

Math 122B
Exam I PRACTICE
February 2014
The University of Arizona

Name: _____

Answers without adequate justification will not receive full credit, including multiple choice. Include units with your answer when appropriate, and box all answers unless an answer line is provided. By signing below I am agreeing to abide by the University of Arizona academic integrity policies and that all work done on this test is my own.

Signature: _____

Tips for Success:

- Look through the entire test before starting to prioritize questions.
- If you get stuck on a question, move on and come back to it later.
- Do a quick reality check after each question: does my answer make sense? Did I include units? Did I show all my work?
- Read over the entire test at the end to make sure you didn't miss anything.
- For each question: take a deep breath, think slowly and deliberately at first, then work quickly once you see what to do.

Requests:

- Show all your steps.
- Please box answers when possible.

1. Compute the derivative of the following functions using differentiation rules:

(a) $f(x) = x^{\pi/2} + 3\sqrt{x} - \frac{2}{x^{1/3}}$

(b) $g(y) = ay^3 + ya^3$

2. Let $g(x)$ be defined as follows:

$$g(x) = \begin{cases} x^2 & x \leq 3 \\ mx + b & x > 3 \end{cases}$$

Find values of m and b so that $g(x)$ is continuous and differentiable everywhere.

3. Let $f(x) = \frac{a}{\sqrt{bx}}$, where a and b are nonzero constant real numbers. *Using the limit definition*, compute $f'(2)$. Leave answer exact, involving a and b .

4. One of the consequences of Einstein's theory of *general relativity* is that the mass m of an object depends on its velocity v according to the following function:

$$m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \text{where } c \text{ is the speed of light}$$

- (a) Evaluate and interpret $m(0)$ and $m(c/2)$ in the context of the problem.

- (b) Evaluate and interpret $\lim_{v \rightarrow c} m(v)$ in the context of the problem.

For the rest of the problem, suppose $m_0 = 10$

- (c) *Approximate* $m'(c/2)$ using your calculator. Start by writing down the difference quotient:

$$\Delta_m(h) = \frac{m(c/2 + h) - m(c/2)}{h} =$$

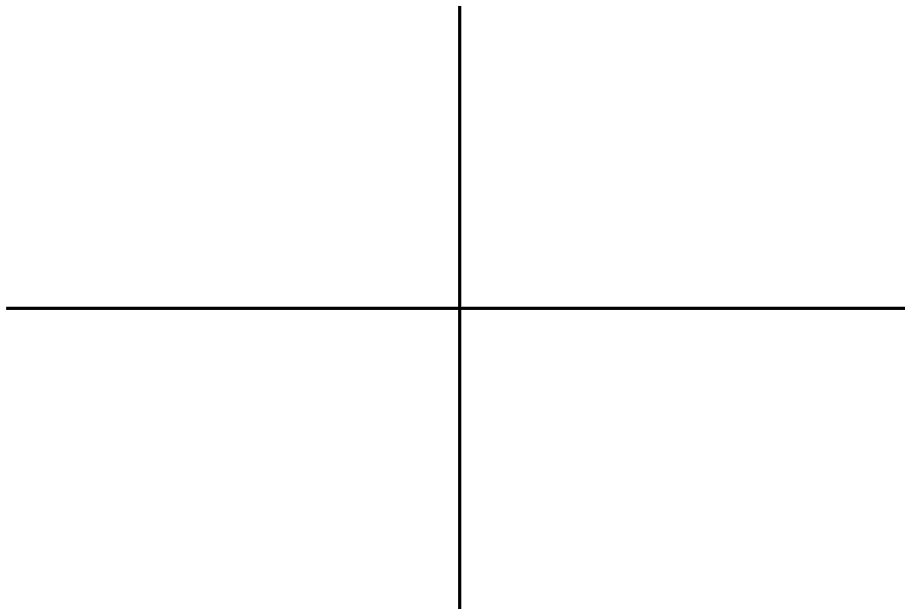
Then, fill in the table below with 4 approximations (round to 3 decimal places):

| | | | | |
|---------------|--|--|--|--|
| h | | | | |
| $\Delta_P(h)$ | | | | |

5. Consider the following function:

$$f(x) = \begin{cases} -x^2 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$

(a) Draw a sketch of $y = f(x)$ for $-2 \leq x \leq 2$ on the axes below:



(b) Show that $f(x)$ is differentiable at 0 by using the limit definition of $f'(0)$.

(c) What is $f'(x)$? (Hint: like f' is a piecewise function).

(d) Does $f''(0)$ exist?

6. For the following questions, give a *brief* explanation and/or sketch to justify for your answer.

(a) If a function is continuous at $x = a$, it is differentiable at a .

TRUE FALSE

(b) If a function is *not* differentiable at $x = a$, then x is *not* continuous at $x = a$.

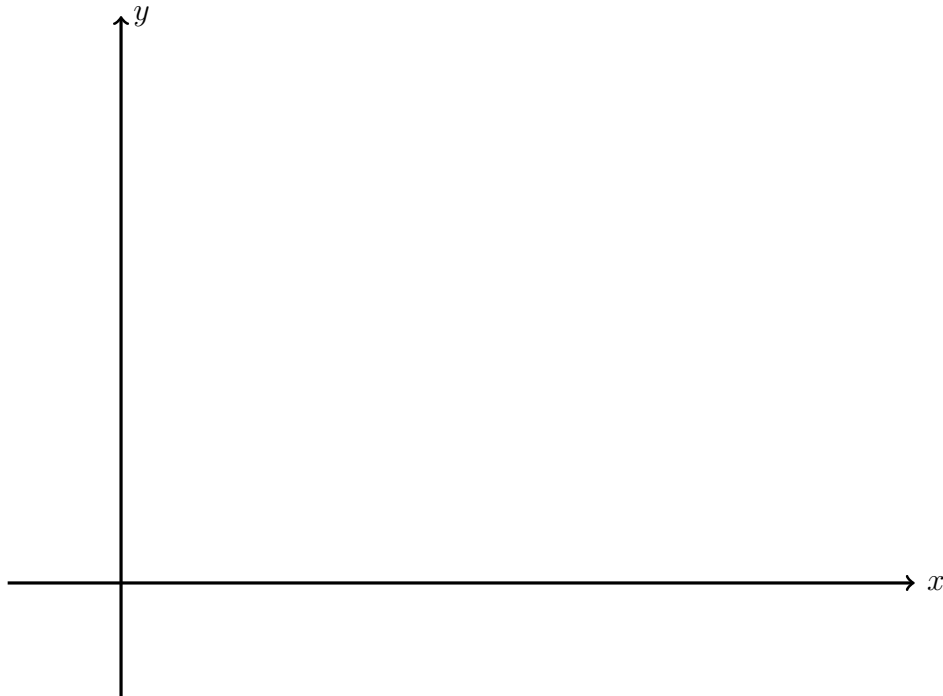
TRUE FALSE

(c) If $f''(a) = 0$, then $f(x)$ changes concavity at $x = a$.

TRUE FALSE

7. On the axes below, draw the graph of a function $y = f(x)$ with the following properties:

- $f(0) = f(6) = 1$
- $f'(0) = f'(5) = 0$
- $f'(x) < 0$ on $(0, 3)$
- $f'(3) = 0$
- $f'' < 0$ on $(0, 2)$ and $(4, 6)$



8. Using the graph of $y = f''(x)$ shown below, sketch a possible graph of $y = f(x)$ on the same axes. You may assume that $f(0) = 0$.

