Math 122B: Differentiation Rules

$$\frac{d}{dx}\left[x^a\right] = ax^{a-1}, \quad a \in \mathbb{R}$$

$$\frac{d}{dx}\left[b^x\right] = \ln(b)b^x, \quad b > 0$$

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + g'(x)f(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

$$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$$

$$\frac{d}{dx}\left[\cos(x)\right] = -\sin(x)$$

$$.\frac{d}{dx}\left[\tan(x)\right] = \sec^2(x)$$

$$\frac{d}{dx}\left[\cot(x)\right] = -\csc^2(x)$$

$$\frac{d}{dx}\left[\sec x\right] = \sec x \tan x$$

$$\frac{d}{dx}\left[\csc(x)\right] = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\arctan(x)\right] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\sinh(x)\right] = \cosh(x)$$

$$\frac{d}{dx}\left[\cosh(x)\right] = \sinh(x)$$

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$$

$$\frac{d}{dx}\left[f(y)\right] = f'(y)\frac{dy}{dx}$$

(This last rule is just the chain rule, but written in a form which is more useful for implicit differentiation. For example $\frac{d}{dx}[y^2] = 2y\frac{dy}{dx}$) .

Derivative Theorems

The Mean Value Theorem If $f(x)$ is continuous on $a \leq x \leq b$ and differentiable on $a < x < b$, then there exists a number c with $a < c < b$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

In words: there is an $x-$value in the interval (a, b) so that the derivative of the function at that location is equal to the slope of the line between $(a, f(a))$ and $(b, f(b))$.