below to evaluate the following:

x	0	1	2	3	4	5
$f(x)$	-1	2	1	5	-2	2.5
$f'(x)$	1.5	4.1	-4.2	7.5	3	-1

(a) $(f^{-1})'(2)$

(b) $Q'(3)$ if $Q(x) = \ln((f(x))^2)$.

4. Let $f(x) = \exp(-\cosh(x))$ (note: $\exp(y) = e^y$).

   (a) Find the local linearization (aka tangent line approximation) for $f(x)$ at $x = 1$.

(b) Is the local linearization an *over* estimate or *under* estimate near $x = 1$? Explain using calculus.

5. Answer the following true/false questions; no explanation is required.

(a) $\frac{d}{dx}[f(x)g(h(x))] = f'(x)g(h(x)) + f(x)g'(h(x))$

TRUE FALSE

(b) Suppose $f(x)$ is differentiable on $(0, 1)$. Then, there exists a c with $0 < c < 1$ such that $f'(c) = f(1) - f(0)$

TRUE FALSE

(c) If $x = y^{2/3}$, then $\frac{dy}{dx} = \frac{3}{2}y^{1/3}$.

TRUE FALSE

6. Suppose we want to approximate $f(x) = e^x$ with a *cubic* function near $x = 0$, i.e.

$$e^x \approx a + bx + cx^2 + dx^3$$

We would like this cubic function to 'agree' with $f(x)$ in the following ways:

1. It should have the same *value* as $f(x)$ at $x = 0$
2. It should have the same *derivative* as $f(x)$ at $x = 0$.
3. It should have the same *second* derivative as $f(x)$ at $x = 0$
4. It should have the same *third* derivative as $f(x)$ at $x = 0$.

Find the values of a, b, c and d so that these four conditions are satisfied.